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Equity between overlapping generations
by

John Quiggin
Australian Research Council Federation Fellow,
University of Queensland

Schools of Economics and Political Science
University of Queensland
Brisbane, 4072
rsmg@uq.edu.au
http://www.uq.edu.au/economics/rsmg

Equity between overlapping generations

John Quiggin
School of Economics,
University of Queensland,
Brisbane 4072 Australia
j.quiggin@uq.edu.au

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Abstract

This note is a demonstration that, in the presence of overlapping generations and under standard conditions for a social welfare ordering (Pareto optimality, transitivity, independence), the only ordering consistent with utilitarianism for all people currently alive at any given point in time is one based on weighting all people equally, regardless of their date of birth. In particular, this implies that, under reasonable conditions, the appropriate choice for the pure rate of social time preference is equal to zero.

1 Introduction

One of the longest running controversies in welfare economics has concerned the appropriateness of discounting future utility. As Schelling (1995) observes, the predominant view in the economics literature is that the appropriate discounting procedure should include a pure rate of time preference, reflecting individual preferences for utility in the present relative to utility in the future. However, beginning with Ramsey (1928), whose work is the starting point for formal analysis of intertemporal choices, many writers have rejected the inherent discounting of future utility as ethically unjustified, and this viewpoint is shared by many philosophical advocates of utilitarianism.

The inclusion or exclusion of a pure rate of time preference in the evaluation of social policy is a matter of great practical significance. With the most common choice of pure time preference rate (3 per cent), the discount factor for a period of 25 years is below 0.5 and for a period of 100 years the discount factor is below 1/16. In particular, assuming that the utility of life as opposed to death does not change over time, this implies that a policy that saved one life today would be justified even if it cost, with certainty, 15 lives a century in the future.

The treatment of pure time preference has critical implications for the assessment of responses to climate change. Stern (2007) argues that the costs of inaction on climate change are high enough to justify a program aimed at stabilising atmospheric levels of CO2 at 550 parts per million or lower. Nordhaus (2007) shows that this conclusion depends critically on the exclusion of inherent time preference from discounting, which, given logarithmic utility, leads to the use of a real discount rate approximately equal to the rate of consumption growth (around 2 per cent a year in Stern's analysis). Inclusion of inherent time preference leads to a preference for a 'climate-policy ramp' in which costly reductions in emissions are deferred to the future.[1]

Much of the debate on the question of whether a pure rate of time preference can be justified is concerned with determining the appropriate way to balance the interests of 'current' and 'future' generations. The central question, in this framing of the problem, is whether, and to what extent, members of the current generation have the right to allocate resources in their own favour, at the expense of unborn future generations.

The central point of this note is to observe that this way of posing the problem is invalid, because members of different generations are alive at the same time. Any policy that discounts future utility must discriminate

[1] Nordhaus justifies the use of a high discount rate, incorporating pure social time preference. However, as noted by Quiggin (2008), the real rate of interest on low-risk bonds is close to the (risk-free) discount rate proposed by Stern. The fact that the average rate of return to capital is significantly higher than the real bond rate is a manifestation of the equity premium puzzle, first noted by Mehra and Prescott (1985).

not merely against generations yet unborn but against the current younger generation. Assuming that members of any given generation are concerned about their own lifetime utility, rather than myopically concerned with current utility alone, a social allocation rule that incorporates pure time preference gives higher weight to the lifetime utility of earlier-born generations. Assuming a 3 per cent pure rate of time preference, as above, and 25 years between generations, the lifetime welfare of those aged 50 or more is valued twice as highly as the welfare of their children, and four times as highly as the welfare of their grandchildren, all of whom may be alive at the same time. This is obviously inconsistent with any form of utilitarianism in which all those currently alive are valued equally.

Furthermore, by the nature of overlapping generations, there is no point at which a coherent distinction between current and future generations can be drawn. In the absence of some general catastrophe, many children alive today will still be alive in 2100, at which time people already alive will reasonably be able to anticipate the possibility of survival beyond 2200.

The main formal contribution of this note is a demonstration that, in the presence of overlapping generations and under standard conditions for a social welfare ordering (Pareto optimality, transitivity, independence), the only ordering consistent with utilitarianism for all people currently alive at any given point in time is one based on weighting all people equally, regardless of their date of birth. In particular, this implies that, under reasonable conditions, the appropriate choice for the pure rate of social time preference is equal to zero. Some implications are derived, followed by some concluding comments.

2 Main Result

Consider an economy in discrete time $t = 1...T$ with overlapping generations, Generation t, born in period t, lives for two periods, t and $t+1$. Consumption for generation t is given by the pair $\mathbf{c}^t = (c^t_t, c^t_{(t+1)})$, where $c^t \in \Re^M$ is a vector of consumption goods. Preferences for generation t are represented by a utility function, additively separable over time, of the form

$$V^t = u_1\left(c^t_t\right) + u_2\left(c^t_{(t+1)}\right), \tag{1}$$

where u_i, $i = 1, 2$ are utility functions for consumption in lifetime period i. The functions u_i are independent of t. The special case

$$u_2(c) = bu_1(c), \forall c \in \Re^M, \tag{2}$$

where $b \leq 1$ is a discount factor, is of particular interest. In this case, consumption preferences do not change over the lifecycle, but individuals prefer consumption in lifetime period 1 (when they are young) to consumption in period 2 (when they are old).

We may also define aggregate utility in period t as

$$W^t = u_1\left(c_t^t\right) + u_2\left(c_t^{t-1}\right).$$

Given the assumption of additive separability, we may summarise social outcomes in terms of lifetime utility vectors $\mathbf{u}^t = \left(u_1^t, u_2^t\right) = \left(u_1\left(c_t^t\right), u_2\left(c_{(t+1)}^t\right)\right)$, $t = 1...T$ so that

$$\begin{aligned} V^t &= u_1^t + u_2^t \\ W^t &= u_1^t + u_2^{t-1}. \end{aligned}$$

It is a matter of convenience whether we undertake analysis

A utility profile $U = \left(\mathbf{u}^1, ., \mathbf{u}^t, ..\mathbf{u}^T\right)$ is a set of utility vectors, one for each generation. We define $(U_{-t}; \mathbf{u}) = \left(\mathbf{u}^1, ...\mathbf{u}^{t-1}, \mathbf{u}, \mathbf{u}^{t+1}...\mathbf{u}^T\right)$ to be the profile U with $\mathbf{u}^t$ replaced by $\mathbf{u}$.

We now consider the problem of defining a social ordering $\succeq$ on sets of utility profiles $\mathbf{U} \in \Re^{2\times T}$, with typical elements $U = \left(\mathbf{u}^1, ...\mathbf{u}^T\right)$. The time horizon may be taken to represent a point beyond which all consumption profiles converge, so that the social ordering is unaffected by the consumption of generations after after T. The ordering must satisfy the following conditions:

Axiom 1 *A.1 Pareto optimality: For $U, \hat{U}$ such that $\hat{u}_1^t + \hat{u}_2^t \geq u_1^t + u_2^t$, $t = 1, ...T$ with strict inequality for at least one t, $\hat{U} \succeq U$.*

Axiom 2 *A.2.Transitivity: If $U' \succeq U$, $U'' \succeq U'$ then $U'' \succeq U$.*

Axiom 3 *A.3 Independence: If $\hat{U} \succeq U$ where $\mathbf{u}^t = \hat{\mathbf{u}}^t$ then, for any $\mathbf{u}$, $\left(\hat{U}_{-t}; \mathbf{u}\right) \succeq (U_{-t}; \mathbf{u})$.*

Axiom 4 *A.4 Utilitarianism within periods: For $U, \hat{U}$ such that*

$$\hat{u}_1^t + \hat{u}_2^{t-1} \geq u_1^t + u_2^{t-1}$$

$t = 1...T$ with strict inequality for at least one t, $\hat{U} \succeq U$.

Axioms A.1–3 are standard. The crucial axiom is A.4 which requires that any change in consumption within a time period t that increases aggregate utility in that period must increase social welfare as measured by the ordering $\succeq$. That is, utilitiarianism must hold within periods. In interpreting this requirement, it is useful to consider the special case of preferences satisfying additive separability with a utility discount factor, given by equation (2). Under these preferences, all individuals derive more lifetime utility from consuming any given bundle when young than from consuming the same bundle when old. Hence, starting from an initial position of equality, a transfer of consumption from the current old generation to the current young generation will yield an increase in social welfare, just like a similar transfer of consumption from old to young in the course of the lifetime of a given generation.

Given these axioms it is straightforward to derive our main result:

Proposition 5 *For a social ordering satisfying 1-4, $\hat{U} \succeq U$ if and only if*
(i) $\sum_t \hat{V}^t = \sum_t \hat{u}_1^t + \hat{u}_2^t \geq \sum_t u_1^t + u_2^t = \sum_t V^t$
(ii)$\sum_t \hat{W}^t = \sum_t \hat{u}_1^t + \hat{u}_2^{t-1} \geq \sum_t u_1^t + u_2^{t-1} = \sum_t W^t$

Proof. Let

$$T^* = \max\left\{t : \mathbf{u}^t \neq \hat{\mathbf{u}}^t\right\}$$

The proof is by induction on T^*. For $T^* = 1$, the result follows immediately from Pareto-optimality. To clarify the exposition, we will first derive the result for the case $T^* = 2$.Choose $\mathbf{u}, \mathbf{u}' \in \Re^{2\times 2}$ such that

$$u_1^1 + u_2^1 + u_1^2 + u_2^2 \geq u_1'^1 + u_2'^1 + u_1'^2 + u_2'^2. \tag{3}$$

Define

$$\begin{aligned}
\mathbf{u}'' &= \left(u_1'^1, u_2'^1, u_1'^2 + \left(u_2'^2 - u_2^2\right), u_2^2\right) \\
\mathbf{u}''' &= \left(u_1'^1, u_2'^1 + \left(u_1'^2 - u_1^2\right) + \left(u_2'^2 - u_2^2\right), u_1^2, u_2^2\right).
\end{aligned}$$

Now

$$\mathbf{u}' \tilde{} \mathbf{u}'' \tilde{} \mathbf{u}'''$$

by A.1 and A.2 respectively. By (3),

$$u_1^1 + u_2^1 \geq u_1'^1 + u_2'^1 + \left(u_1'^2 - u_1^2\right) + \left(u_2'^2 - u_2^2\right).$$

and hence, by A.1, $\mathbf{u} \succeq \mathbf{u}'''$, so, by A.3, $\mathbf{u} \succeq \mathbf{u}'$. Now assume the result holds for $T^* = \tau$. Choose $\mathbf{u}, \mathbf{u}' \in \Re^{2\times(\tau+1)}$ such that

$$\sum_{i=1,2} \sum_{t=1}^{\tau+1} u_i^t \geq \sum_{i=1,2} \sum_{t=1}^{\tau+1} u_i'^t.$$

Following the same approach as in the case $T^* = 2$, we can define $\mathbf{u}'', \mathbf{u}'''$ such that

$$\begin{aligned}
& \mathbf{u}' \tilde{} \mathbf{u}'' \tilde{} \mathbf{u}''' \\
\mathbf{u}^{t+1} &= \mathbf{u}'''^{(t+1)} \\
\sum_{i=1,2} \sum_{t=1}^{\tau} u_i^t &\geq \sum_{i=1,2} \sum_{t=1}^{\tau} u_i'''^t
\end{aligned}$$

and the inductive step now follows from A.4. The equivalence with (ii) is immediate. ■

Proposition 1 contradicts the widely held intuition that if individuals prefer earlier to later consumption, respect for individual preferences implies that a similar preference should be incorporated in social preference orderings. As argued in the introduction, this intuition reflects a confusion between intertemporal preferences and life-cycle effects within generations. In life-cycle terms, a consistent preference for consumption earlier rather than later means a preference for consumption when young over consumption when old. This preference has implications for the allocation of resources over the life-cycle of any given generation, but it does not support a preference for later-born over earlier born generations. In fact, consistent preference for consumption when young over consumption when old implies that (given an initial position of constant consumption) a transfer of consumption in every generation from the currently old to the currently young will raise the welfare of all generations.

Corollary 6 *Consider utility profiles* $\mathbf{u}$ *and* $\hat{\mathbf{u}}$ *such that, for each time period* t*,*

$$u_1^t - \hat{u}_1^t = u_2^{(t-1)} - \hat{u}_2^{(t-1)} = \Delta^t.$$

That is, within each period, the change from $\mathbf{u}$ *to* $\mathbf{u}'$ *affects both generations equally*

Then $\mathbf{u}$ *is preferred to* $\mathbf{u}'$ *if and only if*

$$\sum \Delta^t > 0,$$

that is, if the undiscounted sum of utility differences is positive.

2.1 Additive separability with impatience

The results derived above are of particular interest for the case of additively separable preferences displaying utility discounting, represented in equation (2).

First, consider the social decision problem of setting $C = \left(\mathbf{c}^1, ...\mathbf{c}^T\right)$ to allocate a fixed quantity K of a single consumption good. Assume for simplicity that u displays constant relative risk aversion and define $\theta < 1$ such that, for any c,

$$\frac{u'(c)}{u'(\theta c)} = \beta.$$

It is easy to see that the optimal solution consistent with conditions A.1-4 must set

$$\begin{aligned} \mathbf{c}^t &= (\hat{c}, \theta\hat{c}), t = 1...T \\ \hat{c} &= \frac{K}{T(1+\theta)}. \end{aligned}$$

That is, with a fixed quantity to allocate, impatience means that each generation will choose a lifetime consumption path in which consumption is high in period 1, when individuals are young, and lower when they are old. However, since the lifetime utility of all generations is valued equally, the consumption of each generation is the same. This example illustrates the distinction between inherent preference, within a generation, for consumption early in life (which is consistent with Proposition 5) and inherent preference for one generation over another (which is not).

Next, suppose that the consumption vector c consists of a private consumption good x and a public good y, and that the utility functions are of the form:

$$\begin{aligned} u_1(x, y) &= w(x) + y \\ u_2(x, y) &= \beta(w(x) + y) \end{aligned}$$

for some function w. Now consider a unit transfer of the public good y from period t to period $t+1$. This will reduce the utility of generation t by $1-\beta$ utility units, since this generation is young in period t and old in period $t+1$. But this reduction in social welfare is exactly offset by the differences between the utility gain to generation $t+1$ (young in period $t+1$) and the utility loss to generation $t-1$ (old in period t). Hence, for given x, the optimal policy is that which maximizes the available quantity of the public good, regardless of when it is enjoyed.

Finally, consider the extension of the analysis above to the case of a continuous-time model with finitely-lived individuals. It is apparent that the axioms presented above can be modified to fit the continuous time case, and that the conclusion of Proposition 5 will apply, yielding the Ramsey rule of saving as the optimal social policy.

3 Concluding comments

The central insight of the overlapping generations model is that economic interactions between contemporaneous earlier-born and later-born generations are crucial to the determination of the time paths of output, consumption and investment. Yet discussion of intergenerational issues in discounting is largely based on a distinction between 'current' and 'future' generations that makes no sense in an overlapping generations model. This in turn has contributed to a confusion between individual preferences over current and future consumption, and social preferences regarding the equitable allocation of resources within a given generation.

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