RISK AND SUSTAINABLE
MANAGEMENT GROUP
WORKING PAPER SERIES

TITLE:

**Managing Option Trading Risk
with Greeks when Analogy
Making Matters**

AUTHOR:

Hammad Siddiqi

Working Paper: F13_2

Managing Option Trading Risk with Greeks when Analogy Making Matters

Hammad Siddiqi

h.siddiqi@uq.edu.au

There are various types of risk associated with trading options. Traders typically manage such risks with the help of various partial derivatives of option prices known as Greeks. Experimental and anecdotal evidence suggests that mental accounting matters in the valuation of options. Mental accounting changes the values of Greeks significantly with crucial implications for risk management. I show that for a call option, delta-risk is under-estimated, gamma risk is over-estimated, and the value-decay due to the passage of time is under-estimated. For a put option, all three types of risks are over-estimated. I also show that covered call writing is more profitable when mental accounting influences prices.

Keywords: *Mental Accounting, Greeks, Options, Delta, Gamma, Theta, Covered Call*

JEL Classification: G13, G12

Managing Option Trading Risk with Greeks when Mental Accounting Matters

Framing of assets into mental accounts has crucial implications for how they are perceived. Shefrin and Statman (1993) suggest that features of similarity play an important role in how assets are grouped into mental categories. A call option on the stock of X Company has two key features, one identifying it as a call option and the other identifying it as relating to the stock of the X Company. Investors are more likely to co-categorize the call option with a share of the X Company than with a share of another firm.

In fact, many professional financial traders consider a call option to be a surrogate for the underlying stock and advise investors to consider replacing the underlying with the corresponding call option.[1] It appears that they frame a call option in the same mental account as the underlying. Consequently, similar goals may be set for the two assets. In a series of laboratory experiments, it has been found that mental accounting matters for pricing financial options. The first such experiment in a binomial setting is Rockenbach (2004) who finds that the hypothesis of mental accounting of a call option with the underlying explains the data best, which implies that participants demand the same expected return from a call option as available on the underlying. Experiments reported in Siddiqi (2012) and Siddiqi (2011) explore this further and find that the mental accounting of a call option with its underlying is due to the similarity in payoffs between the two assets as adding a third risky asset with dissimilar payoffs has no effect. It appears that participants in laboratory markets consider a call option to be a surrogate for the underlying without receiving any coaching to this effect due to the similarity in their payoffs. Arguably, investors in financial markets are even more likely to consider a call a surrogate for the underlying as they receive such advice from professional traders.

In this paper, we investigate the implications of mental accounting for management of option trading risk. Framing a call option in the same mental account as the underlying due to similarity in their payoffs has crucial implications for risk management. Traders make us of various

---

[1] As illustrative examples, see the following:
http://ezinearticles.com/?Call-Options-As-an-Alternative-to-Buying-the-Underlying-Security&id=4274772,
http://www.investingblog.org/archives/194/deep-in-the-money-options/,
http://www.triplescreenmethod.com/TradersCorner/TC052705.asp,
http://daytrading.about.com/od/stocks/a/OptionsInvest.htm

option Greeks known as delta, gamma, theta, vega, and rho for managing risk that arises from trading options. I show how mental accounting changes the values of these Greeks. Mental accounting changes the values significantly, potentially leading to large losses due to incorrect hedging. So, if traders believe that mental accounting is influencing prices then they should use the updated values of the Greeks.

Mental accounting, a term coined in Thaler (1980), is a broad concept. Thaler (1999) defines mental accounting as a set of cognitive operations used by individuals to organize, evaluate and keep track of financial activities. There is no comprehensive theory of the set of restrictions people place on the creation of mental accounts. Following Shefrin and Statman (1993), I propose that features of similarity play a crucial role (see Tversky (1977)). To distinguish it from other types of mental accounting, I refer to mental accounting due to similarity judgment as analogy making

The binomial model developed by Cox, Rubinstein, and Ross (1979) is a powerful tool for approximating option prices especially when closed-form solutions are not available. In this paper, I refer to this binomial model as the binomial Black Scholes model to differentiate it from the binomial approximation of the mental accounting model, which I call the binomial mental accounting model. The Greeks (delta, gamma, theta, vega, and rho) are crucial for professional traders as they are the tools used for hedging trading risks. When analytical formulas are not available (for example, for American put options), these Greeks are typically numerically approximated via binomial trees.

I illustrate the changes in the values of the Greeks caused by mental accounting through a binomial model of a European call option that does not pay dividends before expiry. As a closed-form solution is also available for its price, I also provide updated analytical formulas for the Greeks adjusted for mental accounting. I also show that mental accounting via analogy making provides a new behavioral explanation for the popularity of covered call strategy (different from the explanation in Shefrin and Statman (1993), which relies on prospect theory and mental accounting).

Section 2 illustrates the different price predictions of the principle of no-arbitrage (basis for binomial and Black Scholes pricing) and the hypothesis of "mental accounting of call with underlying due to similarity judgment (analogy making)" through a simple example. Section 3 shows that the delta-hedging portfolios grow at different rates under the two cases. Section 4 shows how Greeks change with mental accounting in a binomial setting. Section 5 provides analytical formulas

3

for Greeks adjusted for mental accounting. Section 6 looks at the profitability of covered call strategy under mental accounting vs. no arbitrage pricing. Section 7 concludes.

2. Mental Accounting via Analogy Making: A Simple Example

To understand the role of analogy making in formation of a mental account and its consequences, consider an investor in a two state-two asset complete market world. The investor has initially put his money in the two assets: A stock (S) and a risk free bond (B). The stock has a price of $60 today. In the next period, the stock could either go up to $108 (the red state) or go down to $30 (the blue state). Each state has a 50% chance of occurring. The bond costs $60 today and it also pays $66 in the next period implying a risk free rate of 10%. Suppose a new asset "A" is introduced to him. The asset "A" pays $98 in the red state and $20 in the blue state. How much should the investor be willing to pay for it?

Finance theory provides an answer by appealing to the principle of no-arbitrage: *identical assets should offer the same returns.* Consider a portfolio consisting of a long position in S and a short position in 0.151515 of B. In the red state, S pays $108 and one has to pay $10 due to shorting 0.151515 of B resulting in a net payoff of $98. In the blue state, S pays $30 and one has to pay $10 on account of shorting 0.151515 of B resulting in a net payoff of $20. That is, payoffs from S-0.151515B are identical to payoffs from "A". Hence, according to the no-arbitrage principle, "A" should be priced in such a way that its expected return is equal to the expected return from (S-0.151515B). It follows that the no-arbitrage price for "A" is $50.90909.

In practice, constructing a portfolio that replicates "A" is no easy task. When simple tasks such as the one described above are presented to participants in a series of experiments, they seem to rely on analogy-making to figure out their willingness to pay. See Rockenbach (2004), Siddiqi (2011), and Siddiqi (2012). So, instead of trying to construct a replicating portfolio which is identical to asset "A", people find an actual asset similar to "A" and price "A" in analogy with that asset. That is, they rely on the principle of analogy: *similar assets should offer the same returns* rather than on the principle of no-arbitrage: *identical assets should offer the same returns.*

Asset "A" is similar to asset S. It pays more ($98) when asset S pays more ($108) and it pays less ($20) when asset S pays less ($30). Expected return from S is 1.15 $\left(\frac{0.5\times108+0.5\times30}{60}\right)$. According to the principle of analogy, A's price should be such that it offers the same expected return as S. That is, the right price for A is $51.30435.

In the above example, there is a gap of $0.395 between the no-arbitrage price and the analogy price. Rational investors should short "A" and buy "S-0.151515B". However, if we introduce a small transaction cost of 0.5%, then the total transaction cost of the proposed scheme exceeds $0.395, preventing arbitrage. The transaction cost of shorting "A" is $0.2565 whereas the transaction cost of buying "S-0.1515B" is $0.345 so the total transaction cost is $0.6019. Hence, in principle, the deviation between the no-arbitrage price and the analogy price may not be corrected due to transaction costs even if we assume perfect replication. Note that asset "A" is equivalent to a call option on "S" with a strike price of 10.

3. Binomial Mental Accounting Model vs. Binomial Black Scholes Model

In this section, the difference between the binomial mental accounting model and binomial Black Scholes model is illustrated with a three period numerical example. In the same example, we show how the Greeks take different values in the binomial mental accounting model when compared with the binomial Black Scholes model.

Consider a binomial model with the following parameter values: Up factor=2, Down factor=0.5, no. of binomial periods=3, risk free interest rate per binomial period=0.01, there are no dividends. The objective is to price a European call option with a strike of 90. The probability of up movement is 0.5. The current stock price is 100. Hence, the expected (gross) return per binomial period on the stock is 1.25.

Figure 1 shows the binomial model and the call prices under the two approaches. The price of call under the binomial Black Scholes approach is denoted by Call-NA, whereas the price of call under the mental accounting approach is denoted by Call-MA. The expiration values are denoted by Call. Call-NA is calculated by using the portfolio replication argument and using backward induction. Call-MA is calculated by equating the expected return on call with the expected return on

the underlying and using backward induction. The position in the risk free asset is denoted by B. The delta of Call-NA is denoted by x-NA, whereas x-MA denotes the delta of Call-MA.

Two things should be noted. 1) Call-MA is larger than Call-NA before expiry, if call prices are different from zero. 2) x-MA is larger than x-NA until one period before expiry when x-MA and x-NA become equal to each other.

Up	2
Down	0.5
Prob. Of Up	0.5
Strike	90
Interest Rate	0.01
Stock Price	100
E(Return)	1.25

			Stock	400	**Stock** 800
			x-NA	1	**Call** 710
			B	-89.1089	
			Call-NA	310.8911	
	Stock	200	x-MA	1	
	x-NA	0.912871	Call-MA	328	
	B	-53.7202			**Stock** 200
	Call-NA	128.854			**Call** 110
Stock 100	x-MA	0.946667	**Stock**	100	
x-NA 0.775924	Call-MA	148.8	x-NA	0.733333	
B -26.0701			B	-36.3036	
Call-NA 51.52234			Call-NA	37.0297	
x-MA 0.874667	**Stock**	50	x-MA	0.733333	
Call-MA 66.56	x-NA	0.493729	Call-MA	44	
	B	-12.221			**Stock** 50
	Call-NA	12.46544			**Call** 0
	x-MA	0.586667	**Stock**	25	
	Call-MA	17.6	x-NA	0	
			B	0	
			Call-NA	0	
			x-MA	0	**Stock** 12.5
			Call-MA	0	**Call** 0

Figure 1

6

Suppose one writes this call option. How can this position be hedged? If the binomial Black-Scholes model is correct, then the delta hedging portfolio, $S \times xNA - CallNA$, rebalanced every period leads to a risk-free return. The value of the delta-hedging portfolio at time-0 (when the stock price is 100) is: $100 \times 0.775924 - 51.52234 = 26.07006$. In the next period, the stock price can either go up to 200 or go down to 50. If it goes up, the value of the delta-hedging portfolio created earlier becomes: $200 \times 0.775924 - 128.854 = 26.3308$. If it goes down, the value of the portfolio becomes: $50 \times 0.775924 - 12.46544 = 26.3308$. As $^{26.3308}/_{26.07006} = 1.01$, the delta-hedging portfolio grows at a rate equal to the (gross) risk-free rate per binomial period.

Suppose the stock moves up in the next period, the relevant value of delta is now 0.912871. The value of the rebalanced delta-hedging portfolio is: $200 \times 0.912871 - 128.854 = 53.7202$.

In the following period, the stock price can either go up to 400 or go down to 100. If it goes up to 400, the portfolio value becomes: $400 \times 0.912871 - 310.8911 = 54.2574$. If it goes down, the portfolio value becomes: $100 \times 0.912871 - 37.0297 = 54.2574$. $^{54.2574}/_{53.7202} = 1.01$. Hence, once again, the portfolio grows at the risk-free rate per binomial period.

It is easy to see that the delta hedging portfolio (rebalanced every period) grows at the risk free rate r. Such dynamic hedging, in the continuous limit, leads to the Black Scholes option pricing formula.

If mental accounting determines prices then the relevant delta hedging portfolio is $S \times xMA - CallMA$. The value of delta hedging portfolio at the start when the stock price is 100 is 20.9067. This means that if an investor has written a call option for 66.56 and has shorted 0.874667 units of the underlying, then the value of the portfolio is 20.9067. Suppose, in the next period, the stock price goes up to 200. What is the value of the delta-hedging portfolio created earlier? The value is $200 \times 0.874667 - 148.8 = 26.1334$. If the stock price goes down to 50 instead, then the value is $50 \times 0.874667 - 17.6 = 26.1334$. That is, regardless of which state of nature is realized in the next period, the delta-hedging portfolio created a period earlier grows to the same value of 26.1334. Hence, the rate of growth of the delta hedging portfolio per period under mental accounting is $^{26.1334}/_{20.9067} = 1.25$, which is equal to the expected return on the underlying. This is no co-incidence. If mental accounting determines prices, then the delta-hedging portfolio

grows at a rate equal to the expected return on the stock throughout the binomial period (see Siddiqi (2013)). It has been shown that, if mental accounting determines call prices, then a new option pricing formula is obtained in the continuous limit (see Siddiqi (2013)).

What happens when delta-hedging is done while ignoring mental accounting? If mental accounting is influencing prices, then the true value of delta, at time-0, is 0.874667. If delta is incorrectly estimated to be 0.775924, then the value of the delta-hedging portfolio at time-0 is $100 \times 0.775924 - 66.56 = 11.0324$. In the next period, if the stock price goes up to 200, the value becomes: $200 \times 0.775924 - 148.8 = 6.3848$. If the stock price goes down to 50, the value becomes: $50 \times 0.775924 - 17.6 = 21.1962$. So, under-hedging creates a potential for loss if the stock price moves up.

4. Estimating Greeks in a Binomial Setting

To illustrate how the Greeks change when mental accounting influences prices, delta, gamma, and theta are numerically approximated under both the binomial Black Scholes model and the binomial mental accounting model. The same binomial tree is used as in the previous section. Let's denote an upward movement on a binomial tree with a superscript '+' and the downward movement on a binomial tree with a superscript '-'. Consecutive movements are denoted with consecutive signs. That is, '+-+' means an upward tick, followed by a downward tick, then an upward tick. It is well known that in the binomial tree considered here, the formulas for delta, gamma, and theta are:

$$delta = \frac{C^+ - C^-}{S^+ - S^-} \tag{1}$$

$$gamma = \frac{\left(\frac{C^{++} - C^{+-}}{S^{++} - S^{+-}}\right) - \left(\frac{C^{+-} - C^{--}}{S^{+-} - S^{--}}\right)}{\frac{S^{++} - S^{--}}{2}} \tag{2}$$

$$theta = \frac{C^{+-} - C}{2\Delta t} \tag{3}$$

The above formulas are used to calculate the values of respective Greeks and the results are reported in table 1.

Table 1		
Numerically approximated Partial Derivatives		
Greeks	**Binomial Mental Accounting**	**Binomial Black Scholes**
Delta	0.874667	0.775924
Gamma	0.00048	0.00056
Theta	-11.325	-7.24632

As table 1 shows, if mental accounting determines call prices and the binomial Black Scholes model is used to estimate the Greeks for hedging purposes, serious problems arise. As delta is underestimated, the naked call writing position would be under-hedged. The extent of time decay of the option is also significantly underestimated, whereas gamma is over-estimated.

As gamma of the stock is zero, buying or selling the stock does not affect it. So, a trader, interested in hedging away gamma risk, has to take an appropriate position in other options, often on the wrong side of the market. That is, the trader may have to buy overpriced options or sell underpriced ones to hedge away gamma risk. If gamma is over-estimated, such positions may be more aggressive than what is required to create an effective hedge, adversely affecting profitability. Hence, over-estimating gamma may trigger unnecessary trade in options.

If mental accounting is ignored, magnitude of theta is under-estimated. That is, the extent of time decay is under-estimated. So, a long position in a call option will lose value faster than what is anticipated with the passage of time.

5. Analytical Formulas for Greeks

As seen in section 3, the delta-hedging portfolio grows at a rate equal to the expected growth rate of the underlying. Siddiqi (2013) shows that, in the continuous limit, this results in the following option pricing formulas for a European call with no dividends:

$$C = SN(d_1^M) - Ke^{-(r+\delta)(T-t)}N(d_2^M) \tag{4}$$

$$\text{where } d_1^M = \frac{ln(S/K) + (r + \delta + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$\text{and } d_2^M = \frac{ln\left(\frac{S}{K}\right) + \left(r + \delta - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

S is the price of the underlying, K is the strike price, (T-t) is time to expiry, σ is the volatility of the underlying's returns, r is the risk-free rate, and δ is the risk premium on the underlying. The only difference between the Black Scholes formula for European call and the above formula is the appearance of δ in the above formula. That is, if the risk premium on the underlying is zero, the above formula converges to the Black Scholes formula.

The price of a European put option with mental accounting can be obtained via put-call parity:

$$Ke^{-(r+\delta)(T-t)}N(-d_2^M) - SN(-d_1^M) \tag{5}$$

The appearance of the risk premium on the underlying, δ, is the only difference between mental accounting and Black Scholes formulas.

Analytical formulas for Greeks with mental accounting are obtained by taking the appropriate partial derivatives of the above formulas.

5.1. Delta Risk and Consequences of Under-estimating it

Let C denote the price of a call option and let P denote the price of a put option. We use superscript M to denote mental accounting, and superscript B to denote pricing under Black Scholes assumptions. The following are the formulas for deltas under mental accounting:

$$\frac{\partial C}{\partial S} = N(d_1^M) \tag{6}$$

$$\frac{\partial P}{\partial S} = N(d_1^M) - 1$$

Where $d_1^M = \frac{ln(S/K) + (r + \delta + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$

The formulas for deltas under Black Scholes assumptions are:

$$\frac{\partial C}{\partial S} = N(d_1^B) \tag{7}$$

$$\frac{\partial P}{\partial S} = N(d_1^B) - 1$$

$$d_1^B = \frac{ln(S/K) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

As can be seen, the only difference between the two formulas is that the risk premium on the underlying appears in the mental accounting formulas. As long as the risk premium on the underlying is positive, mental accounting deltas are higher than the Black Scholes deltas (if we consider absolute values, then the delta of put under mental accounting is lower than the delta of put under Black Scholes). Consequences for individual options are immediate. For a call writing position, using Black Scholes delta leads to under-hedging as a smaller quantity of the underlying is bought. For a put writing position, using Black Scholes delta leads to over-hedging as a larger quantity of the underlying is shorted.

Often, traders are interested in portfolio deltas instead of individual deltas. The delta of a portfolio of option on the same underlying stock is equal to the sum of the deltas of each component option multiplied by the number of options held. A common market practice is to hold delta neutral portfolios Delta neutral portfolios are those portfolios whose values are not affected

by a relatively small change in the underlying's price. Incorrect estimation of delta has serious consequences for a typical strategy designed to take advantage of option mispricing.

To illustrate the risks involved, suppose a trader has created a position as shown in table 2 to take advantage of perceived mispricing. He believes that A and C are under-priced, whereas B is over-priced. He has purchased 5 units of A, 10 units of C, and has written 25 units of B to take advantage of mispricing. Hopefully, in the near future, the prices will converge to their theoretical values resulting in a large profit for him. Table 2 also shows hypothetical deltas under Black Scholes and mental accounting.

Table 2				
Delta Calculation for a Hypothetical Options Portfolio				
Option	**Type**	**Quantity Held**	**Delta (Black Scholes)**	**Delta (Mental Accounting)**
A	Call	5	0.6	0.63
B	Call	-25	0.65	0.6825
C	Put	10	-0.4	-0.37
$Portfolio\ delta\ (Black\ Scholes) = 5 \times 0.6 - 25 \times 0.65 + 10 \times (-0.4) = -17.25$				
$Portfolio\ delta\ (Mental\ Accounting) = 5 \times 0.63 - 25 \times 0.6825 + 10 \times (-0.37) = -17.6125$				

The trader, in our example, faces significant risks. As the portfolio delta is negative, if the stock price moves up before the price discrepancy in options is corrected, he stands to lose a significant amount of money. Suppose, he decides to protect himself from such risk by buying the underlying stock in sufficient quantity so that the portfolio becomes delta neutral. If each option is over 100 units of the underlying, he needs to buy 1725 units of the underlying under the Black Scholes values, whereas he is required to buy 1761.25 units under mental accounting to make his portfolio delta neutral. If mental accounting is influencing prices, and he uses Black Scholes delta values then his delta risk is not eliminated by buying 1725 units of the underlying and he can lose a significant amount of money if the stock price moves up slightly.

5.2. Gamma Risk and Consequences of Over-estimating Gamma

From put-call parity, it follows that the gammas for corresponding call and put options are the same. The formula for gamma under mental accounting is given by:

$$\gamma = \frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1^M)}{S\sigma\sqrt{T-t}} = \frac{\partial^2 P}{\partial S^2} \tag{8}$$

$$d_1^M = \frac{ln(S/K) + (r + \delta + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

If $\delta = 0$, then the mental accounting formula converges to the Black Scholes formula. If $\delta > 0$ then the gamma value under mental accounting is lower than the gamma value under Black Scholes. So, if mental accounting influences prices and the Black Scholes model is used to calculate gamma, the gamma values are overestimated.

Consider a hypothetical portfolio shown in table 3. Suppose the portfolio has been made delta neutral by taking an appropriate position in the underlying.

Table 3				
Gamma Calculation for a Hypothetical Options Portfolio				
Option	Type	Quantity Held	Gamma (Black Scholes)	Gamma (Mental Accounting)
A	Call	-50	0.015	0.011
B	Call	25	0.013	0.009
C	Put	-10	0.015	0.011
$Portfolio\ Gamma\ (Black\ Scholes) = -50 \times 0.015 + 25 \times 0.013 - 10 \times 0.015 = -0.575$				
$Portfolio\ Gamma\ (Mental\ Accounting) = -50 \times 0.011 + 25 \times 0.009 - 10 \times 0.011 = -0.435$				

As can be seen from the table, portfolio gamma has been estimated as -0.575 under the Black Scholes model, whereas the mental accounting value is -0.435. Even though the portfolio is delta neutral, there is gamma risk. As perceived by a risk manager using the Black Scholes gamma, if the stock price moves up slightly, the portfolio will be risk equivalently short by 57.5 stocks. If the stock price moves down slightly, the portfolio will be risk equivalently long by 57.5 stocks. Such a risk will

not be acceptable to a risk manager who has a tolerance for smaller units of stock equivalent risk. Hence, he may be forced to trade options on the wrong side of the market. That is, he may be forced to buy overpriced options and/or sell underpriced options adversely affecting profitability.

The underlying stock has a gamma of zero, so it cannot be used to eliminate gamma risk. Hence, one has little choice but to buy/sell expensive options to reduce gamma risk. Overestimating gamma may cause the risk to appear larger than what it actually is. This may result in unnecessary trade in expensive options in an attempt to reduce gamma risk.

5.3 Theta Risk and Consequences of Under-Estimating Theta Risk

The mental accounting formula for theta of a European call with no dividends is:

$$\frac{\partial C}{\partial t} = -\frac{SN'(d_1^M)\sigma}{2\sqrt{T-t}} - (r+\delta)Ke^{-(r+\delta)(T-t)}N(d_2^M) \tag{9}$$

$$d_1^M = \frac{ln(S/K) + (r + \delta + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2^M = \frac{ln(S/K) + (r + \delta - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

The mental accounting formula for theta of a European put with no dividends is:

$$\frac{\partial P}{\partial t} = -\frac{SN'(d_1^M)\sigma}{2\sqrt{T-t}} + (r+\delta)Ke^{-(r+\delta)(T-t)}N(-d_2^M) \tag{10}$$

Theta of a call option under mental accounting typically has a larger absolute value when compared with the Black Scholes theta, whereas, mental accounting theta of a put option is typically smaller (in absolute value) than the corresponding Black Scholes theta. Such miss-estimation leads to the following:

1) Call options decline in value with the passage of time faster than their anticipated decline rate under the Black Scholes assumptions. So, more upward movement in the underlying stock is needed to overcome the effect of passage of time if a long call is held. In contrast, if a short position in a call is created, and the underlying is not expected to do much, then the portfolio value rises faster than anticipated. So, by under-estimating the effect of passage of time on call options, a long call

14

position falsely appears more attractive, whereas, a short call position incorrectly appears less attractive than what it actually is.

2) For a put option, if mental accounting is influencing prices, then the time-decay is typically over-estimated under Black Scholes. So, a long position is a put option incorrectly appears less attractive and a short position falsely appears more attractive then what it should be. In fact, for deep in-the-money put options, there is a stronger chance that theta is positive than the corresponding chance under the Black Scholes model.

3) Strategies that combine a long position in call with a short position in a put option lose value at a faster rate than what is expected under the Black Scholes world. The Black Scholes model under-estimates the time decay in a call option and over-estimates the time decay in a put option so the strategy, Call – Put, incorrectly appears more attractive. Hence, a long synthetic (long call + short put with the same strike) loses value at a faster rate, especially in a sluggish market, than what is anticipated in the Black Scholes model.

4) The fact that time-decay of a call option is theoretically under-estimated implies that strategies that involve a short position in a call option are more profitable in practice than what is theoretically expected, especially when the underlying stock does not do move much. Perhaps, this can partly explain the popularity of covered call strategy. A covered call involves combining a writing position in a call option with a long position on the underlying. Under mental accounting, return from covered call is larger than the expected return from covered call under the Black Scholes model.

5.4 Vega and Rho Risks

Under mental accounting, as in the Black Scholes model, vega for a call option is equal to the vega of a corresponding put option:

$$\frac{\partial C}{\partial \sigma} = S\sqrt{T-t} \times N'(d_1^M) = \frac{\partial P}{\partial \sigma} \tag{11}$$

Where $d_1^M = \frac{ln(S/K)+(r+\delta+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$

15

As long as $\delta > 0$, vega under mental accounting is smaller than vega under the Black Scholes model with mental accounting vega converging to Black Scholes vega when $\delta = 0$.

Under mental accounting, call and put option rhos are:

$$\frac{\partial C}{\partial r} = (T - t)e^{-(r+\delta)(T-t)}KN(d_2^M) \tag{12}$$

$$\frac{\partial P}{\partial r} = -(T - t)e^{-(r+\delta)(T-t)}KN(-d_2^M)$$

Where $d_2^M = \frac{ln(S/K)+(r+\delta-\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$

In practice, vega and rho are not very useful risk measures as it is illogical to expect that a change in interest rates or a change in volatility does not change the price of the underlying.

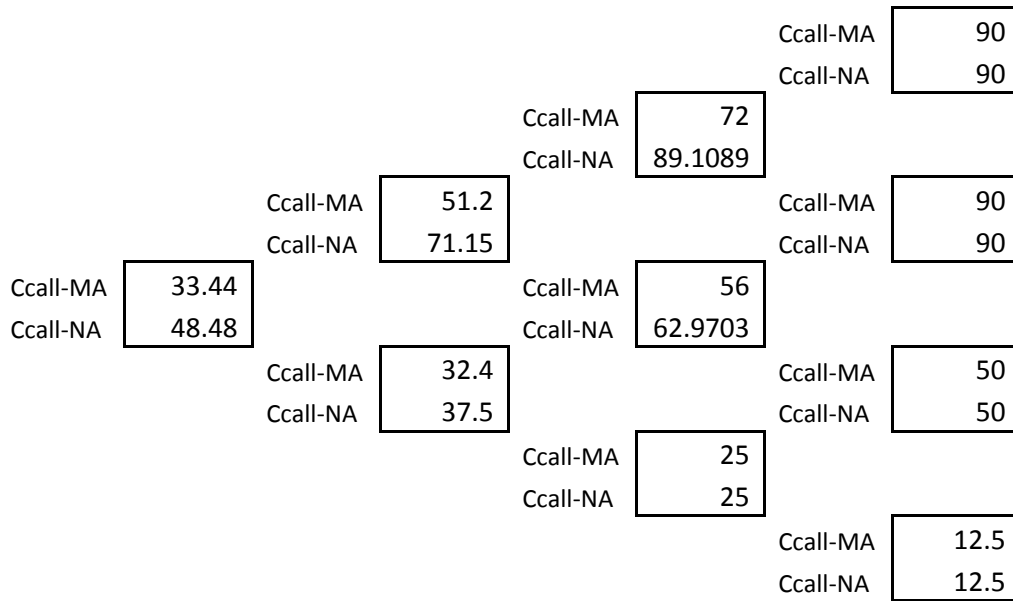## 6. The Profitability of the Covered Call Strategy

Covered call writing is surely one of the most popular investment strategies (if not the most popular strategy), widely used by both institutional as well as individual investors. It involves combining a long position on the underlying with a short position on a call option on the underlying. According to Lakonishok, Lee, Pearson and Poteshman (2007), a large percentage of calls are written as part of covered call strategies.

In the Black Scholes context, the popularity of covered call writing is puzzling. Covered call writing typically reduces risk, which implies a lower expected return if markets are efficient. Black writes, "*it is not correct to say that an investor can increase his expected return by writing a call option. In fact, he reduces his expected return because he creates a position in which he will come out ahead only if the stock does not move much.*" Black (1975, page 39).

Earlier empirical studies that investigated the profitability of covered call strategy found evidence confirming this view. See Booth et al (1985), Bookstaber and Clarke (1984), and Merton et al (1978). However, more recent studies have been more favorable to the covered call strategy and found that it can improve portfolio performance. See Morad and Naciri (1991), Whaley (2002), Feldman and Roy (2004), Constantinides et al (2008) among others.

Mental accounting of call with its underlying, as discussed here, provides an explanation for the popularity of covered call writing. Under mental accounting, covered call writing is equivalent to generating liquidity without affecting expected returns.

Figure 2 shows how the value of a covered call position changes along a binomial tree under mental accounting as well as under the no-arbitrage argument. Covered call under mental accounting is denoted by Ccall-MA, whereas covered call under no-arbitrage assumption is denoted by Ccall-NA. The same binomial tree as shown in figure 1 is used to calculate these values. The expected return per binomial period under mental accounting from the covered call strategy remains equal to 1.25 throughout the tree. Note that this is the same expected return as obtained from the buy and hold strategy in which only the underlying is bought. However, with covered call, less initial outflow is required, and the money saved can be invested elsewhere. So, a covered call strategy under mental accounting is equivalent to relaxing the liquidity constraint without affecting expected returns.

| | | | | | | Ccall-MA | 90 |
| | | | | | | Ccall-NA | 90 |

| | | | Ccall-MA | 72 | | | |
| | | | Ccall-NA | 89.1089 | | | |

| | Ccall-MA | 51.2 | | | | Ccall-MA | 90 |
| | Ccall-NA | 71.15 | | | | Ccall-NA | 90 |

| Ccall-MA | 33.44 | | Ccall-MA | 56 | | | |
| Ccall-NA | 48.48 | | Ccall-NA | 62.9703 | | | |

| | Ccall-MA | 32.4 | | | | Ccall-MA | 50 |
| | Ccall-NA | 37.5 | | | | Ccall-NA | 50 |

| | | | Ccall-MA | 25 | | | |
| | | | Ccall-NA | 25 | | | |

| | | | | | | Ccall-MA | 12.5 |
| | | | | | | Ccall-NA | 12.5 |

Figure 2: Covered Call Strategy under Mental Accounting vs. No Arbitrage Pricing

In contrast, under the no-arbitrage argument of the Black Scholes type, the covered call strategy generates considerably smaller returns. In figure 2, the expected return from covered call under no-arbitrage pricing varies from 1.01 to 1.25.

To understand the difference in profitability across the two approaches, suppose a long position on one unit of underlying is combined with a short position on one unit of a call option on the underlying. Assume that the call is written on one unit of the underlying. The value of such a covered call writing position at the time of its creation is given by:

$$V = w_1 S - w_2 C \tag{13}$$

Where w_1 and w_2 are positive weights such that $w_1 > 1$ and $w_1 - w_2 = 1$

Under Black Scholes assumptions, equation 11 becomes:

$$w_1 S - w_2 \{ SN(d_1) - Ke^{-r(T-t)} N(d_2) \}$$

$$=> (w_1 - w_2 N(d_1)) S + w_2 KN(d_2) e^{-r(T-t)} \tag{14}$$

To consider the impact of covered call writing in the Black Scholes world, initially assume that only the underlying is bought and held. This is equivalent to putting $w_1 = 1$ and $w_2 = 0$ in equation (14). Now, consider a situation in which a call option is also written on the underlying stock. As equation (14) shows, this amounts to reducing the weight of the stock and increasing the weight of the risk free asset in the portfolio. That is, the weight of the stock in the portfolio is now $w_1 - w_2 N(d_1)$ instead of 1, whereas the weight of the risk-free asset is now $w_2 N(d_2)$ instead of 0. The return on the risk-free asset is typically lower than the return on the stock, which means that the expected return from covered call writing should typically be lower than the expected return form just holding the underlying stock. With the passage of time and changes in the stock price, the respective weights of the underlying and the risk-free asset change, however, as long as there is a positive weight on the risk-free asset, the expected return from covered call should be lower than the expected return from just holding the underlying. As covered call writing is expected to reduce returns, the popularity and widespread use of covered call writing is quite puzzling in the Black Scholes context.

Under mental accounting, the formula for the price of a call option is given in equation (4). Substituting from (4) into (13) and simplifying leads to:

$$(w_1 - w_2 N(d_1^M))S + w_2 KN(d_2^M)e^{-(r+\delta)(T-t)} \tag{15}$$

As equation 15 shows, under mental accounting, covered call writing is equivalent to reducing the weight on the underlying, and increasing the weight on a hypothetical risk-free asset that offers the same return as the expected return on the stock. With the passage of time and changes in the stock price, the weights change, however, the expected return from covered call always equals the expected return from the underlying. Hence, if mental accounting determines call prices, then the popularity of the covered call strategy is not puzzling as it provides liquidity without sacrificing expected return.

7. Conclusions

A call option is typically considered a surrogate for the underlying stock. Such consideration may result in a call option being framed in the same mental account as the underlying. Consequently, similar returns may be expected from the two assets. In this article, I investigate the implications of mental accounting of a call with its underlying for risk management. There are various types of risks associated with option trading. These risks are expressed in the form of various partial derivatives of option prices known as Greeks. I show that if mental accounting is influencing prices and the Black Scholes model is used, then call delta-risk in under-estimated, call gamma-risk is over-estimated, and call time-decay is under-estimated. For a put option, delta-risk is over-estimated, gamma-risk is over-estimated, and put time-decay is over-estimated.

The article also provides a new explanation for the popularity of covered call writing. Under the Black Scholes assumptions, adding a call writing position to a long position on the underlying, known as covered call writing, typically lowers expected returns. That is, covered call writing provides liquidity by sacrificing some returns. In this context, the popularity of covered call writing is quite puzzling. If mental accounting is influencing call prices, then covered call writing generates liquidity without sacrificing expected return. That is, covered call writing is a significantly better strategy if mental accounting is influencing prices.

References

Black, F. "Fact and Fantasy in the Use of Options." *Financial Analysts Journal*, 31, (1975), pp. 36–41, pp. 61–72.

Bookstaber,R., and R.G.Clarke. "Option Portfolio Strategies: Measurement and Evaluation." *Journal of Business*, 57, (1984), pp. 469–492

Booth, J. R., A. Tehranian, and G. L. Trennepohl. "Efficiency Analysis and Option Portfolio Selection." *Journal of Financial and Quantitative Analysis*, 20, (1985), pp. 435–450.

Constantinides,G.M., J. C. Jackwerth, M. Czerwonko, and S. Perrakis. (2011), "Are Options on Index Futures Profitable for Risk Averse Investors? Empirical Evidence." *Journal of Fianance,* Vol. LXVI, No. 4, pp. 1407-1437.

Cox, J. C, Ross, J. S., and Rubinstein, M. (1979), "Option Pricing: A Simplified Approach", *Journal of Financial Economics,* pp.229-263.

Feldman, B., and D. Roy. (2004), *Passive Options-based Investment Strategies: The Case of the CBOE S&P 500 Buy Write Index.* Chicago: Ibbotson Associates.

Lakonishok, J., I. Lee, N.D. Pearson, and A.M. Poteshman. "Option Market Activity." *The Review of Financial Studies*, 20, (2007), pp. 813–857.

McIntyre, M. L., and D. Jackson. "Great in Practice, Not in Theory: An Empirical Examination of Covered Call Writing." *Journal of Derivatives and Hedge Funds*, 13, (2007), pp. 66–79.

Merton, R. C., M. S. Scholes, and M. L. Gladstein. "The Returns and Risk of Alternative Call Option Portfolio Investment Strategies." *Journal of Business*, 51, (1978), pp. 183–242.

Morard, B., and A. Naciri. "Options and Investment Strategies." *Journal of Futures Markets*, 10, (1990), pp. 505–517.

Rockenbach, B. (2004), "The Behavioral Relevance of Mental Accounting for the Pricing of Financial Options". *Journal of Economic Behavior and Organization,* Vol. 53, pp. 513-527.

Siddiqi, H. (2011), "Does Coarse Thinking Matter for Option Pricing? Evidence from an Experiment" *IUP Journal of Behavioral Finance*, Vol. VIII, No.2. pp. 58-69

Siddiqi, H. (2012), "The Relevance of Thinking by Analogy for Investors' Willingness to Pay: An Experimental Study", *Journal of Economic Psychology,* Vol. 33, Issue 1, pp. 19-29.

Siddiqi, H. (2013), "Mental Accounting: A Closed-Form Alternative to the Black Scholes Model", available at SSRN: http://ssrn.com/abstract=2340903 or http://dx.doi.org/10.2139/ssrn.2340903

Shefrin, H., and Statman, M. (1993), "Behavioral Aspects of the Design and Marketing of Financial Products", *Financial Management,* Vol. 22, No.2, pp. 123-134.

Thaler, R. (1999), "Mental Accounting Matters", *Journal of Behavioral Decision Making,* 12, pp. 183-206.

Thaler, R. (1980) "Toward a positive theory of consumer choice" *Journal of Economic Behavior and Organization*, 39-60 .

Tversky, A. (1977), "Features of Similarity", *Psychological Review,* pp. 327-352.

Whaley, R. E. "Return and Risk of CBOE Buy Write Monthly Index." *Journal of Derivatives*, 10, (2002), pp. 35–42.

PREVIOUS WORKING PAPERS IN THE SERIES

FINANCE

F12_1   Government Safety Net, Stock Market Participation and Asset Prices by Danilo Lopomo Beteto (2012).

F12_2   Government Induced Bubbles by Danilo Lopomo Beteto (2012).

F12_3   Government Intervention and Financial Fragility by Danilo Lopomo Beteto (2012).

F13_1   Analogy Making in Complete and Incomplete Markets: A New Model for Pricing Contingent Claims by Hammad Siddiqi (September, 2013).