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Bargaining power and efficiency in principal--agent relationships

Robert G. Chambers
Professor and Adjunct Professor, respectively, University of Maryland and University of Western Australia

and

John Quiggin
Australian Research Council Federation Fellow, University of Queensland

Schools of Economics and Political Science
University of Queensland
Brisbane, 4072
rsmg@uq.edu.au
http://www.uq.edu.au/economics/rsmg

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John Quiggin[1] and Robert G. Chambers[2]
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[1]School of Economics, University of Queensland
[2]Department of Agricultural and Resource Economics, University of Maryland College Park

Abstract

Insurance contracts are frequently modelled as principal–agent relationships. Although it is commonly assumed that the principal, in this case the insurer, has complete freedom to design the contract, the problem formulation in much of the principal–agent literature presumes that the contract is constrained-Pareto-efficient. In the present paper, we consider the implications of a richer specification of the choices available to clients. In particular, we consider the entire spectrum of possible power differentials in the contracting relationship between insurers and clients. Our central result is that the agent can exploit information asymmetries to offset the bargaining power of the insurer, but that this process is socially costly.

1 Introduction

A wide variety of economic relationships have been modelled as contracts between a principal and an agent, made under conditions of imperfect and asymmetric information. Examples include contracts between employers and employees, landlords and sharecroppers, or regulators and firms. In most, though not all, cases, such contracts involve the provision of some form of risk-sharing between the principal and the agent. Hence, insurance may be regarded as the paradigmatic case of a principal–agent relationship.

Insurance contracts typically involve some element of bargaining, frequently under conditions of unequal bargaining power. Important concerns of bargaining theory are to formulate precise notions of bargaining power and to formalize the intuition that parties with greater bargaining power or lower levels of risk aversion will secure more favorable outcomes. Kihlstrom and Roth (1982) analyze bargaining over insurance in the case where clients facing known risks bargain with a monopolistic insurer. They show that the insurer will prefer to bargain with a more risk-averse client. This result was also derived independently by Schlesinger (1984). The same result is obtained, with a more sophisticated model of bargaining, by Viaene, Veugelers and Dedene (2002).

In all of these analyses, the loss to be insured is fixed, and the equilibrium bargain is Pareto-efficient. In many cases of interest, however, the client has the capacity to take action which will affect the occurrence, and magnitude of gains and losses. This arises most obviously in the case of agricultural insurance, where the loss to be insured is a loss of production due to climatic shocks or insect infestation, and clients may undertake such actions as the application of fertiliser and pesticides (Horowitz and Lichtenberg 1993, Miranda and Glauber 1997, Chambers and Quiggin 2002). However, the same issues arise whenever clients have the capacity to undertake self-protection (Ehrlich and Becker 1972, Lewis and Nickerson 1989, Quiggin 2002). When clients are engaged in production or self-protection, the insurance contract will affect their productive decisions, even in the absence of the kind of private information that produces moral hazard problems. Hence, the distribution of bargaining power may affect the efficiency of the insurance contract.

The purpose of this paper is to examine the interaction between differential bargaining power and the efficiency of insurance contracts. The analysis is undertaken in a framework of state-contingent production, which allows modelling of self-protection and self-insurance by the client. Our central result is that the client can exploit information asymmetries to offset the bargaining power of the insurer, but that this process is socially costly. Hence, where the client has private information, an increase in her bargaining power will, in general, enhance welfare.

In the case where the insurer has all the bargaining power, we show that the client engages in costly self-protection to enhance her subsequent bargaining position vis-a-vis the insurer. This results in a loss of efficiency relative to the case in which the services provided by the insurer are in competitive supply, subject to a zero-expected-profit constraint. Finally, for the general case of Nash bargaining, we show how the client can benefit from the existence of asymmetric information.

2 State-contingent production

We use upper-case letters to denote state-independent scalars such as the expected output Z and the insurer's expected profit P, lower-case letters to denote state-dependent scalars such as output z_s in state s and boldface to denote vectors such as the state-contingent output vector $\mathbf{z}$.

Production is undertaken by the client, who uses a vector of inputs $\mathbf{x} \in \Re^N$ to produce a vector of state-contingent outputs $\mathbf{z} \in \Re^{M \times S}$. Thus, the technology may be summarized by the family of feasible output sets $Z(\mathbf{x})$. The output z_s, observed if state of nature s is realized, is, in general, an element of $\Re^M$. To simplify notation, we will focus on the case $M = 1$, $S = 2$. The general properties of state-contingent production technologies are discussed by Chambers and Quiggin (2000).

2.1 The effort cost function

The client's *ex post* preferences are of the net returns form

$$w(y, \mathbf{x}) = u(y - g(\mathbf{x})),$$

where u is a differentiable, concave, strictly increasing von Neumann–Morgenstern utility function, y is the return to the client, and g is a strictly convex and increasing function. This objective function, referred to as *utility of net returns*, differs from that commonly used in the literature on principal–agent relationships, in which the objective function is additively separable in income and effort (Grossman and Hart 1983; Quiggin and Chambers 1998). The alternative formulations of the objective function are equivalent if the utility function displays constant absolute risk aversion, as in Holmstrom and Milgrom (1987).

Chambers and Quiggin (1996; 2000) show that, under plausible conditions on g , there exists a nondecreasing and convex effort-cost function $C(z_1, z_2)$ which represents the minimum of $g(\mathbf{x})$ consistent with $\mathbf{x}$ producing (z_1, z_2). Thus, the client's maximum expected utility, given state-contingent payments y_1 and y_2, and consistent with producing the state-contingent output vector (z_1, z_2), is

$$E[w(y, \mathbf{x})] = \pi_1 u(y_1 - C(z_1, z_2)) + \pi_2 u(y_2 - C(z_1, z_2)),$$

where π_s, $s = 1, 2$, is the client's subjective probability of state s, and E is the expectations operator, taken with respect to the probability vector (π_1, π_2). As is shown by Chambers and Quiggin (1996; 2000), standard conditions on the technology set and the effort cost function ensure that C is well-behaved.

Assumption 1: The effort-cost function $C{:}\Re^2 \to \Re$ is convex, strictly increasing and twice differentiable in each argument.

Following Chambers and Quiggin (2000), we define a state-contingent output vector (z_1, z_2) as *inherently risky* if

$$C(z_1, z_2) \leq C(\bar{\mathbf{z}}) \tag{1}$$

where

$$\bar{\mathbf{z}} = (\pi_1 z_1 + \pi_2 z_2, \pi_1 z_1 + \pi_2 z_2).$$

Here the terminology reflects the fact that, at a given level of cost for inherently risky outputs, the client must sacrifice expected output to remove uncertainty from production. Notice, in particular, that if this condition is not satisfied, a risk-averse client can always costlessly self-insure by choosing to produce the riskless output $\bar{\mathbf{z}}$ which yields the same expected output as the risky (z_1, z_2), but at lower cost. By the monotonicity of the client's preference function and his risk aversion, the riskless output vector will thus always be strongly preferred to the risky output vector.[1]

[1] It may appear that all stochastic technologies are inherently risky. This is false. Chambers and Quiggin (2000) define a class of stochastic technologies (the generalized Schur convex) which are nowhere inherently risky.

We define (z_1, z_2) as *monotonic* if $z_1 \leq z_2$ and impose

Assumption 2: Any inherently risky (z_1, z_2) is monotonic.

To guarantee the existence of a non-trivial optimum we require

Assumption 3: There exists $\mathbf{z}$ such that:

$$Z = \pi_1 z_1 + \pi_2 z_2 > C(\mathbf{z}).$$

3 The production problem

3.1 The client's problem without contracting

We first consider the problem where the client is the residual claimant. In a general principal–agent interpretation, this is the case where no contracting takes place. The client receives net return

$$n_s = z_s - C(\mathbf{z})$$

in state s, occurring with probability π_s.

Thus for the case of two states of nature, the client seeks to maximize

$$W(\mathbf{n}) = \pi_1 u(n_1) + \pi_2 u(n_2).$$

This problem has been analyzed by Quiggin and Chambers (2000) for general preferences and S states of nature. In this section some crucial results are summarized. Denoting $\partial C/\partial z_s$ by C_s, the client's first-order conditions are of the form

$$\pi_s u'(z_s - C(z_1, z_2)) - (\pi_1 u'(z_1 - C(z_1, z_2)) + \pi_2 u'(z_2 - C(z_1, z_2)))\, C_s = 0 \quad s = 1, 2$$

with equality at an interior solution, and are illustrated in Figure 1 by a tangency between the client's indifference curve and isocost curve. Under the stated conditions, a unique interior optimum will exist. We define

$$\hat{\mathbf{z}} = \arg\max W(\mathbf{z} - C(\mathbf{z})\mathbf{1})$$

to be the solution to the client's maximization problem, and denote the associated vector of net returns by $\hat{\mathbf{n}}$.

We first observe:

Lemma 1 : *Under the stated conditions, the optimal choice $(\hat{z}_1, \hat{z}_2)$ is inherently risky and monotonic.*

Proof: That $(\hat{z}_1, \hat{z}_2)$ is inherently risky follows from the fact that preferences preserve second-order stochastic dominance, since for $(\hat{z}_1, \hat{z}_2)$ not inherently risky, the vector $(z_1 - C(\mathbf{z}), z_2 - C(\mathbf{z}))$ is dominated by $(\hat{Z} - C(\bar{\mathbf{z}}), \hat{Z} - C(\bar{\mathbf{z}}))$ where

$$\hat{Z} = \pi_1 \hat{z}_1 + \pi_2 \hat{z}_2.$$

Monotonicity follows by Assumption 2.■

Adding and rearranging the first-order conditions yields the arbitrage condition

$$C_1 + C_2 = 1.$$

Suppose on the contrary, that $C_1 + C_2 < 1$. Then it would be possible to increase output in both states by one unit, while increasing costs by less than one unit. Hence, net returns would increase in both states, and the client would be better off with probability1. A symmetrical argument applies when $C_1 + C_2 > 1$. Chambers and Quiggin (2000) define the set of vectors $\mathbf{z}$ satisfying the arbitrage condition as the *efficient set.*

A risk-neutral client chooses $\mathbf{z}$ to maximize expected net return

$$\begin{aligned} N(\mathbf{z}) &= Z - C(\mathbf{z}) \\ &= \pi_1 z_1 + \pi_2 z_2 - C(z_1, z_2). \end{aligned}$$

The risk-neutral optimum choice of $\mathbf{z}$ is denoted $\mathbf{z}^{RN}$ and the associated expected profit is denoted N^{RN}. Visually it coincides with a tangency between the fair-odds line, with slope $-(\pi_1/\pi_2)$, and the client's isocost curve. It will also be useful to define, for any cost level C,

$$\mathbf{z}^{RN}(C) = \arg\max_{\mathbf{z}} \{\pi_1 z_1 + \pi_2 z_2 : C(z_1, z_2) \leq C\},$$

the output vector that maximizes expected revenue, conditional on cost level C.

3.2 Self-protection and self-insurance

A central theme of the principal–agent literature is that seemingly inefficient contractual mechanisms may be explained as devices to spread risk.

In the standard principal–agent model, the risk faced by the agent is exogenous except for a scaling factor determined by the agent's effort level.

Using agricultural production as an example. Chambers and Quiggin (2000) show how the choice of more or less risky technologies can be modelled in the state-contingent framework. Crop diversification provides an example. In the single-output technology modelled here, crop diversification cannot be modelled explicitly. However, if z is regarded as a generic output, diversification may be seen as a particular sort of risk-reducing technology. If the states of nature represent more or less favorable conditions for the production of the primary cash crop, self-protection may be undertaken by increasing the allocation of effort and other resources to the production of subsistence crops, thereby reducing the variability of returns.

The client's cost of self-protection is defined as the loss in expected net return associated with the choice of a given $\mathbf{z}$ and is given by

$$\begin{aligned} \Delta_1(\mathbf{z}) &= N^{RN} - N(\mathbf{z}) \\ &= (Z^{RN} - C(\mathbf{z}^{RN})) - (Z - C(\mathbf{z})). \end{aligned}$$

It is useful to partition this cost into two components:

$$\begin{aligned} \Delta_1(\mathbf{z}) &= \Delta_z(\mathbf{z}) + \Delta_c(\mathbf{z}) : \text{ and} \\ \Delta_z(\mathbf{z}) &= Z^{RN}(C(\mathbf{z})) - Z, \text{ where} \\ \Delta_c(\mathbf{z}) &= (Z^{RN} - C(\mathbf{z}^{RN})) - \left(Z^{RN}(C(\mathbf{z})) - C(\mathbf{z})\right). \end{aligned}$$

With this division, Δ_z may be referred to as the pure cost of self-protection. Δ_z reflects the loss in expected output Z arising from the choice of a less risky state-contingent output vector $\mathbf{z}$ in preference to $\mathbf{z}^{RN}(C(\mathbf{z}))$, which yields the maximum expected output level attainable for the given cost level $C(\mathbf{z})$. Alternatively, and dually, Δ_z may be seen as measuring the cost of resources diverted from increasing the expected level of output to reducing the riskiness of output. The second component, Δ_c represents the scale effect of risk aversion.

By the definitions of $Z^{RN}(C)$ and Z^{RN}, both Δ_z and Δ_c are non-negative for optimal choices $\hat{\mathbf{z}}$, even in cases where $C(\hat{\mathbf{z}}) \geq C(\mathbf{z}^{RN})$. If the technology is smooth and preferences are strictly risk-averse, Δ_z will be strictly positive. Chambers and Quiggin (2000) show that the cost level $C(\hat{\mathbf{z}})$ will be independent of risk attitudes, and therefore Δ_c will be zero, if the technology displays constant absolute riskiness, that is, if for any $\mathbf{z}$, $\delta < 0$

$$\Delta_z(\mathbf{z}) = \Delta_z(\mathbf{z} + \delta\mathbf{1}).$$

Self-insurance will also not be modelled explicitly. Rather the utility function u will be assumed to incorporate the effects of self-insurance. With this convention, the cost of self-insurance for any $\mathbf{z}$ may be defined as

$$\Delta_2(\mathbf{z}) = Z - C(\mathbf{z}) - CE(\mathbf{n}).$$

where $CE(\mathbf{n})$ is the certainty equivalent net income associated with the risky net return vector $\mathbf{z}$ and is determined by

$$CE(\mathbf{n}) = u^{-1}\left(W(\mathbf{z}{-}C(\mathbf{z})\mathbf{1})\right).$$

Risk aversion implies that $\Delta_2(\mathbf{z}) \geq 0$, for all $\mathbf{z}$.

The client's problem, therefore, may be reinterpreted as that of choosing $\mathbf{z}$ to minimize the cost of self-protection and cost of self-insurance:

$$\begin{aligned}\Delta_1(\mathbf{z}) + \Delta_2(\mathbf{z}) &= N^{RN}{-}CE(\mathbf{n}) \\ &= Z^{RN} - C(\mathbf{z}^{RN}){-}CE(\mathbf{n}).\end{aligned}$$

Since the maximum profit available to a risk-neutral client is exogenously given by the technology, and since u is monotone increasing, this is exactly the same as maximizing $W(\mathbf{z}{-}C(\mathbf{z})\mathbf{1}) = u(CE(\mathbf{n}))$. Hence, the optimal choice is $\hat{\mathbf{z}}$, yielding certainty equivalent net returns $CE(\hat{\mathbf{n}})$.

3.3 The stochastic production function case

The importance of self-protection may be seen by considering the special case of a stochastic production function, which has been employed in most of the literature on principal–agent models. In this case,

$$z_s = f(\mathbf{x}, s),$$

or, allowing for free disposal,

$$Z(\mathbf{x}) = \{\mathbf{z} : z_s \leq f(\mathbf{x}, s)\, \forall s\}.$$

The most common approach to stochastic production functions suppresses reference to any underlying state space, focusing instead on a function $f(\mathbf{x}, \varepsilon)$ where ε is a random variable (that is, a measurable function $\varepsilon : S \rightarrow \Re$) taken to represent climatic and other conditions relevant to production. This representation is intuitively appealing if the assumption of a stochastic production function is valid, but presents obstacles to understanding in general.

It is particularly useful to focus on the case of input separability

$$z_s = f\left(m\left(\mathbf{x}\right), s\right),$$

where $m : \Re^N \rightarrow \Re$ is a index of input use. This case includes the most common form of the stochastic production function, that of a single input. Under the assumption of constant returns to scale, m can be chosen to be linearly homogeneous in $\mathbf{x}$, and f is then linear in m.

Under the assumption of a stochastic production function with input separability, the state-contingent output vector $\mathbf{z}$ is completely determined by the level of the input index $m\left(\mathbf{x}\right)$. Hence, assuming cost minimization and the choice of a nondominated output $\mathbf{z}$, $\Delta_z\left(\mathbf{z}\right) = 0$. Thus, under input separability, the stochastic production function technology allows no role for pure self-protection. The only economic choice is the value of the input index $m\left(\mathbf{x}\right)$ which is determined by a trade-off between $\Delta_c\left(\mathbf{z}\right)$, the cost of suboptimal scale, and $\Delta_2\left(\mathbf{z}\right)$, the cost of self-insurance.

A particularly interesting case, commonly examined in empirical work, is that of an additive disturbance

$$f\left(\mathbf{x}, \varepsilon\right) = g\left(\mathbf{x}\right) + h\left(\boldsymbol{\varepsilon}\right).$$

Chambers and Quiggin (2002) show that stochastic production functions of this form display what they refer to as *constant absolute riskiness.* Visually, constant absolute riskiness implies that successive isocost curves have points of equal slope as one proceeds in a direction parallel to the 45^o line in (z_1, z_2) space. It follows that $\Delta_c\left(\mathbf{z}\right) \equiv 0$, and hence also $\Delta_1\left(\mathbf{z}\right) \equiv 0$, so that risk aversion has no effect on output choices.

As Newbery (1977) shows, under conditions of constant returns to scale, and assuming all producers have access to the same stochastic production function technology, a set of competitive factor markets will yield an equilibrium that is constrained Pareto-efficient, in the sense that no improvement can be realized by any set of share contracts. Newbery's argument depends solely on concavity and is therefore applicable to the more general technology considered here.

From a state-contingent perspective, the necessary condition in Newbery's result is that the relevant states of nature should be the same for all producers. It is not sufficient that the technology $f(x, \varepsilon)$ and the distribution of ε should be the same for all producers. In addition, the stochastic input ε should be the same, or at least perfectly correlated. In the present paper, it is assumed that the state of nature s reflects 'idiosyncratic' risks specific to individual clients. Hence, an insurer dealing with many clients may be regarded as risk-neutral without significant loss of generality.

4 Contracting

We now consider the principal–agent problem that arises when a risk-neutral insurer contracts with a risk-averse client who is engaged in production under uncertainty. The insurer has the right to specify contract provisions involving a payment y to a client for an observed output z, with the insurer receiving $z - y$. Hence, if the contract is accepted, the client receives a state-contingent payment vector $\mathbf{y}(\mathbf{z})$ and the insurer receives the state-contingent income vector $\mathbf{z} - \mathbf{y}$. The client is free to take the contract offered by the insurer or to reject it. If the client rejects the contract, he retains the rights to the state-contingent output vector $\mathbf{z}$. In our framework, therefore, the contracting problem reduces to one of simultaneously picking a state-contingent output vector for the client and a state-contingent payment vector for the insurer. The approach, therefore, is general enough to permit any degree of interlinkage of contract stipulations between the client and the insurer.

We consider two polar cases in relation to the insurer's objective function. In the competitive case, we assume that competition among potential insurers drives expected profit to zero. Hence, the problem is one of designing a contract to maximize the client's expected utility subject to the constraint that the insurer must make zero expected profit. In the other polar case, we assume that the insurer has complete monopoly power. Thus the problem is one of maximizing the insurer's expected profit, subject to the constraint associated with the client's right to reject the contract proposed by the insurer and receive instead the state-contingent output vector $\mathbf{z}$.

This interaction is represented as an extensive-form game, and we focus on incentive-compatible subgame-perfect equilibria. We consider three possible information structures. In all cases the client can observe, *ex post,* the state of nature s. In the *first-best* case, the insurer can observe the state of nature *ex post*, and can commit in advance of the game to offer a payment schedule $\mathbf{y}(\mathbf{z})$ if the client chooses state-contingent output vector $\mathbf{z}$. The timing is as follows:

1. The insurer commits to a payment schedule $\mathbf{y}^{FB}$ contingent on the client producing $\mathbf{z}^{FB}$.

2. The client accepts or rejects the insurer's contract (rejection is represented as setting $\mathbf{y} = \mathbf{z}$).

3. The client chooses a state-contingent output vector $\mathbf{z} = (z_1, z_2)$.

4. Nature chooses $s \in \{1, 2\}$.

5. The client and the insurer observe the state of nature s.

6. If the client accepted the contract at stage 2 and produced $\mathbf{z}^{FB}$, she receives $n_s^{FB} = y_s^{FB} - C(\mathbf{z}^{FB})$, and the insurer receives $z_s^{FB} - y_s^{FB}$. If the contract was rejected, the client receives $n_s = z_s - C(\mathbf{z})$ and the insurer receives zero.

In the second-best or *symmetric-information* case, the insurer can observe the state of nature s, but the client chooses the output vector $\mathbf{z}$ before the insurer can commit to a payment schedule. Thus, the bargaining sequence is:

1. The client chooses a state-contingent output vector $\mathbf{z}^{SB} = (z_1^{SB}, z_2^{SB})$.

2. The insurer offers a payment schedule $\mathbf{y}^{SB} = (y_1^{SB}, y_2^{SB})$ for output $\mathbf{z}^{SB}$.

3. The client accepts or rejects the insurer's contract.

4. Nature chooses $s \in \{1, 2\}$.

5. The client and the insurer observe the state of nature s.

6. If the client accepted the contract at stage 3, she receives $n_s^{SB} = y_s^{SB} - C(\mathbf{z}^{SB})$, and the insurer receives $z_s^{SB} - y_s^{SB}$. If the contract was rejected, the client receives $n_s = z_s^{SB} - C(\mathbf{z})$ and the insurer receives zero.

Except where the insurer has no bargaining power, the second-best case gives rise to a hold-up problem for the client, who must choose the state-contingent output vector $\mathbf{z}^{SB}$ before the insurer determines the payment schedule $\mathbf{y}^{SB}$. This is exactly analogous to the classic hold-up problem analyzed by Klein, Crawford and Alchian (1978), in which one party makes a fixed investment whose value depends on the subsequent decisions of a specific contracting partner. An excellent summary of the hold-up literature is given by Holmstrom and Roberts (1998).

In the third-best or *asymmetric-information* case, the insurer can observe *ex post* output z_s, but not the state of nature. Hence the contract offered by the insurer must be incentive-compatible. The timing is:

1. The client chooses a state-contingent output vector $\mathbf{z}^{TB}=(z_1^{TB}, z_2^{TB})$.

2. The client announces an output plan $\tilde{\mathbf{z}}^{TB} = (\tilde{z}_1^{TB}, \tilde{z}_2^{TB})$ (in incentive-compatible subgame-perfect equilibria, $\mathbf{z}^{TB}= \tilde{\mathbf{z}}^{TB}$).

3. The insurer offers a payment schedule $\mathbf{y}^{TB}=(y_1^{TB}, y_2^{TB})$ for output $\tilde{\mathbf{z}}^{TB}=(\tilde{z}_1^{TB}, \tilde{z}_2^{TB})$.

4. The client accepts or rejects the insurer's contract (rejection is represented as setting $\mathbf{y}^{TB}= \mathbf{z}^{TB}$).

5. Nature chooses $s \in \{1, 2\}$.

6. The client observes the state of nature s.

7. The client reports state $\tilde{s}$, (in incentive-compatible subgame-perfect equilibria $\tilde{s}=s$).

8. The insurer observes the *ex post* output z_s^{TB}.

9. If the client accepted the contract at stage 4 and produced $z_s^{TB} = \tilde{z}_{\tilde{s}}^{TB}$, the client receives $n_s^{TB} = y_s^{TB} - C(\mathbf{z}^{TB})$, but if $z_s^{TB} \neq \tilde{z}_{\tilde{s}}^{TB}$ the client receives an arbitrarily large negative payoff. If the contract was rejected, the client receives $n_s = z_s^{TB} - C(\mathbf{z})$, and the insurer receives zero.

The focus of our analysis is on the interaction between the game structure and the relative bargaining power of the insurer and client. We first observe the following result, which is valid for any of the information structures considered in this paper.

Proposition 2 *Suppose $z_1 \le z_2$. Then any contract which is acceptable to the client and which yields non-negative profits to the insurer must satisfy*

$$z_1 \le y_1, y_2 \le z_2.$$

Proof Suppose to the contrary that $y_2 > z_2$. Then the contract can only be profitable if $y_1 < z_1$ and

$$\pi_1 y_1 + \pi_2 y_2 < \pi_1 z_1 + \pi_2 z_2.$$

This means that (z_1, z_2) second-order stochastically dominates (y_1, y_2) so that acceptance of the contract would make the client worse off. Other violations of the conditions can be dealt with similarly. ■

5 Monopolistic insurers

In the monopolistic case, a single insurer contracts with clients by specifying an output vector (z_1, z_2) and payment vector (y_1, y_2). Clients must choose whether to produce the output vector (z_1, z_2) and receive the payment vector (y_1, y_2) proposed by the insurer, or to produce some other output vector (in which case they must self-insure). Then, after committing to (z_1, z_2), clients have the opportunity to accept or decline the contract offered by the insurer.

The problem faced by the client is the need to commit to a state-contingent production vector in the knowledge that she will subsequently deal with a insurer who possesses monopoly bargaining power and who, therefore, has the capacity to capture all available rents. In all such cases, unless the insurer can commit *ex ante* to guarantee the client some minimum utility level, the client must choose her output vector to maximize the utility of her outside option.

5.1 First-best case

In the first-best case, the insurer can commit in advance to providing the client with a given utility level, conditional on accepting the proposed contract. If the insurer's proposed contract yields the client less than $CE(\hat{\mathbf{n}})$, the client's dominant strategy is to reject the offer and produce $\hat{\mathbf{z}}$. If the contract yields at least $CE(\hat{\mathbf{n}})$, the client's weakly dominant strategy is to produce the output proposed by the insurer andto accept the contract.

Hence, the insurer's problem is:

$$\max_{\mathbf{y}}\{\ \pi_1(z_1 - y_1) + \pi_2(z_2 - y_2)\}$$

subject to the constraint

$$\pi_1 u(y_1 - C(z_1, z_2)) + \pi_2 u(y_2 - C(z_1, z_2)) \geq u\left(CE(\hat{\mathbf{n}})\right).$$

The insurer will, therefore, choose (z_1^{FB}, z_2^{FB}) to maximize

$$P = \pi_1 z_1 + \pi_1 z_2 - C(z_1, z_2)$$

and make a state-independent payment Y^{FB} such that

$$Y^{FB} - C(\mathbf{z}^{FB}) = CE(\hat{\mathbf{n}}).$$

It is obvious that $Z^{FB} = Z^{RN}$, so that the insurer's expected profit is

$$\begin{aligned} P^{FB} &= Z^{RN} - Y^{FB} \\ &= \Delta_1(\hat{\mathbf{z}}) + \Delta_2(\hat{\mathbf{z}}). \end{aligned}$$

The solution is illustrated in Figure 2, by having the client produce at the point of tangency between the fair-odds line and the isocost curve for $C\left(\mathbf{z}^{RN}\right)$ and then having the insurer define an implicit indemnity structure that leaves the client at the point of intersection between the fair-odds line and the client's indifference curve through $\hat{\mathbf{n}}$.

5.2 Second-best (symmetric information) case

We next consider the case when the insurer can observe the state of nature, but cannot commit in advance to a conditional payment $\mathbf{y}(\mathbf{z})$. Hence, the client commits to the state-contingent production vector (z_1, z_2) before negotiating with the insurer. The insurer must then offer a payment vector (y_1, y_2) which the client can either accept or decline. Since the insurer can observe the state of nature, the client must announce $\tilde{s}{=}s$ and must therefore announce $\tilde{\mathbf{z}} = \mathbf{z}$. Given that the client has committed to, and announced, the state-contingent output $\mathbf{z}$, the dominant strategy for the insurer is to offer the client exactly $W(\mathbf{z}{-}C(\mathbf{z})\mathbf{1})$, the utility the client would get from consuming the output (z_1, z_2) chosen in stage 1. Hence the iteratively dominant strategy for the client is to choose the output $\hat{\mathbf{z}}$ that maximizes this utility. Hence, this game has a unique subgame-perfect equilibrium, which we now explore in detail.

The insurer's objective function is:

$$\max_{\mathbf{y}} \ \pi_1(y_1 - z_1) + \pi_2(y_2 - z_2)$$

subject to the constraint

$$\pi_1 u(y_1 - C(z_1, z_2)) + \pi_2 u(y_2 - C(z_1, z_2)) \geq u\left(CE(\hat{\mathbf{n}})\right)$$

which requires that the client, having committed to the (z_1, z_2), vector will find the insurer's contract at least as attractive as the alternative of consuming (z_1, z_2).

In the optimal solution, the insurer will offer a state-independent payment of $\hat{Y}$, where

$$\hat{Y} - C(\hat{\mathbf{z}}) = CE(\hat{\mathbf{n}}).$$

The insurer's expected profit is

$$
\begin{aligned}
\hat{P} &= \hat{Z} - \hat{Y} \\
&= P^{FB} - \Delta_1(\hat{\mathbf{z}}) \\
&= \Delta_2(\hat{\mathbf{z}})
\end{aligned}
$$

where

$$\Delta_1(\hat{\mathbf{z}}) = [Z^{FB} - C(z_1^{FB}, z_2^{FB})] - [\hat{Z} - C(\hat{z}_1, \hat{z}_2)],$$

which is the cost of self-protection by the client, and

$$\Delta_2(\hat{\mathbf{z}}) = \hat{P} - CE(\hat{\mathbf{n}})$$

is the client's risk premium. Under monopoly, the symmetric information case involves a welfare loss of $\Delta_1(\hat{\mathbf{z}})$ relative to the first-best. This loss reflects the cost of self-protection undertaken by the client in anticipation of the hold-up problem associated with the insurer's use of monopoly power.

Note that this welfare loss does not arise in the case of a stochastic production function with additive disturbances.

5.3 Third-best (asymmetric information) case

The asymmetric information monopolistic case will be referred to as the third-best, since both the insurer's monopoly power and the client's private information reduce aggregate welfare relative to the first-best The insurer's dominant strategy, given an announced output $\tilde{\mathbf{z}}$ is to offer a payment schedule $\mathbf{y}$ yielding the client $W(\tilde{\mathbf{z}} - C(\tilde{\mathbf{z}})\mathbf{1})$ if $\tilde{\mathbf{z}}$ is produced, and $W \leq W(\tilde{\mathbf{z}} - C(\tilde{\mathbf{z}})\mathbf{1})$ if any $\mathbf{z} \neq \tilde{\mathbf{z}}$ is produced. Hence, in any subgame-perfect equilibrium, the client produces and announces $\hat{\mathbf{z}}$, yielding certainty-equivalent outcome $CE(\hat{\mathbf{n}})$.

The outcome in state s is that the insurer's payoff is $z_s - y_s$, and the client's payoff is $y_s - C(z_1, z_2)$. Hence the insurer's problem becomes

$$\max_{\mathbf{y}} \ \pi_1(y_1 - \hat{z}_1) + \pi_2(y_2 - \hat{z}_2)$$

subject to constraints analogous to those in the competitive case:

$$
\begin{aligned}
\pi_1 u(y_1 - C(\hat{z}_1, \hat{z}_2)) + \pi_2 u(y_2 - C(\hat{z}_1, \hat{z}_2)) &\geq u\left(CE(\hat{\mathbf{n}})\right); \\
\pi_1 u(y_1 - C(\hat{z}_1, \hat{z}_2)) + \pi_2 u(y_2 - C(\hat{z}_1, \hat{z}_2)) &\geq u(y_1 - C(\hat{z}_1, \hat{z}_1)); \\
\pi_1 u(y_1 - C(\hat{z}_1, \hat{z}_2)) + \pi_2 u(y_2 - C(\hat{z}_1, \hat{z}_2)) &\geq u(y_2 - C(\hat{z}_2, \hat{z}_2)); \text{ and} \\
\pi_1 u(y_1 - C(\hat{z}_1, \hat{z}_2)) + \pi_2 u(y_2 - C(\hat{z}_1, \hat{z}_2)) &\geq \pi_1 u(y_2 - C(\hat{z}_2, \hat{z}_1)) + \pi_2 u(y_1 - C(\hat{z}_2, \hat{z}_1)).
\end{aligned}
$$

Assuming $z_1 \leq z_2$,the incentive compatibility constraints clearly require $y_1 \leq y_2$ with strict inequality whenever $z_1 < z_2$. Hence we obtain the following Corollary to Proposition 1.

Corollary 3 *Any solution to the asymmetric information problem with non-negative expected profit for the insurer must have*

$$z_1 < y_1 < y_2 < z_2.$$

Since $y_2 < z_2$, the option of producing (z_2, z_2) and receiving (y_2, y_2) under the insurance contract is dominated by the alternative of producing $\mathbf{z} = (z_2, z_2)$ and setting $\mathbf{y} = \mathbf{z}$. Also, since (z_2, z_1) is not inherently risky, the option of producing (z_2, z_1) and receiving (y_2, y_1) is dominated by the alternative of setting $\mathbf{z} =(y_2, y_1)$ and setting $\mathbf{y} = \mathbf{z}$. In each case, the dominating alternative is dominated by the trivial contract in which the client produces $\hat{\mathbf{z}}$ and receives payment $\mathbf{y} = \hat{\mathbf{z}}$, yielding net returns $\hat{\mathbf{n}}$. Noting that this contract satisfies all the constraints, we observe that the set of feasible contracts yielding $W \geq W(\hat{\mathbf{z}}-C(\hat{\mathbf{z}})\mathbf{1})$ is non-empty. Assuming that C is 'sufficiently' convex, the set of feasible contracts will also be compact. Hence, we have:

Lemma: There exists an optimal pair $(\mathbf{y}, \mathbf{z})$ satisfying the constraints $(T.1)$ to $(T.4)$. For this pair, $(\mathbf{y}, \mathbf{z})$, the constraints $(T.3)$ and $(T.4)$ are not binding.

We have proved the following result, previously derived by Grossman and Hart (1983) for the case of an objective function additively separable in income and effort.

Proposition 4 *In the asymmetric information problem with competitive insurance and a net returns objective function, the equilibrium will yield the client reservation utility.*

We can derive an explicit solution to the insurance problem. Let u_i denote $u(y_i - C(\hat{z}_1, \hat{z}_2))$, $i = 1, 2$. Then the solution to the problem takes the form

$$\begin{aligned} \pi_1 u_1 + \pi_2 u_2 &= u\left(CE(\hat{\mathbf{n}})\right) \\ &= u(y_1 - C(\hat{z}_1, \hat{z}_1)). \end{aligned}$$

Hence,

$$y_1 - C(\hat{z}_1, \hat{z}_1) = CE(\hat{\mathbf{n}})$$

or

$$\begin{aligned} y_1(\hat{\mathbf{z}}) &= CE(\hat{\mathbf{n}}) + C(\hat{z}_1, \hat{z}_1) \\ &= \hat{Y} + C(\hat{z}_1, \hat{z}_1) - C(\hat{z}_1, \hat{z}_2) \end{aligned}$$

and

$$\begin{aligned} y_2(\hat{\mathbf{z}}) - C(\hat{z}_1, \hat{z}_2) &= u^{-1}\left(\frac{u\left(CE(\hat{\mathbf{n}})\right) - \pi_1 u(y_1(\hat{\mathbf{z}}) - C(\hat{z}_1, \hat{z}_2))}{\pi_2}\right) \\ &= u^{-1}\left(\frac{u\left(CE(\hat{\mathbf{n}})\right) - \pi_1 u(CE(\hat{\mathbf{n}}) + C(\hat{z}_1, \hat{z}_1) - C(\hat{z}_1, \hat{z}_2))}{\pi_2}\right). \end{aligned}$$

Thus, the insurer's expected profit is

$$\begin{aligned} P^{TB} &= \hat{z} - \pi_1 y_1(\hat{z}_1, \hat{z}_2) - \pi_2 y_2(\hat{z}_1, \hat{z}_2) \\ &= P^{FB} - \Delta_1(\hat{\mathbf{z}}) - \Delta_2^{TB}(\hat{\mathbf{z}}), \end{aligned}$$

where Δ_1 is the client's cost of self-protection as before, and

$$\Delta_2^{TB}(\hat{\mathbf{z}}) = (\pi_1 y_1(\hat{\mathbf{z}}) + \pi_2 y_2(\hat{\mathbf{z}}) - C(\hat{\mathbf{z}})) - CE(\hat{\mathbf{n}})$$

is the client's risk premium associated with the requirement for incentive-compatibility. Moreover, we note that $0 \leq \Delta_2^{TB}(\hat{\mathbf{z}}) \leq \Delta_2(\hat{\mathbf{z}})$ and

$$P^{TB} = \Delta_2(\hat{\mathbf{z}}) - \Delta_2^{TB}(\hat{\mathbf{z}}).$$

The existence of asymmetric information prevents the insurer from fully insuring the client and capturing the entire risk premium.

The incentive-compatibility constraint implies:

$$CE^{TB} = \hat{y}_1 - C(\hat{z}_1, \hat{z}_1)$$

so

$$\begin{aligned} \Delta_2^{TB} &= \pi_1 \hat{y}_1 + \pi_2 \hat{y}_2 - C(\hat{z}_1, \hat{z}_2) - CE^{TB} \\ &= \pi_2(\hat{y}_2 - \hat{y}_1) - (C(\hat{z}_1, \hat{z}_2) - C(\hat{z}_1, \hat{z}_1)). \end{aligned}$$

6 Competitive insurance

The most important differences between the state-contingent production approach adopted here and the standard model based on a stochastic production function technology arise in the competitive case, where the insurer's expected profit is zero. In the symmetric case, the absence of a hold-up problem arising from the need to deal with a monopolistic insurer means that the client does not need to commit to costly self-protection prior to contracting. Hence, the first-best outcome is achieved. Under asymmetric information, the problem of inadequate insurance is mitigated by the capacity of the client to bear more risk than would be the case in the presence of the hold-up problem, though less than in the presence of full insurance. The essential difference between the general state-contingent representation and the stochastic production function approach is the treatment of production risk as a decision variable rather than as a by-product of input choices.

6.1 First-best and symmetric information cases

In the competitive case, the insurer must offer the contract that maximizes the client's utility, subject to the insurer making zero expected profit. In the first-best case, the insurer will therefore choose (z_1^{FB}, z_2^{FB}) to maximize

$$N^{FB} = \pi_1 z_1 + \pi_1 z_2 - C(z_1, z_2)$$

and make the payment N^{FB} in both states of nature. It is obvious that $N^{FB} = N^{RN}$ so that the contract yields the client a welfare gain of

$$N^{RN} - CE(\hat{\mathbf{n}}) = \Delta_1(\hat{\mathbf{z}}) + \Delta_2(\hat{\mathbf{z}})$$

relative to the equilibrium without insurance.

Competition among insurers ensures that the insurer must offer the most appealing possible contract to the client, subject to the zero-expected-profit constraint. Hence, even in the absence of an *ex ante* commitment by the insurer, the first-best contract is achievable provided that the state of nature is observable. That is, the symmetric information equilibrium is the same as the first-best. This common outcome is the same as in the first-best monopoly case, except that all the benefits of the contract go to the client rather than the insurer.

Relative to the monopolistic symmetric information case, the client is better off and the insurer is worse off, as would be expected. However, unlike the monopolistic case, the outcome in the competitive symmetric information case is Pareto-efficient.

6.2 Asymmetric information case

The asymmetric information case arises when the insurer cannot observe, or at least contract on, either the state of nature or the state-contingent production vector. Hence it is possible for the client to misrepresent the state-contingent production vector to which she has committed, and support this misrepresentation by misreporting the state of nature where necessary. For example, the client might commit to (z_1, z_1) but report that she has committed to (z_1, z_2). Whatever state of nature actually occurred, the client would produce z_1 and report the occurrence of state 1.[2] We may confine attention to incentive-compatible equilibria, in which such misrepresentation does not occur.

For given $\mathbf{z}$, the optimal payment vector must satisfy

$$\begin{aligned}
(T.1) \quad \pi_1 y_1 + \pi_2 y_2 &= \pi_1 z_1 + \pi_2 z_2; \\
(T.2) \quad \pi_1 u(y_1 - C(z_1, z_2)) + \pi_2 u(y_2 - C(z_1, z_2)) &\geq u(y_1 - C(z_1, z_1)); \\
(T.3) \quad \pi_1 u(y_1 - C(z_1, z_2)) + \pi_2 u(y_2 - C(z_1, z_2)) &\geq u(y_2 - C(z_2, z_2)); \text{ and} \\
(T.4) \quad \pi_1 u(y_1 - C(z_1, z_2)) + \pi_2 u(y_2 - C(z_1, z_2)) &\geq \pi_1 u(y_2 - C(z_2, z_1)) \\
&\quad + \pi_2 u(y_1 - C(z_2, z_1))
\end{aligned}$$

As in the monopoly case, only the first and second constraints will bind in equilibrium.

Thus, for any announced (z_1, z_2), competition will induce the insurer to offer an output-dependent payment $\mathbf{y}$ that maximizes the client's utility subject to the zero profit constraint

$$(T.1) \quad \pi_1 y_1 + \pi_2 y_2 = \pi_1 z_1 + \pi_2 z_2$$

[2] This is the only relevant possibility, assuming $z_1 \leq z_2$. Since the insurance contract must have $y_2 \leq z_2$ by Proposition 1, the option of producing (z_2, z_2) and receiving (y_2, y_2) under the insurance contract is dominated by the alternative of not contracting and receiving (z_2, z_2). Under the assumption of constant returns to scale, the option of producing (z_2, z_1) is dominated by a convex combination of the returns available by producing (z_2, z_2), yielding $z_2 - C(z_2, z_2)$, and (z_1, z_1), yielding $z_1 - C(z_1, z_1)$.

and the incentive-compatibility constraint

$$(T.2) \quad \pi_1 u(y_1 - C(z_1, z_2)) + \pi_2 u(y_2 - C(z_1, z_2)) \geq u(y_1 - C(z_1, z_1)).$$

Let the optimal solution to this problem be denoted $\mathbf{y}(\mathbf{z})$. We now consider some characteristics of the equilibrium pair $(\mathbf{y}, \mathbf{z})$.

Consider first $\mathbf{y}(\hat{\mathbf{z}})$. Since $z_1 < y_1 < y_2 < z_2$ and

$$\pi_s u'(z_s - C(z_1, z_2)) - (\pi_1 u'(z_1 - C(z_1, z_2)) + \pi_2 u'(z_2 - C(z_1, z_2)))\, C_s = 0 \quad s = 1, 2$$

we must have

$$\begin{aligned} \pi_1 u'(y_1 - C(z_1, z_2)) - (\pi_1 u'(y_1 - C(z_1, z_2)) + \pi_2 u'(y_2 - C(z_1, z_2)))\, C_1 &< 0 \quad s = 1, 2 \\ \pi_2 u'(y_2 - C(z_1, z_2)) - (\pi_1 u'(y_1 - C(z_1, z_2)) + \pi_2 u'(y_2 - C(z_1, z_2)))\, C_2 &> 0 \quad s = 1, 2. \end{aligned}$$

Hence, the client would benefit from a change which increased z_2 and y_2, and reduced z_1 and y_1 in such a manner as to hold $C(z_1, z_2)$, $z_2 - y_2$ and $z_1 - y_1$ constant. Such a change would leave the expected profit equal to zero. Moreover, totally differentiating the right-hand side of the incentive-compatibility constraint yields the following expression for the change in the client's utility conditional on producing (z_1, z_1) :

$$u'(y_1 - C(z_1, z_1)\, [dy_1 - (C_1(z_1, z_1) + C_2(z_1, z_1))\, dz_1] \, .$$

Observing that $C_1(z_1, z_1) + C_2(z_1, z_1) \leq 1$, and $dy_1 = dz_1 < 0$, the right-hand side declines, while the left-hand side increases. Hence, the incentive-compatibility constraint is satisfied after the change. It follows that the client will prefer to choose a state-contingent output vector $\mathbf{z}$ such that

$$\begin{aligned} \pi_1 z_1 + \pi_2 z_2 &> \pi_1 \hat{z}_1 + \pi_2 \hat{z}_2 \\ z_1 &< \hat{z}_1 < \hat{z}_2 < z_2. \end{aligned}$$

That is:

Proposition 5 *The optimal output* $\mathbf{z}$ *in the competitive asymmetric information solution is derived from a mean-increasing spread of* $\hat{\mathbf{z}}$*.*

Having derived this result it is possible to characterize the welfare losses in the competitive asymmetric information solution relative to the first-best. The client's problem at stage 1 is to choose $\mathbf{z}^{CAS}$ to maximize

$$\pi_1 u(y_1(\mathbf{z}) - C(z_1, z_2)) + \pi_2 u(y_2(\mathbf{z}) - C(z_1, z_2)),$$

yielding net returns $\mathbf{n}^{CAS}$. Denote the expected output and net returns by Z^{CAS}, N^{CAS}.

Relative to the first-best, the client incurs a cost of self-protection

$$\Delta_1(\mathbf{z}^{CAS}) + \Delta_2(\mathbf{z}^{CAS}) = N^{RN} - N^{CAS},$$

and a cost of incomplete insurance

$$\Delta_2(\mathbf{z}^{CAS}) = N^{CAS} - CE(\mathbf{n}^{CAS}).$$

As noted above, $\mathbf{z}^{CAS}$ is riskier than $\hat{\mathbf{z}}$, and $E[\mathbf{n}^{CAS}] \geq E[\hat{\mathbf{n}}]$. Hence, $\Delta_1(\mathbf{z}^{CAS}) \leq \Delta_1(\hat{\mathbf{z}})$. Moreover, $CE(\mathbf{n}^{CAS}) \geq CE(\hat{\mathbf{n}})$. Hence,

$$\Delta_1(\mathbf{z}^{CAS}) + \Delta_2(\mathbf{z}^{CAS}) \leq \Delta_1(\hat{\mathbf{z}}) + \Delta_2(\hat{\mathbf{z}}).$$

As in the monopoly case, the incentive-compatibility constraint implies that:

$$CE(\mathbf{n}^{CAS}) = y_1^{CAS} - C(z_1^{CAS}, z_1^{CAS})$$

so

$$\begin{aligned}\Delta_2(\mathbf{z}^{CAS}) &= \pi_1 y_1^{CAS} + \pi_2 y_2^{CAS} - C(z_1^{CAS}, z_2^{CAS}) - CE^{CAS} \\ &= \pi_2(y_2^{CAS} - y_1^{CAS}) - (C(z_1^{CAS}, z_2^{CAS}) - C(z_1^{CAS}, z_1^{CAS})).\end{aligned}$$

Since the client was free to choose the output level $\hat{\mathbf{z}}$,

$$\Delta_1^{TB} + \Delta_2^{TB} \geq \Delta_1^{CAS} + \Delta_2^{CAS}.$$

7 Bargaining solutions

In the monopolistic solutions considered above, the insurer's monopoly power allows him to capture the entire rent. Compared to the competitive case, however, the insurer's profit is less than the reduction in the certainty-equivalent income of the client, and there is, therefore, a net social loss. In both the symmetric information and asymmetric information cases, the client must precommit to an inefficient production vector to secure his reservation utility. In asymmetric information problems, there is an additional loss relative to the first-best arising from the insurer's need to offer an incentive-compatible contract.

We now consider the possibility of co-operative solutions, in which the client and insurer can contract, *ex ante,* so as to avoid one or both of these sources of divergence from the first-best. The solution concept applied is that of a Nash bargaining solution. The disagreement point is either the symmetric information solution or the asymmetric information solution derived above for the monopoly case. The agreement point may be either the first-best or an asymmetric information solution in which the insurer commits to a payment schedule based on observed output, but the state of nature is not contractible.

Co-operative bargaining solutions may arise either because clients gain an increase in bargaining power relative to monopolistic insurers or because the externality associated with the client's private information is partially internalized. As an example of the former process, individual bargaining with a monopoly insurer may be replaced by collective bargaining. In an employment relationship, for example, workers may be represented by unions. Alternatively, policies such as employee stock ownership plans may produce some commonality of interest between clients and insurers and thereby lead to the internalization of externalities.

7.1 First-best case

We first consider the case where the insurer and client can reach the first-best outcome through bargaining. The disagreement point is one in which the client chooses some $\tilde{\mathbf{z}}$, yielding the reservation certainty-equivalent income $CE(\tilde{\mathbf{n}})$. No contracting takes place and the insurer therefore receives zero.[3] The agreement point is one in which the client produces the first-best output $\mathbf{z}^{FB} = \mathbf{z}^{RN}$ and receives a nonstochastic payment Y, yielding net income $Y - C\left(\mathbf{z}^{FB}\right)$. Bargaining therefore determines the payment Y received by the client and the insurer's profit $Z - Y$.

[3]Note that the insurer may contract with otherclients, so that his income in the event of disagreement is not equal to zero. The existence of outside income will be reflected in relative bargaining power.

Analysis of bargaining problems requires a cardinal specification of the utility of income under certainty. Diminishing marginal utility of certain income is not necessarily equivalent to risk-aversion under certainty even though both may be represented by concavity of the utility function. For simplicity, we assume that utility for both parties is linear in certainty-equivalent income. (For the risk-neutral insurer, certainty-equivalent income is equal to expected income.)

The relative bargaining power of the two parties is represented by a parameter α [4] . Thus, the bargaining problem is to choose Y to maximize

$$\hat{V} = \left(\left(Y - C\left(\mathbf{z}^{FB}\right)\right) - CE(\tilde{\mathbf{n}})\right)^{\alpha} P^{1-\alpha},$$

where $P = Z - Y$.

The first-order condition on Y is:

$$\alpha\left(\left(Y - C\left(\mathbf{z}^{FB}\right)\right) - CE(\tilde{\mathbf{n}})\right)^{\alpha-1} P^{1-\alpha} = (1-\alpha)\left(\left(Y - C\left(\mathbf{z}^{FB}\right)\right) - CE(\tilde{\mathbf{n}})\right)^{\alpha} P^{-\alpha}$$

or:

$$\frac{\left(\left(Y - C\left(\mathbf{z}^{FB}\right)\right) - CE(\tilde{\mathbf{n}})\right)}{P} = \frac{\alpha}{(1-\alpha)}.$$

As in the analysis of Kihlstrom and Roth (1982), the greater the bargaining power of the client, the higher is the payment Y.

Totally differentiating with respect to $CE(\tilde{\mathbf{n}})$ and rearranging yields

$$(1-\alpha) = (1-\alpha)\frac{\partial Y}{\partial CE(\tilde{\mathbf{n}})} - \alpha\frac{\partial P}{\partial CE(\tilde{\mathbf{n}})},$$

or, since

$$\frac{\partial Y}{\partial CE(\tilde{\mathbf{n}})} + \frac{\partial P}{\partial CE(\tilde{\mathbf{n}})} = 0$$

$$\frac{\partial Y}{\partial CE(\tilde{\mathbf{n}})} = (1-\alpha).$$

Hence, the client's final share of income is increasing in $CE(\tilde{\mathbf{n}})$, and the optimal choice for the client is $\tilde{\mathbf{z}} = \hat{\mathbf{z}}$. However, since the actual output is $\mathbf{z}^{FB}$, the choice of $\tilde{\mathbf{z}}$ only affects the division of the surplus. The analysis of

[4] If utility functions display diminishing marginal utility of income, commonly referred to in the bargaining literature as risk-aversion, the curvature of the cardinal utility functions may be incorporated in the determination of α.

the first-best case confirms the result derived by Bell (1989) in the context of tenancy contracts, that, under costless monitoring, the insurer–agent and Nash bargaining solutions, assuming an affine payment structure, are identical up to a side payment.

Since, the more risk-averse is the client, the lower is $CE(\hat{\mathbf{n}})$,we obtain the result that, the more risk-averse is the client, the better off is the insurer. Note that this is not the standard bargaining theory result: the less risk-averse party, that is, the one with the less concave cardinal utility of wealth, has more bargaining power. In the present case, both the insurer and the client have cardinal utility linear in certainty-equivalent wealth. The result arises because, the more risk-averse is the client, the greater are the gains from insurance. Since these gains are shared in proportion to bargaining power, the insurer is better off. On the other hand, since the client receives only part of the gains from insurance, a reduction in $CE(\hat{\mathbf{n}})$ leaves her strictly worse off whenever $\alpha < 1$.

7.2 Bargaining solution with symmetric information

In the symmetric information case, the disagreement point, as before, is one in which the client chooses some $\tilde{\mathbf{z}}$, receiving $CE(\tilde{\mathbf{n}})$ and the insurer receives zero. The agreement point is one in which the output $\tilde{\mathbf{z}}$ is produced and the insurer offers full insurance, giving the client a non-stochastic payment Y and receiving the profit

$$P(\tilde{\mathbf{z}},Y) = E[\tilde{\mathbf{z}}] - Y.$$

Thus, the bargaining problem is to choose Y to maximize

$$\hat{V} = ((Y - C(\tilde{\mathbf{z}})) - CE(\tilde{\mathbf{n}}))^{\alpha} P(\tilde{\mathbf{z}},Y)^{1-\alpha},$$

which, as before, yields the solution condition

$$\frac{((Y - C(\tilde{\mathbf{z}})) - CE(\tilde{\mathbf{n}}))}{P(\tilde{\mathbf{z}},Y)} = \frac{\alpha}{(1-\alpha)}.$$

Totally differentiating with respect to $\tilde{\mathbf{z}}$ and rearranging yields

$$\begin{aligned}(1-\alpha)\nabla_{\tilde{\mathbf{z}}}(Y - C - CE(\tilde{\mathbf{n}})) &= \alpha\nabla_{\tilde{\mathbf{z}}}P(\tilde{\mathbf{z}},Y) \\ &= \alpha\left(\nabla_{\tilde{\mathbf{z}}}E[\tilde{\mathbf{z}}] - \nabla_{\tilde{\mathbf{z}}}Y\right),\end{aligned}$$

where $\nabla_{\tilde{\mathbf{z}}}$ denotes the gradient with respect to the subscripted vector. Hence,

$$\nabla_{\tilde{\mathbf{z}}}Y - (1-\alpha)\nabla_{\tilde{\mathbf{z}}}C = (1-\alpha)\nabla_{\tilde{\mathbf{z}}}CE(\tilde{\mathbf{n}}) + \alpha\nabla_{\tilde{\mathbf{z}}}E[\tilde{\mathbf{z}}].$$

In the case $\alpha = 0$, we have

$$\nabla_{\tilde{\mathbf{z}}}(Y - C) = \nabla_{\tilde{\mathbf{z}}}CE(\tilde{\mathbf{n}}),$$

and the client will maximize $Y - C(\tilde{\mathbf{z}}) = CE(\tilde{\mathbf{n}})$ by choosing $\tilde{\mathbf{z}} = \hat{\mathbf{z}}$. On the other hand, if $\alpha = 1$,

$$\nabla_{\tilde{\mathbf{z}}}(Y - C) = \nabla_{\tilde{\mathbf{z}}}E[\tilde{\mathbf{z}}] - \nabla_{\tilde{\mathbf{z}}}C,$$

and the client will maximize $Y - C(\tilde{\mathbf{z}}) = E[\tilde{\mathbf{z}}] - C(\tilde{\mathbf{z}})$ by choosing $\tilde{\mathbf{z}} = \mathbf{z}^{FB}$. More generally, the greater the value of α, the greater the optimal value of $E[\tilde{\mathbf{z}}] - C(\tilde{\mathbf{z}})$ and therefore the greater the total surplus. Thus, if the client chooses the output vector $\mathbf{z}$ before the insurer can commit to a payment schedule, bargaining power matters not only to the division of the surplus but to the size of the surplus. It is straightforward to show, however, that an increase in α cannot make the insurer better off, so that the bargaining solution is always constrained-Pareto-efficient.

7.3 Bargaining solution with asymmetric information

In the asymmetric information case, the agreement point is one in which the output $\tilde{\mathbf{z}}$ is produced and the insurer offers an incentive compatible payment schedule $\mathbf{y}$ receiving profit

$$P(\tilde{\mathbf{z}}, N) = E[\tilde{\mathbf{z}}] - E[\mathbf{y}(\mathbf{z}, N)],$$

where N is the client's certainty-equivalent net income

$$N = u^{-1}(W(\mathbf{y} - C(\mathbf{z}))).$$

Suppose that the client's preferences display constant absolute risk aversion. Then

$$\frac{\partial P(\tilde{\mathbf{z}}, N)}{\partial N} = 1.$$

Thus, the bargaining problem is to choose N to maximize:

$$\hat{V} = (N - CE(\tilde{\mathbf{n}}))^{\alpha} P(\tilde{\mathbf{z}}, N)^{1-\alpha},$$

which, assuming constant absolute risk aversion, yields the solution condition

$$\frac{(N - CE(\tilde{\mathbf{n}}))}{P(\tilde{\mathbf{z}}, u)} = \frac{\alpha}{(1-\alpha)}.$$

Totally differentiating with respect to $\tilde{\mathbf{z}}$ and rearranging yields

$$\begin{aligned}(1-\alpha)\nabla_{\tilde{z}}\left(N - CE(\tilde{\mathbf{n}})\right) &= \alpha\nabla_{\tilde{z}}\frac{\partial P(\tilde{\mathbf{z}}, u)}{\partial \tilde{\mathbf{z}}} \\ &= \alpha\nabla_{\tilde{z}}\left(E[\tilde{\mathbf{z}}] - E[\mathbf{y}(\mathbf{z})]\right).\end{aligned}$$

In the case $\alpha = 0$, we have

$$\nabla_{\tilde{z}}\left(N - CE(\tilde{\mathbf{n}})\right) = 0,$$

and the client will maximize N by choosing $\tilde{\mathbf{z}} = \hat{\mathbf{z}}$. On the other hand, if $\alpha = 1$,the client will maximize N by choosing $\tilde{\mathbf{z}} = \mathbf{z}^{TB}$. Thus, once again, the greater the client's bargaining power, the greater the total surplus.

In the model of socially costly exploitation analyzed by Chambers and Quiggin (2000), the presence of asymmetric information makes the agent (a tenant farmer) better off, by reducing the return to efforts by the principal (a landlord) aimed at reducing the tenant's reservation utility. By contrast, in the present case, asymmetric information never improves the welfare of either party, and makes both the insurer and the client strictly worse off whenever $0 < \alpha < 1$.

8 Concluding comments

This paper has explored the contracting behavior of clients and insurers under conditions of asymmetric information and differential bargaining power. The main focus of attention has been the interaction between differential bargaining power and two potential sources of departure from the first-best. The first, which is applicable to a wide variety of contracting situations, is that clients anticipating the need to deal with a insurer with monopoly power (or, more generally, with substantial bargaining power) will undertake costly self-protection to improve the outside option that will form the basis of subsequent bargaining. The second is the problem of moral hazard, in which the client has private information about the state of nature.

The crucial result is that differential bargaining power will affect not only the distribution of surplus but the total surplus generated.

The analysis has been undertaken using a state-contingent representation of production, developed by Chambers and Quiggin (2000). A key advantage of the state-contingent approach is the ease with which self-protection and asymmetric information can be represented. A variety of extensions of the analysis undertaken here might be considered. Problems of adverse selection might be modelled by allowing for a richer state space, with the client observing a signal, represented by a partition of the state space, prior to contracting. In addition, the possibility of action by the insurer designed to reduce the attractiveness of the outside option as in Chambers and Quiggin (2000) might be considered.

More generally, the analysis above suggests that principal–agent theory may usefully be applied to problems involving collective action, using the concept of state-contingent production. The allocation of risk is a central issue in collective choice, and the state-contingent framework makes this issue explicit.

9 References

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