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Games without Rules

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Abstract

We introduce the notion of an outcome space, in which strategic interactions are embedded. This allows us to investigate the idea that one strategic interaction might be an expanded version of another interaction. We then characterize the Nash equilibria arising in such extensions and demonstrate a folk-type theorem stating that any individually rational element of the outcome space is a Nash equilibrium.
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1 Introduction

The idea of representing economic outcomes as Nash equilibria of games has revolutionized economics, notably the study of industrial organization and particularly the theory of oligopoly. Game-theoretic approaches have displaced, and largely discredited, the older structure-conduct-performance approach to industrial organization and models of oligopoly.

However, a number of developments have cast doubt on the capacity of game theoretic approaches to provide clear-cut solutions to industrial organization problems.[1] Perhaps the most important has been the 'Folk Theorem' showing that any individually rational outcome may emerge as a Nash equilibrium for an infinitely repeated game (Aumman and Shapley, 1976). Attempts to resolve this multiplicity problem through refinements of the Nash equilibrium concept have generated some interesting insights, but no generally accepted solution.

A similar problem has emerged in the study of one-shot oligopoly games. Klemperer and Meyer (1989) showed that, if the strategy space in a standard one-shot oligopoly game is assumed to consist of all possible supply schedules, any outcome in which no firm earns negative profits is a Nash equilibrium. More precisely, Klemperer and Meyer showed that any set of outputs for oligopoly firms, along with the associated market clearing price, is a Nash equilibrium if each firm commits to a supply schedule ensuring that any other price would generate lower profits for its rivals. The essential requirement is a commitment to respond aggressively to any attempt by a rival firm to increase its sales. Klemperer and Meyer have also proposed an equilibrium concept that restores uniqueness under certain conditions. This proposal will be discussed below.

There is some similarity between the methods used to prove the Folk Theorem and the argument of Klemperer and Meyer showing that any non-negative profit outcome is an equilibrium in supply schedules. The basic argument behind the Folk Theorem is that the players can ensure a particular cooperative outcome by committing themselves to a 'Grim' strategy of permanent noncooperation in the event of any deviation.

Another example that illustrates the inability of game theory to provide clear-cut solutions comes from the lobbying or contests literature. It is commonly assumed in this literature

[1] The game-theoretic approach has also been challenged by empirical studies of industrial organization. In a seminal book, Sutton (1991) argues that the "new generation of game-theoretic models appears to place only limited constraints on the data." His point is that the theory establishes that any outcome is possible, and indeed this is what happens.

that agents, who are disputing some rent or prize, have only one instrument to influence the outcome's choice by the decision maker. For example, a manufacturers' association could offer money to a bureaucrat to levy a tariff on competing imports (Grossman and Helpman, 1994) or offer contributions to political parties (Hillman and Ursprung, 1988).[2] A typical result of this literature is that in a Tullock's rent-seeking model (Tullock, 1980), where the probability of a player winning the game depends on expenditures to some power $R > 2$, full dissipation of rents occurs in equilibrium when the strategy space is continuous. (See, for example, Baye, Kovenock and De Vries, 1994). However, these results depend on the assumption that players are confined to strategies consisting of fixed monetary payments. Quite different results are obtained if players can adopt strategies in which the supply of lobbying funds varies as the total amount committed by other players changes. There are many such strategies that do not necessarily involve the capacity to observe and respond to the strategies of other players. For example, suppose that players adopt strategies based on the purchase of some fixed amount of advertising in relevant media or on offering retainers to expert witnesses. As other players increase their expenditure the price of such scarce goods will rise, and expenditure will rise also.

More generally, several authors have pointed out that there are many other instruments available to players in a lobbying or rent-seeking game. In the example of the manufacturers' association above, Epstein and Hefeker (2002) argue that "they could place advertisements in newspapers, try to influence experts, join forces with environmental groups who see imports as a threat to environmental conditions, or support groups that are worried about the negative effects of globalization." These authors then obtain different results in terms of rent dissipation by examining the case where agents might have a second instrument to influence decision makers. Similarly, Haan and Schoonbeek (2003) examine a rent-seeking contest where players spend both money and effort trying to influence decision makers and again obtain different results. In the same vein, Konrad (2000) considers the interaction of standard effort and sabotage (effort that reduces particular rivals' performance) and obtains additional results on rent dissipation.

Underlying these common themes is a more fundamental point. Many of the economic problems that have been analyzed using a game-theoretic framework are 'games without rules'. Unlike, say, poker or chess, or the formalized interactions of an auction, there is no rulebook that specifies the strategies an oligopolist can adopt or the strategies that interest groups disputing a rent might follow. Hence, any proposed game-theoretic representation is, in essence, a 'conjectural variations' model, in which the conjecture is that other players will

[2] For a survey of the lobbying or rent-seeking contests, see Nitzan (1994).

choose to hold constant some particular variable, described as their 'strategy'.[3]

If strategies are not normally specified in problems of economic interest, how can strategic interactions be modelled? We argue that, in economic analysis, outcomes, and not strategies, are the natural primitives. Economists can normally observe market prices and aggregate quantities with a high degree of accuracy. It is often possible to obtain fairly good information on the output levels, costs and profits of individual firms. With somewhat greater difficulty, it may be possible to use a combination of observed data on costs and 'engineering' data concerning technology to make inferences about the production possibility sets faced by firms and to use repeated observations to derive estimates of market demand. By contrast, information about the strategic choices available to firms and other market participants is rarely observable directly. Hence, the same observed outcome may be consistent with quite different representations of the strategic interactions between firms.

The objective of this paper is to develop this point in a general setting, using the concept of an outcome space, in which strategic interactions are embedded. Using this approach, it is possible to give a natural definition of the concept that one strategic interaction is an expanded version of another, and to characterize the potential Nash equilibria arising in extensions of a given strategic interaction. The crucial result is that, in the absence of structural information about the 'rules of the game', any individually rational element of the outcome space is a Nash equilibrium. The relationship of this result to the Folk Theorem for repeated games and to the work of Klemperer and Meyer is discussed.

The restriction of the result to individually rational outcomes is important, since it indicates that, even in the absence of any information about the rules of the game (interpreted as beliefs held by players about the strategies available to others), it is not true that 'anything can happen'. Individual rationality is sufficient to ensure that no outcome is sustainable if it yields a lower return to some player than that player could obtain unilaterally, for example by withdrawing from the interaction altogether. Information on the outcomes players can obtain unilaterally is commonly available from direct inspection of the relevant market and the associated set of feasible outcomes, without consideration of strategic issues. More generally, closer analysis of the outcome space may yield restrictions on the set of feasible strategies that do not depend on arbitrary assumptions drawn, for example, from the choice of descriptive variables used by modellers.

[3] In the preface of his new book, McAfee (2002) lists twenty six different strategic variables that might be chosen by firms.

2 Background

In most formal representations of game theory, problems are represented in terms of a payoff matrix, representing a mapping from a Cartesian product of strategy spaces to a Cartesian product of utility spaces. Since the 'rules of the game' are assumed to be known in advance, the specification of outcomes, arising from the interaction of the strategic choices of the players, and giving rise to their payoffs expressed in terms of profit or utility, is logically superfluous.

The typical game-theoretic representation of an economic problem represents the outcome as a function $f(x_1, x_2...x_n)$ where x_i is a sufficient statistic for the decisions taken by individual i, given the values of x_{-i}. It follows then that the possible values of x_i are assumed to represent the strategies available to player i, and the analysis focuses on Nash equilibria of the resulting game.

In the language of conjectural variations, this procedure amounts to the claim that individual i believes that other individuals $\{-i\}$ will leave their own choices of x_{-i} unchanged regardless of the choice of x_{-i}. At a minimum, this assumption requires an assumption analogous to that of rational expectations in macroeconomics, namely that all individuals in the real world believe the same model as that proposed by the analyst. But the Nash equilibrium assumption is considerably stronger than a requirement for model-consistent expectations. Individuals must not only share the analyst's model, but must also suppose that every other individual j is passive in their choice of the sufficient statistic x_j, which is, in many cases, chosen arbitrarily by the modeller. As will be argued below, in the absence of specific information about the rules of the game, this approach is completely uninformative, in the sense that any individually rational outcome can be derived as a Nash equilibrium.

3 Setup

3.1 Outcomes

We denote the space of possible outcomes as Y, with elements y. We assume that for each participant i we can define a (known) preference ordering over Y and an associated utility function $u_i : Y \rightarrow \Re$.[4]

The triple $(Y, I, \mathbf{u})$ consisting of the space Y, the set of agents I and the vector of utility functions u_i is said to be a *problem representation.* In this paper, we will focus on the case $I = \{1, 2\}$, except where otherwise specified.

[4] Here the utility function is ordinal. If uncertainty is present, the space of outcomes is taken to consist of probability distributions over some space of consequences. For the present, there will be no need to consider the additional structure that will arise in this case.

Example 1: Consider an exchange economy and let the set of outcomes Y consist of allocations of consumption $\mathbf{C} = (\mathbf{c}_1, \mathbf{c}_2)$ consistent with the original endowments, in the sense that aggregate consumption of every good is less than or equal to the aggregate endowment. Agent i's preferences are represented by a utility function $u_i, i = 1, 2$. The problem representation for this exchange economy is given by the triple $(\mathbf{C}, I, \mathbf{u})$, where $I = \{1, 2\}$ and $\mathbf{u} = (u_1, u_2)$.

Example 2: Consider the allocation of an object that is worth V to one of two players, 1 and 2. The player who gets the object has to make a payment to the seller who values the object at zero. This player who makes the highest offer wins the object and pays the equivalent to her offer. The problem representation is as follows:

$$Y = \{(x_1, x_2, p_1, p_2)\}$$

$$I = \{1, 2\}$$

$$\pi_i = (V - p_i)x_i, i = 1, 2$$

where x_i denotes the probability that player i is allocated the object and p_i the offer made by player i with $x_1 + x_2 \leq 1$, $p_1 \geq 0$ and $p_2 \geq 0$. Note that if offers are identical then the object is allocated with equal probability to each one of the bidders.

The game described in Example 2 above is a first-price auction. A variant of this auction format that is commonly used in the lobbying or rent-seeking literature, the all-pay auction, is described in the next example.

Example 3: Consider a rent-seeking contest where two agents, 1 and 2, are disputing a prize worth V. These players spend resources (bribing officials, advertising their cause, trying to influence experts, and so on) to win the competition. The player who spends the most resources wins the prize. The problem representation is as follows:

$$Y = \{(x_1, x_2, p_1, p_2)\}$$

$$I = \{1, 2\}$$

$$\pi_i = Vx_i - p_i, i = 1, 2$$

where x_i denotes the probability that player i wins the prize and p_i the amount of resources spent by player i, with $x_1 + x_2 \leq 1$, $p_1 \geq 0$ and $p_2 \geq 0$.. Note again that if both players spend the same amount of resources then the prize is allocated with equal probability to each one of the bidders.

The notion of an outcome space here is defined by the data available to an outside observer. Different observers may organize their data differently, leading to descriptions of the problem that are superficially different. Two problem representations $(Y, I, \mathbf{u})$ and $(Y^*, I^*, \mathbf{u}^*)$ are equivalent if there exist 1-1 mappings $\phi : Y \leftrightarrow Y^*$ and $\gamma : I \leftrightarrow I^*$ such that for all $y, y' \in Y$, $u_i(y) \geq u_i(y')$ if and only if $u^*_{\gamma(i)}(\phi(y)) \geq u^*_{\gamma(i)}(\phi(y'))$. Equivalently, there exist monotone increasing mappings $\phi_i : \Re \rightarrow \Re$ such that Diagram 1 commutes. A trivial example arises when ϕ is the identity mapping on Y and the ϕ_i are any monotonic real-valued functions.

Example 4: Suppose there are two duopolists producing a single good. We denote the output of participant i by z_i and assume that each participant has a strictly convex cost function $c_i : \Re \rightarrow \Re$ and that the market price is determined as $p = D^{-1}(Z)$ where $Z = z_1 + z_2$ and D^{-1} is a well-behaved inverse demand function. Finally we assume that each participant is concerned only about her own profit and prefers more profit to less. A natural choice of problem representation, is therefore

$$\begin{aligned} Y &= \{(z_1, z_2, p) : p = D^{-1}(Z), Z = z_1 + z_2\} \\ I &= \{1, 2\} \\ u_i(y) &= \pi_i \\ &= pz_i - c_i(z_i) \quad i = 1, 2. \end{aligned}$$

However, this is by no means the only possible representation. Suppose we define the market share for participant i

$$s_i = \frac{z_i}{Z} \tag{1}$$

and the unit markup over average cost as

$$\theta_i = \frac{pz_i - c_i(z_i)}{c_i(z_i)}. \tag{2}$$

Then we can define the problem representation $Y^* = \{(s_1, \theta_1, s_2, \theta_2, Z)\}$ where

$$\begin{aligned} s_1 + s_2 &= 1 \\ \frac{(1+\theta_1)c_1(s_1 Z)}{s_1 Z} &= \frac{(1+\theta_2)c_2(s_2 Z)}{s_2 Z} \\ Z &= D((1+\theta_1)c_1(s_1 Z)/s_1 Z)\} \end{aligned}$$

$$\begin{aligned} I &= \{1, 2\} \\ u^*_i &= \pi_i = \theta_i c_i(s_i Z) \quad i = 1, 2. \end{aligned}$$

There is a 1-1 mapping between Y and $Y*$ given by (1) and (2), and the identity mapping on $\Re$ makes Diagram 1 commute.

To an economist, the problem representation (Y, I, u) seems natural and obvious, while (Y^*, I^*, u^*) may appear complicated and obscure. However, market analysts frequently use the representation Y^* or some variant on it (a common variant is to report the markup over average variable costs).

One reason for preferring the representation Y^* is that various shocks may be expected either to leave the outcome y^* largely or wholly unchanged. For example a fully anticipated monetary shock would be expected to have no impact on the outcome in the representation (Y^*, I^*, u^*) , but would imply a price increase in the representation (Y, I, u). More generally, the market shares and markups over average cost for major companies in many industries have remained quite stable over long periods, whereas prices and quantities have changed over time.

As this example shows, a range of equivalent representations may be available for any given problem. The choice between representations is a matter of analytical convenience. It does not, in itself, imply anything about the beliefs of participants or about equilibrium outcomes.

3.2 Actions and choices

Strategic behavior is defined using the dual concepts of actions and choice sets. An action a_i taken by an agent i defines a subset $Y(a_i) \subseteq Y$ of feasible outcomes. Conversely, the choice set for agent i, conditional on the action vector $\mathbf{a}_{-i}$ for the other agents is given by

$$C_i(\mathbf{a}_{-i}) = \cap_{j \neq i} Y(a_j).$$

In particular, in the case where there are two agents, i and j, $C_j(a_i) = Y(a_i)$. That is, the action taken by agent i defines the choice set available to agent j and vice versa. As discussed below, the term 'choice set' reflects the fact that an equilibrium outcome must be an undominated member of the choice set C_j, the set of all available choices for player j given the actions of other players. Thus, *in equilibrium,* each agent may consider themselves as choosing the best outcome available to them, conditional on the actions of the other agents.

Both objective and subjective interpretations of the choice set are available. In the objective interpretation, the choice set available $Y(a_i)$ is objectively determined, independent of the beliefs of agent j . For example, in the oligopoly case, if firm i closes down, firm j can produce any output and receive the price given by the demand curve.

In the subjective interpretation, the choice set is a description of the beliefs of agent i about the behavior of the other agents $-i$. Thus, in a duopoly interaction, if firm i imputes Cournot behavior to firm j, the observation of an output q_j implies a belief that the choice set available to i consists of all market clearing outcomes in which firm j produces q_j.[5] Note

[5] Under this subjective interpretation it is possible to model the dynamics of the process of belief formation

that this subjective interpretation is exactly the opposite of that found in some presentations of game theory. In equilibrium, it does not matter how agents conceive of their own strategies. What matters is their beliefs about the choice set presented to them by other agents. In the Cournot duopoly example, the best element of the choice set $C_i(q_j)$ is the same whether firm i conceives of themselves as selecting an optimal output, or price, or markup. What matters is that the choice set for i is believed (by i) to consist of outcomes where firm j produces q_j.

The set of actions available to individual i is described as *complete* if for any $y \in Y$, there exists an action $a \in A_i$ with $y \in Y(a)$. That is, completeness implies that there are no outcomes that are inconsistent with any action that individual i can undertake. In many contexts, such as that of Example 1, it is possible to interpret $Y(a_i)$ as a statement made by agent i regarding acceptable outcomes.

A vector of actions $(a_1, ..., a_I)$, one for each player, is *consistent* if the associated outcomes have a nonempty intersection and *exact* if this intersection has a single member. Thus, for a constistent vector of actions we have

$$\bigcap_{i \in I} Y(a_i) \neq \emptyset$$

and consistency implies that $C_i(\mathbf{a}_{-i}) \neq \emptyset$.

Clearly, a specification that allows inconsistent actions is unsatisfactory. A specification that is not exact leaves an element of indeterminacy. Hence, attention will be focused on problems representations in which all action vectors are consistent and exact.

The concepts of completeness, consistency and exactness may be illustrated for the examples given above.

Example 1 (continued): Consider the case of two agents, 1 and 2, with agent i having endowment $\mathbf{e}_i$. An action for agent i may be interpreted as a statement of the trades that player i is prepared to accept, and may therefore be represented by a set $Y(a_i)$ of consumption bundles for agent i with the remainder of the aggregate endowment being consumed by individual $j \neq i$. Alternatively, and more consistent with a game-theoretic interpretation, an action for agent i may be interpreted as determining the choice set $Y(a_i)$ for agent j.

Assuming agents cannot be forced to trade, any choice set for agent i must have the consumption bundle $\mathbf{e}_i$ as an element. Hence, if $Y(a_i)$ and $Y(a_j)$ are consumptio bundles for i, j respectively, their intersection must contain the element $(\mathbf{e}_i, \mathbf{e}_j)$ and hence must be non-empty. It follows that any set of actions will be consistent.

explicitly. See, for example, Camerer, Ho and Chong (2001). This avoids the usual criticism of the conjectural variations approach: “The problem is that it involves a kind of pseudo-dynamics pasted on top of inherently static models.” (Varian, 1992, p.303).

There are many different possible specifications of the set of actions available to an agent. The following are of particular interest:

(a) The set of proposals consisting of all pairs of the form $(\mathbf{c}, \mathbf{e})$. Thus each action may be interpreted as offering the other agent the choice of a proposed allocation c and the endowment or disagreement point e.
(b) The set of all subsets of C of the form $\{\mathbf{c} : \mathbf{p} \bullet \mathbf{c} \leq \mathbf{p} \bullet \mathbf{e}\}$ for some $\mathbf{p}$. Each action may be seen as announcing a set of prices at which the agent is willing to trade.
(c) The set of all subsets of C containing the endowment point $\mathbf{e}$.

In the absence of further restrictions on the offers that can be made, the set of actions (a) and (c) are complete. The set of actions (b) is not complete, since trade cannot bring the agent to an allocation $\mathbf{c} > \mathbf{e}$. However, if the outcome space is restricted to the Edgeworth–Bowley box, (b) is complete.

Example 2 (continued): In the common value auction example with complete information, if $A_i = \Re_+$ (as it is commonly assumed) for $i = 1, 2$, then the set of actions is complete.

Example 4 (continued): In the duopoly case, the set of actions is *not* complete if firms are restricted to specify a strictly positive quantity and the output space is $[0, \infty)$.

3.3 Strategic interactions

A strategic representation Σ consists of a problem representation $(Y, I, \mathbf{u})$ and a set of actions A_i for each agent. Observe that, for any two equivalent problem representations $(Y, I, \mathbf{u})$ and $(Y^*, I^*, \mathbf{u}^*)$, any strategic representation Σ for $(Y, I, \mathbf{u})$ gives rise, in a natural fashion, to a strategic representation Σ^* for $(Y^*, I^*, \mathbf{u}^*)$. Hence, two strategic representations are defined as *equivalent* when there exists an equivalence between the problem representations such that the mapping from Y to Y^* induces a 1-1 mapping from the actions of Σ to those of Σ^*.

Some applications of game theory proceed from a problem representation to a game-theoretic analysis on the implicit assumption that the strategies available to the participants may be read directly from the problem representation. Thus, given that the analyst has adopted the representation Y for the duopoly problem above, Cournot behavior may be assumed, and attention may be focused on the associated equilibrium outcome and its comparative statics. On the other hand, if the analyst adopted the representation Y^*, it might be assumed that the Nash equilibrium in markups (Grant and Quiggin, 1994) was the outcome of interest. The fact that equivalent problem representations give rise to an equivalence between strategic representations indicates that this approach is unwarranted. *No inferences about behavior can be drawn from an essentially arbitrary choice of representation of the problem.*

If all action vectors are consistent, there is a natural mapping from the Cartesian product space $\prod A_i$ to the outcome space Y. Under fairly weak conditions this mapping will be a surjection onto the outcome space. However, the mapping will not, in general, be 1-1. That is, different action vectors may have the same intersection. This point has been obscured by the fact that, when a game-theoretic analysis is derived directly from a problem representation in the manner described above, a mapping between Y and $\prod A_i$ arises automatically.

Example 4 (continued): Given the representation of the outcome space Y for the oligopoly problem in terms of a vector of outputs, one for each firm, and a market-clearing price, there exists a 1-1 correspondence between Y and the Cartesian product space $\prod A_i$ where for each i, $A_i = \{z_i : z_i \geq 0\}$ (the Cournot strategic representation). There is a similar 1-1 correspondence arising between the set Y^* and the Cartesian product space $\prod A_i^*$ where for each i, $A_i^* = \{\theta_i : \theta_i \in \Re\}$ (the markup representation). Since, Y and Y^* are isomorphic, there are also 1-1 correspondences between $\prod A_i$ and Y^* and between $\prod A_i^*$ and Y. Suppose, however, that firms can adopt either a fixed output or a fixed markup. There is an obvious mapping from $\prod (A_i \cup A_i^*)$ onto Y (or Y^*), but it is not 1-1.

The distinction between the outcome space, which is given by the problem representation prior to the consideration of any strategic issues, and the Cartesian product space $\prod A_i$ is crucial to the argument developed in this paper.

3.4 Extensions

In the example in the previous section, we considered the oligopoly situation in which firms might choose either fixed quantities or fixed markups. We now develop this idea more generally. For a given strategic representation Σ, with problem representation $(Y, I, \mathbf{u})$, an *extension* of Σ is defined as a strategic representation Σ' with problem representation $(Y, I, \mathbf{u})$ such that for each agent i, $A_i \subseteq A_i'$.

Example 2 (continued): A strategic representation of the common value auction in which bidders are allowed to make contingent offers is an extension of strategic representations in which bidders are restricted to bid a nonnegative number.

Example 3 (continued): A strategic representation of the lobbying game in which players are allowed to choose two instruments (for example, physical effort and money) is an extension of the strategic representations in which agents are restricted to bid a nonnegative number.

Example 4 (continued): A strategic representation of the oligopoly problem in which firms' actions are described by arbitrary supply schedules is an extension of strategic representations in which firms' actions specify their quantity, price or markup

An extension is an *augmentation* if, for each $a' \in A_i'$, there exists $a \in A_i$ such that $Y(a) \subseteq Y(a')$. Thus, all new actions represent an expansion of the choice set for some existing action.

Example 4 (continued): A strategic representation of the oligopoly problem in which firms specify a nonnegative quantity is an augmentation of strategic representations in which firms' actions are described by strictly positive quantities.

The concept of an extension illustrates the crucial role of the outcome space in the approach proposed here, and the fundamental difference between this approach and that usually adopted in game theory. Beginning with a standard game-theoretic representation, it is obviously possible to add new strategies for one or more players. The crucial distinction is that the addition of new strategies in a game-theoretic representation automatically implies the addition of new rows and columns in the Cartesian product $\prod A_i$. By contrast, in the setting analyzed here, the outcome space is given, prior to any consideration of possible actions. Hence, the shift from a strategic representation to an extension of that representation does not change the outcome space.

4 Equilibrium concepts

Concepts of equilibrium are most simply defined using the notation of choice sets. An equilibrium consists of a family of choice sets C_i and a selected element $y^* \in \cap C_i$. For the case of independent strategic representations, including games, attention is normally focused on the Nash definition of equilibrium which requires that:

(i) for each i, $y^* \in \arg\max\{u_i(y) : y \in C_i\}, i = 1, 2$.

Where cooperative action is possible, attention is normally focused on the core definition of equilibrium:

(ii) there exists no $y^{**} \in C_i$ with $u_i(y^{**}) \geq u_i(y^*)$ for all i and strict inequality for at least one i.

We define the set K of equilibria of type (ii) and E of equilibria of type (i), that is, $y \in Y$ satisfying (i) and (ii) respectively for a given strategic representation. Observe that E □ K. For the case of two agents $E = K$.

For any strategic representation and any agent i the maximin utility level is defined as

$$m^i = \max_{a \,\in\, A_i} \{\min\{u_i(y) : y \in Y(a)\}\}.$$

The minimax is defined as

$$m_i = \min_{a \,\in\, A_{-i}} \{\max\{u_i(y) : y \in C(a)\}\}.$$

That is, the maximin is the highest utility level the player can guarantee by a unilateral action. The minimax is the lowest utility level the agent can be forced to accept. The following lemma provides a utility ranking of these two equilibria concepts from the viewpoint of player i.

Lemma 1: For any consistent strategic representation, $u_i(m^i) \leq u_i(m_i)$.

Proof: Define

$$a^i = \arg \max_{a \,\in\, A_i} \{\min\{u(y) : y \in Y(a)\}\}$$

$$a_i = \arg \min_{a \,\in\, A_{-i}} \{\max\{u(y) : y \in C(a)\}\}.$$

Choose $y \in C(a^i) \cap Y(a_i)$, and observe $u_i(m^i) \leq u_i(y) \leq u_i(m_i)$.$\square$

The next lemma establishes that the two equilibrium sets of an extension of a strategic representation extend the original equilibrium sets in a natural way. The proof is omitted.

Lemma 2: If Σ is a strategic representation with equilibrium sets K, E, and Σ' is an extension of Σ with equilibrium sets K', E', then $E' \,\square\, E, K' \,\square\, K$.

We are now ready to prove our main result.

Proposition 1: If Σ is a consistent strategic representation with two agents i, j, then for any $y^* \in Y$, $u_i(y^*) \geq m_i$, $u_j(y^*) \geq m_j$ there exists an extension Σ' such that $y^* \in K'$.

Proof: (If) Define y_i, y_j such that $u_j(y_i) = m_j$, $u_i(y_j) = m_i$ and consider an extension Σ' of Σ such that the agents have available the actions

$$a_j = \{y^*\} \cup \{y_j\}$$

$$a_i = \{y^*\} \cup \{y_i\}.$$

Then the family of actions $\{a_i, a_j\}$ and the outcome y^* satisfy the definition of an equilibrium of type (ii).

□

The basic logic of this result is simple. The given outcome y^* can be 'enforced' as an equilibrium by a mechanism that ensures that any agent i who deviates from y^* will be forced to their minimum utility m_i by the coordinated actions of the other agents. There is an obvious kinship with the Folk Theorem for repeated games, which will be explored below.

Example 1 (continued): For the case of an exchange economy, Proposition 1 describes the subset of the Edgeworth-Bowley box consisting of individually rational outcomes. For any allocation $\mathbf{c}$ yielding gains from trade to both parties (that is, for which $u_i(c_i) \geq u_i(m_i) \quad \forall i$), and any Σ containing the actions $a_i = \{\mathbf{c}, \mathbf{e}\} \forall i$, it is evident that $\mathbf{c} \in K$. Since any Σ can be augmented to contain $\{\mathbf{c}, \mathbf{e}\}$, the augmentation Σ' has $\mathbf{c} \in K$. This represents an equilibrium in which both parties agree on the allocation $\mathbf{c}$, but defection by either individual leads to the breakdown of the agreement and a return to the pre-trade point.

Example 2 (continued): In the common value auction example, Proposition 1 guarantees that any outcome that yields at least zero profits to both bidders can be implemented by an appropriate extension.

Example 3 (continued): In the lobbying game, the above proposition indicates that any individually rational outcome can be attained in equilibrium.

Example 4 (continued): In an oligopoly situation, any cartel agreement can be represented using Proposition 1. It seems reasonable to suppose that each firm is free to adopt as an action any supply schedule that yields non-negative profit for every price p. Hence, any cartel member can adopt the 'competitive' strategy of maximizing output at every price subject to the profitability constraint. In the simple case where all firms have U-shaped cost curves with the same minimum average cost, the adoption of this strategy by any firm ensures that the market-clearing price is equal to average cost, and therefore that all firms make zero profit. Denote the resulting equilibrium by C. Hence, any market-clearing outcome $\left(\mathbf{z}, D^{-1}(Z)\right)$ with non-negative profit for all firms can be reached if each firm adopts the action $(\mathbf{z}, C)$. That is, any firm's deviation from its agreed output z_i is punished by the adoption of competitive behavior by all other members of the cartel.

Precisely the same analysis is possible for the representation in terms of market shares.

However, in this case, there is a salient choice for the 'natural' equilibrium. The equilibrium C is associated with a vector of market shares, Θ, an aggregate output Z^c and a market-clearing price $D^{-1}(Z^c)$. The natural cartel equilibrium combines the same vector of market shares, Θ, with the monopoly price and aggregate output level.

Example 1 (continued): Any given price vector $\mathbf{p}$ defines a budget set $C_i(\mathbf{p})$ for each individual. Define the choice set for individual i as

$$C_i(\mathbf{p}) = \{c : c_i \in C_i(\mathbf{p})\}.$$

Then if an equilibrium exists for the family of choice sets $C_i(\mathbf{p})$ it is a competitive equilibrium in the usual sense.

4.1 Games

Any game gives rise to a problem representation. A strategic representation is a *non-cooperative game* if:
(i) the class of actions available to each agent i is complete; and
(ii) the intersection of any set of actions, one for each agent, has exactly one member.

Note that condition (ii) implies that any set of actions, one for each agent, is consistent.

The canonical problem representation for a non-cooperative game is:

$$Y = \prod A_i$$

with the u_i being defined in the obvious fashion by the payoff matrix. Alternative labellings of the strategies available in a given game give rise to equivalent problem representations. The converse is not true, however. Two different games may give rise to equivalent problem representations.

It is also possible for a game to be given a non-canonical problem representation. For example, given the standard problem representation, in terms of quantities for oligopoly, the actions available to the agents may be defined in terms of supply schedules that preserve a fixed markup, as in Grant and Quiggin (1994).

4.2 Equilibria for games

In this section, we present a series of variations on Proposition 1. The basic point is the same in each case. If we are given only the problem representation, any outcome consistent with individual rationality can be represented as the Nash equilibrium of a noncooperative game.

Proposition 2: If Σ is a two-player noncooperative game with players i, j then for any $y^* \in Y$ such that $u_i(y^*) \geq m_i, u_j(y^*) \geq m_j$ there exists a complete, consistent extension Σ' of Σ, such that $y^* \in E'$.

Proof: Take $y^* \in Y$ such that $u_i(y^*) \geq m_i, u_j(y^*) \geq m_j$. Let

$$C_j = a_i = \{y^*\} \cup \left(\cup_{a \in A_j} \arg\max\{u_j(y) : y \in Y(a)\}\right),$$

and $C_i = a_j$ similarly. This action consists of the desired equilibrium y^* along with the minimal outcomes available to the other player from unilateral actions under Σ. Hence, it is consistent. Completeness of Σ' follows from completeness of Σ. Finally, it is obvious that

$$y^* \in \arg\min\{u_i(y) : y \in C_i\}$$

and similarly for Cj. Hence $y^* \in E'$.□

The next example illustrates the power of Proposition 2.

Example 2 (continued): Consider the first-price common value auction with full information. It is well known that, if $A_i = \Re_+$, then this game admits only mixed strategy equilibria (and there are multiple mixed strategy equilibria).

Now take $y^* = (1/2, 1/2, 0, 0)$. Note that $\pi_i(y^*) = \frac{1}{2}V > m_i = 0$, $i = 1, 2$. Proposition 2 guarantees that by defining a set of actions A_i' containing A_i we can obtain y^* as an equilibrium of the extended game. This can be accomplished by allowing bidders to make enforceable side contracts. Indeed, auction designers are particularly concerned about ruling out the possibility of collusion, either by introducing design features that make collusion more difficult (for example, by introducing a second stage (see Klemperer 2002-a)) or by making sure that the penalties for collusion are severe and that effective monitoring occurs.

Example 3 (continued): Consider the lobbying game. Baye, Kovenock and de Vries (1994) show that this game only admits mixed strategy equilibrium when $A_i = \mathbb{R}_+$ (and again there are multiple mixed strategy Nash equilibria).

Now take $y^* = (1, 0, V, 0)$. Note that $\pi_i(y^*) = m_i = 0$, $i = 1, 2$. Proposition 2 assures us that by appropriately defining a set of actions A_i' containing A_i we can 'implement' y^* as a Nash equilibrium of the extended game. Indeed, this can be accomplished by extending the action set so that players can make contingent bids. Then one can check that y^* can be supported by player 1 making the following contingent offer: "I will bid

V if player 2 bids zero and if player 2 bids $0 < b_2 < V$, I will bid $b_2 + \varepsilon$", and player 2 making the following contingent bid: "I bid 0 if 1 bids V and if player 1 bids $b_1 < V$, I bid $b_1 + \varepsilon$."

Note that Proposition 2 does not guarantee exactness. Under the definitions adopted here, it is possible that player j has available two actions a_1, a_2 such that $y_1 \in Y(a_2)$, where:

$$y_1 = \arg\min\{u_j(y) : y \in Y(a_1)\},$$

$$y_2 = \arg\min\{u_j(y) : y \in Y(a_2)\}.$$

Note that, in this case,

$$u(y) \geq m_j \geq u_j(y_1) \ \geq u_j(y_2).$$

Hence, any divergence from exactness will be unimportant. Exactness of Σ' may be obtained in Proposition 2 by imposing the following additional condition on Σ :

The empty intersection condition: Any two distinct actions for a given player have empty intersection.

Observe, however, that even if Σ satisfies this condition, the extension Σ' will not necessarily satisfy it.

We can modify Proposition 2 to require that the extension be an augmentation, at the cost of requiring that the equilibrium outcome yield each player at least maximin rather than minimax.

Proposition 2a: If Σ is a two-player noncooperative game with players i, j, then for any $y^* \in Y$ such that $u_i(y) \geq m_i, u_j(y) \geq m_j$ there exists a complete, consistent augmentation Σ' of Σ, such that $y^* \in E'$.

Proof: Let

$$a_i = \arg \min_{a \,\in\, A_{-i}} \{\max\{u_i(y) : y \in C_i(a)\}\},$$

$$Y(a_i') = Y(a_i) \cup \{y^*\}$$

and similarly for j. Then $y^* \in E'$.□

Example 4 (continued): In the oligopoly case, Proposition 3 shows that if Σ includes the option of closing down and earning zero profits, any equilibrium must yield non-negative profits for every firm. Proposition 2 shows that, in the duopoly case, any allocation satisfying this condition can be an equilibrium.

4.3 More than two agents

The empty intersection condition is more important in permitting relaxation of the restriction to two agents in Proposition 2.

Proposition 3: If Σ is a noncooperative game satisfying the empty intersection condition then for any $y^* \in Y$ such that $u_i(y^*) \geq m_i, \forall i$, there exists an extension Σ' of Σ, also a noncooperative game, such that $y^* \in E'$.

Proof: Define

$$a_i = \{y^*\} \cup (\cup_{j \neq i} (\cup_{a \in A_j} \arg\min\{u_j(y) : y \in a\}) \forall i.$$

Under the empty intersection condition, if all individuals choose a_i, then

$$C_i = \cup_{j \neq i} a_i = \{y^*\} \cup (\cup_{a \in Ai} \arg\min\{u_i(y) : y \in a\}),$$

so that

$$y^* \in \arg\max\{u_i(y) : y \in C_i\}.$$

Completeness of Σ' follows from completeness of Σ. Finally, it is obvious that $y \in \arg\max\{u_i(y) : y \in C_i\}$, and similarly for C_j.

Proposition 2 can be generalized to the class of aggregative games (Cornes, Hartley and Sandler 1999). A noncooperative game is *aggregative* if the payoff of every player can be expressed as a function of that player's own choice variable and the unweighted sum of every player's choice variable. In aggregative games, for any given participant, therefore, the opponents 'look like' a single player.

Proposition 4: If Σ is a n-player aggregative game, then for any $y^* \in Y$ such that $u_i(y^*) \geq m_i, i = 1, ..., n$, there exists a complete, consistent extension Σ' of Σ, such that $y^* \in E'$.

Proof: Take $y^* \in Y$ such that $u_i(y^*) \geq m_i$ for every i. Define for all i, $i = 1, ..., n$:

$$C_{I-\{i\}} = a_i = \{y^*\} \cup \left(\cup_{j \neq i} \cup_{a \in A_j} \arg\min\{u_j(y) : y \in Y(a)\}\right).$$

This action consists of the desired equilibrium y^* along with the minimal outcomes available to the other players from unilateral actions under Σ. Hence, it is consistent. Completeness of Σ' follows from completeness of Σ. Finally, given that Σ is aggregative, player i effectively only faces one (aggregated) opponent. Thus, we can label $I - \{i\} = j$, and the proof continues in the same fashion of that of Proposition 2. That is, we can assert that

$$y^* \in \arg\max\{u_i(y) : y \in C_i\}$$

and similarly for Cj. Hence $y^* \in E'$. Note that Σ' might not be aggregative. □

It should be clear by now that examining nonaggregative games with $n \geq 3$ will be a very difficult task. The main reason is that we would need to know more about the interaction amongst the actions available for the various possible coalitions. This would require imposing additional structure and it is left for future research.

5 Restriction to individually rational outcomes

Results 1-4 have shown that, under a range of conditions, any individually rational outcome in a representation Σ may be obtained as a Nash equilibrium of an extension Σ'. We may obtain the following partial converse.

Proposition 5: If Σ is a consistent strategic representation with equilibrium set E, and Σ' is a consistent extension of Σ with equilibrium set E', then, for all $y \in E'$ and all $i \in I, u_i(y) \geq m_i$.

Proof: For any i, consider the action $a \in \alpha_\iota$ defined by

$$a = \arg\min_{a \in A_i}\{\max\{u(y) : y \in a\}\}.$$

Since Σ' is an extension of Σ, $a \in A'_i$. Since Σ' is consistent, the equilibrium choice set C_i has nonempty intersection with a. Let $y_0 \in C_i \cap a$ and observe that

$$\begin{aligned} m_i &= min\{u(y) : y \in a\} \\ &\leq u(y_0) \\ &\leq u(y) \end{aligned}$$

where the last inequality follows from the definition of equilibrium.□

It might be thought that, in the absence of a fixed set of strategies, 'anything can happen'. Proposition 5 shows that this is not the case. Individual rationality is sufficient to ensure that no outcome is sustainable if it yields a lower return to some player than that player could obtain unilaterally, for example by

withdrawing from the interaction altogether. Information on the outcomes players can obtain unilaterally is commonly available from direct inspection of the relevant market and the associated set of feasible outcomes, without consideration of strategic issues. For the case of $n > 2$ agents, it seems likely that, under reasonable conditions, it will be possible to restrict the set of feasible outcomes to those in the core.

Example 5: If Σ is the prisoners dilemma, the minimax option m_i for both players is that obtained when both confess. Propositions 2 and 3 may be interpreted as saying that if both players have available the strategy of holding out, conditionally on the other's also holding out, the Pareto-superior outcome in which neither confesses is an equilibrium. Proposition 5 shows that here exists no extension of the game in which the outcome where only one player confesses is an equilibrium.

6 The Folk Theorem and Implementation

The central argument of this paper is similar to that underlying various versions of the Folk Theorem for repeated games. The Folk Theorem shows that, in the absence of discounting, any outcome that Pareto-dominates a Nash equilibrium of a one-shot game may be obtained as a Nash equilibrium of the same game indefinitely repeated without discounting. The proof relies on the idea that, if an individual deviates from the desired equilibrium, other players can 'punish' her by enforcing the less desirable Nash equilibrium.

Consider a strategic interaction with outcome space Y and the countably infinite Cartesian product $\bar{Y} = Y^{\mathcal{N}}$ where $\mathcal{N}$ denotes the natural numbers, with typical element $\mathbf{y} = (y_1, ...y_t, ...)$. In the absence of discounting, the utility function for each agent is given by :

$$\bar{u}_i(\mathbf{y}) = \lim \inf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} u_i(y_t).$$

We now consider the set of actions available to agents. Beginning with a set of actions A_i and associated subsets $Y(a_i) \subseteq Y$, it is natural to identify each a_i with the constant action $\bar{a}_i$ identified with

$$Y(a_i) \times Y(a_i) \times Y(a_i) ... \subseteq Y^{\infty}.$$

We will focus on the case when the interaction defined by $(A_1, ...A_i....A_I)$ is a non-cooperative game. In particular, we assume exactness, so that for any $\mathbf{a} = (a_1, ...a_i, ...A_I)$ there is a unique $y \in \cap_i Y(a_i)$. Let $\bar{\mathbf{y}}$ be the infinite sequence with all entries equal to y. For all i,

$$\bar{u}_i(\bar{\mathbf{y}}) = u_i(y).$$

Thus, any strategic representation $(Y, I, \mathbf{u}, A)$ can trivially be extended to a strategic representation $\left(\bar{Y}, \bar{I}, \bar{\mathbf{u}}, \bar{A}\right)$ in which the only actions are constant actions. Moreover, with the obvious notation, we have

$$m^i = \bar{m}^i$$

$$m_i = \bar{m}_i.$$

Hence, Propositions 2 to 4 apply, yielding the standard Folk Theorem result that any individually rational outcome may be obtained as the Nash equilibrium of an appropriate extension of $\left(\bar{Y}, \bar{I}, \bar{\mathbf{u}}, \bar{A}\right)$.

The literature on the Folk Theorem strengthens this result by imposing various restrictions on the permissible extensions of $\left(\bar{Y}, \bar{I}, \bar{\mathbf{u}}, \bar{A}\right)$. Typically, the only allowable actions associated with the triple $\left(\bar{Y}, \bar{I}, \bar{\mathbf{u}}\right)$ consist of actions of the form $(a_{i1}, ...a_{it}....)$, where $a_{it} \in A_i$ may be conditional on the basis of no information other than the past actions of others and, in the case of mixed strategies, the realization of some randomizing device. In the absence of discounting, restrictions of this kind do not have any impact.

The literature on refinements of equilibrium was motivated, in part, by the desire to find plausible restrictions on the set of permissible actions that would overturn the conclusion of the Folk Theorem, and therefore also of Propositions 2 to 4. However, no such restriction has found general acceptance. The central argument of this paper is that restrictions capable of overturning Propositions 2 to 4 might be sought in extrinsic 'rules of the game', rather than in the intrinsic characterization of equilibrium. [6]

It is also worth pointing out the relationship of Propositions 2 to 4 to the theory of implementation. As in this paper, implementation theory is concerned primarily with outcomes. The question it intends to address is whether we can achieve a certain outcome by finding messages (or actions) that will be chosen by individuals as part of an equilibrium. Maskin (1999) shows that any social choice correspondence that is monotonic and satisfies a no veto power condition can be implemented in Nash equilibrium. Monotonicity is satisfied in our setting as preferences are fixed. The 'no veto power' condition corresponds in our setting to the individual rationality condition.

Thus Results 2-4 can be interpreted in terms of implementability. The novelty here is that we achieve these results by extending an exogenously given set of actions rather than by specifying the set of actions available to individuals. By contrast, in the Maskin result, the entire set of actions is specified so as to yield a desired mapping from preferences to

[6] For an anti-folk theorem in a repeated surplus-splitting game involving a weakening of subgame consistency, see Abreu and Pearce (2000).

outcomes. The intersection between the two analyses is the trivial case of the present model when the action set is initially empty.

7 The Klemperer-Meyer analysis

Propositions 1 and 2 may usefully be interpreted by considering a basic strategic representation consisting of objectively available actions, and an extension defined in terms of subjectively interpreted choice sets. The analysis of oligopoly presented by Klemperer and Meyer (1989) fits naturally into this pattern. Regardless of any assumptions about the behavior of other firms, it is reasonable to assume that each firm i has the option of closing down and earning zero profits and confining the class of possible outcomes to those with $q_i = 0$. Hence we may confine attention to extensions of the 'basic' strategic representation Σ, in which each firm has available the action of closing down. On the other hand, the question of whether a firm can credibly commit itself to a given supply schedule is a question about the beliefs of other firms.

Klemperer and Meyer (1989) obtain our Proposition 2 with some modications arising from the additional structure imposed by their problem setting, that of oligopolists producing a homogenous good. In this setting, outcomes may be described by a set of outputs, one for each firm, with the associated price being determined by the market demand curve. (As has been argued already, this description is not unique). For this outcome space, Klemperer and Meyer show that the result of Proposition 2 holds in the case where strategies are required to be supply curves defined by one-dimensional manifolds. The Klemperer-Meyer result holds for arbitrary numbers of firms, reflecting the fact that this is an aggregative game.

Klemperer and Meyer go on to propose an alternative concept of equilibrium in supply schedules, based on the assumption of exogenous random shocks to industry demand. In this setup, the optimal supply schedule traces out the most preferred price-quantity pairs in the stochastic Nash equilibrium outcome.Klemperer and Meyer argue:

> In the absence of uncertainty, the motivation for modelling firms as competeting via supply functions is not compelling, Without uncertainty, a firm knows its residual demand with certainty, and it therefore has a single profit-maximising point, which it could achieve by choosing either a fixed price or a fixed quantity. It gains nothing from the ability to choose a more general supply function, so the use of these more general strategies would not be robust even to tiny costs of maintaining the greater flexibility they embody.

This argument is highly problematic. In particular, it appears to imply that firms should be indifferent between the options of specifying a fixed quantity (Cournot) and specifiying a fixed price (Bertrand), and should not be willing to pay even a tiny cost to switch from Bertrand to Cournot. This is not correct (see, for example, Kreps and Scheinkman 1983).

As has already noted, it is not the firm's own interpretation of its decision variable that matters but the interpretation conveyed to other players. If a firm convinces others that it is committed to a Cournot strategy, it will be faced with a more attractive residual demand curve in equilibrium than if it convinces others that it is committed to a Bertrand strategy.

This is not to say that the Klemperer-Meyer equilibrium concept should be rejected. As they note, it is a very plausible basis for equilibrium in circumstances where firms must empower an agent to react in a profit-maximising fashion to price changes without knowing whether these changes arise from shocks to industry demand or from the strategic actions of competitors. It is precisely this kind of detail that differentiates a game with exogenously determined rules from a general problem representation for an economic interaction with unknown rules. It is to the former class of problems that we now turn.

8 Games with rules

The analysis above has shown that no useful statements about equilibrium can be derived simply from a specification of the outcome space as a Cartesian product of summary statistics for the actions of individual agents. This issue did not arise in early applications of game theory, since analysis was applied to games such as chess and poker. In these games the rules specifying the permissible strategies are either written (as in chess and standard versions of poker), or agreed by custom (as in nonstandard versions of poker). Leaving aside the possibility of cheating, the epistemic status of the strategy space is not an issue for games of this kind.

Some economic interactions have similarly well-defined rules. For example, a sealed-bid second-price auction in which communication between bidders is prohibited has a very simple action (or strategy) space, in which each bidder's set of possible actions consists of the possible values for their bids.[7]

At the other extreme, consider one of the first fields in which attempts were made to apply game theory, that of military strategy. For the zero-sum case of 'total war', the maximin solution of von Neumann and Morgenstern (1944) is perfectly satisfactory and consistent with the results presented above. But if neither party aims at the annihilation of the other, they share a common interest in minimizing the casualties associated with any given territorial or political outcome of war. Hence, some elements of the outcome space Pareto-dominate others. Given an appropriate set of strategies, it would be natural to seek Nash equilibria.

[7] Of course, as with games like poker, individuals may choose to break the rules, incurring the risk of a penalty.

But although 'rules of war' may preclude the use of particular weapons or tactics, there is, in general, no way of prescribing a set of strategies for the parties that is sufficiently limited to impose significant restrictions on the set of possible equilibrium outcomes. Thus, game theory predictions based on the Nash equilibrium concept have limited practical military applications.[8]

In general, it is necessary to bring to bear extrinsic information about the 'rules of the game' if useful predictions about outcomes are to be obtained. Such information may be either institutional or behavioral. Institutional information may be related to knowledge about the political and economic environment (for example, some 'actions' might be ruled out by law or social norms). **In particular, if the institutional structure allows agents to achieve some outcomes unilaterally, for example by withdrawal, individual rationality provides bounds on the set of feasible outcomes, as shown in Proposition 5.**

Behavioral information may involve knowledge about the expectations or decision-making procedures adopted by participants. Perhaps the most useful contribution along these lines is that of Sutton (1997). Sutton argues that empirical evidence on the relationship between the size of the market and the 'toughness' of competition may be used to bound the range of feasible outcomes.

9 Concluding comments

The crucial observation of this paper is that the application of game-theoretic methods to model an economic interaction requires information about the actions that are feasible for participants. If the set of actions available to agents is complete, noncooperative and exact, the space of possible outcomes may be represented, in the usual fashion, as the Cartesian product of the individual action sets. In the absence of extrinsic information about the actions that are feasible for participants, it is not possible to work in the opposite direction. Contrary to the implicit assumption underlying many applications of game theory, the fact that the feasible outcome space may be represented as a Cartesian product of individual choice variables does *not* imply that those choice variables define a strategy space.

On a positive note, our approach highlights that economics and, in particular, game theory, can be used in a more effective way by incorporating into the problem all the data observed by the analyst. Interestingly, recent debate on the successes and failures of 3G telecom auctions around the world and, in particular, in Europe, has highlighted the need to

[8] Nevertheless, game theory remains a useful conceptual tool for thinking through military strategic problems, and is taught regularly around the world in military academies.

tailor the design of an auction to the specific country's context. [9] In this sense, our analysis suggests that the potential usefulness of a mechanism design approach where the economic environment is embedded in the mechanism rather having the analyst make decisions about the relevant variables that ignore such environment. Relatively little work has been done on the design of mechanisms for agents acting strategically in their dealings with each other, as well as with the principal. The analysis presented in this paper suggests that this is a promising area for exploration.

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[9] See, for example, Binmore and Klemperer (2002) and Klemperer (2002-a,b).

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