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# Government Safety Net, Stock Market Participation and Asset Prices

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## Abstract

This paper studies the effect on equilibrium prices adventing from the presence of a safety net during financial crises. It is shown that, by inflating prices with the insurance provided through its intervention policy, a government might be sowing the seeds of a crisis that its intention is to prevent in the first place. The model developed is one with risk-neutral agents facing a static decision problem, under different (i) frameworks - with and without the possibility of intervention - and (ii) informational scenarios - imperfect, perfect and common prior information. Intervention occurs whenever the combined return of those in the market goes below a certain threshold. By having different frameworks and scenarios, the impact of the safety net on a diverse class of assets can be studied. Equilibrium prices are shown to be always higher in the framework with the possibility of intervention, regardless of the informational scenario. There is a limit on price inflation, however, since an equilibrium fails to exist in those instances where high prices indicate an intervention to be imminent. The effect on prices of an increase in the degree of uncertainty turns out to be dependent on the supply level of the asset.

**Keywords:** Financial crises; government intervention; market participation; bubbles.

**JEL Classification:** E32; E44; G01.

# 1 Introduction

During crises episodes, either financial or economic related, the course of action of the government is always subject to a great deal of controversy. No consensus is ever achieved between those who, on the one side, champion the idea of government intervention and those who, on the other side, believe in the self-correcting capacity of markets. From an *ex post* point of view, government intervention might be required, if the state of affairs is not to be aggravated; from an *ex ante* perspective, if government intervention is taken for granted whenever bad outcomes happen, risks might be incurred in excess.

The way the government behaves in crises episodes has an impact on the payoff of virtually any asset. The reason is that, at least to some degree, there is always a correlation between an asset's payoff and the *state of fundamentals* of the economy - those factors that indicate how well the economy is performing, and the very same factors the government aims at when intervening. Affecting the payoff, the government turns out to be important for the investment decision of agents, which in turn affects the demand for the asset and, consequently, its price. The asset's price is part of the state of fundamentals, causing the action of the government to feed back into itself. Figure 1 illustrates this process.

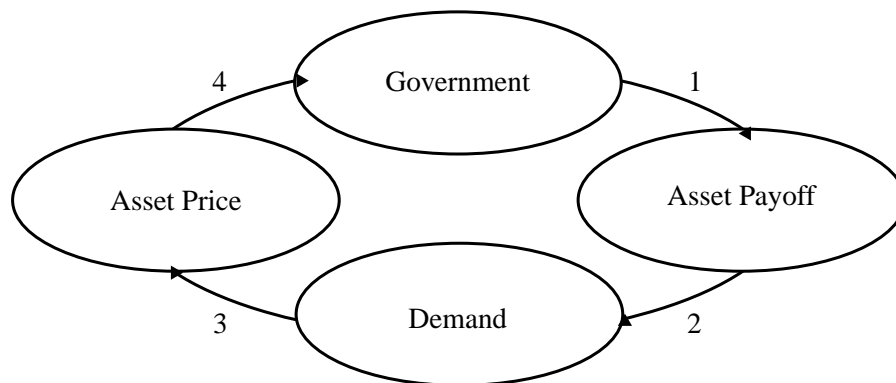


Figure 1: Impact of government intervention and feedback.

An example of the aforementioned process is the case of mortgage-backed securities (MBS) in the financial crisis episode of 2007-2009. MBS are securities whose payoff derives from a pool of mortgages, assembled together and issued as a single asset, in what is known as *securitization*. Securitization basically creates a secondary market for loans, which helps in channeling the necessary funds for financial institutions to create new mortgages.

Of prominent role in this secondary market is Fannie Mae, a company created by

the US government in 1938, with the goal of fostering the level of home ownership in the country. Initially established as a *government-sponsored enterprise* (GSE), it was converted into a publicly traded company in 1968. This change of ownership altered its government guarantee status, from being *explicit* pre-1968 to being *implicit* post-1968. Quoting the Wikipedia entry for Fannie Mae<sup>1</sup>,

Originally, Fannie had an “explicit guarantee” from the government; if it got in trouble, the government promised to bail it out. This changed in 1968. Ginnie Mae was split off from Fannie. Ginnie retained the explicit guarantee. Fannie, however, became a private corporation, with only an “implied guarantee”. There was no written documentation, no contract, and no official promise that the government would bail it out. The industry, government officials, and investors simply assumed it to be so.(...)

This implied government guarantee would constitute arc number 1 in Figure 1. Since the payoff of MBS are attached to the payment of the loans themselves, it is an instrument with a great deal of *credit risk*. Therefore, the support from the US government conferred to the securities traded by Fannie Mae a lot of appeal, increasing the demand for them - arc number 2. From Wikipedia,

However, the implied guarantee, as well as various special treatments given to Fannie by the government, greatly enhanced its success.

As the story goes, with an active secondary market for loans - and a fierce competition for new customers, in a business deemed profitable at the time - a plethora of credit became available to those willing to assume a mortgage. With the availability of credit came a decrease in the lending standards, setting the stage for a disaster: the increased demand for houses inflated a *bubble* - the boom, arc number 3 - and, when the high level of prices could not be sustained anymore - the bust - those disqualified borrowers had no ways of fulfilling their obligations. By the time, a great fraction of the mortgage market was owned by Fannie Mae, whose collapse would pose a serious threat to all those invested in assets like MBS - making government intervention inevitable, arc number 4.

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<sup>1</sup>[http://en.wikipedia.org/wiki/Fannie\\_Mae](http://en.wikipedia.org/wiki/Fannie_Mae).

(...)Fannie and Freddie underpinned the whole U.S. mortgage market. As recently as 2008, Fannie Mae and the Federal Home Loan Mortgage Corporation (Freddie Mac) had owned or guaranteed about half of the U.S.'s \$12 trillion mortgage market. If they were to collapse, mortgages would be harder to obtain and much more expensive. Fannie and Freddie bonds were owned by everyone from the Chinese Government, to Money Market funds, to the retirement funds of hundreds of millions of people. If they went bankrupt, all those investments would go to 0, and there would be mass upheaval on a global scale.(...)

As far as government intervention is concerned, not only the payoff of assets in a particular segment - e.g., mortgage securities held by Fannie Mae - but also the payoff of assets associated with a broader measure of the performance of the economy - e.g., stock market indices - matter. For, the latter has the potential of dragging the economy down - through the *expectations* channel - that is not necessarily present in the former. If the Dow Jones index succumbs to a large fall, for example, the *potential* losses that might result from the economy dipping into a recession - a situation that could arise as investors lose confidence and withdraw from the market - pose a threat serious enough to demand government intervention, regardless of how important the *realized* losses are.

In this way, government intervention can be associated with different classes of assets - all that matters is them being important from a systemic point of view, posing a risk that cannot be diversified away by the individual action of agents. The overall impact of the presence of the government - on the behavior of investors and, therefore, prices - is determined by the interaction of the characteristics of assets in different classes *with* that possibility of intervention. For, just to pick two possible dimensions, consider (i) payoff uncertainty and (ii) overall level of investment in the asset.

Regarding (i), investors might have a varying degree of information across assets in different classes - less information about the likes of mortgage securities, more information about the likes of the Dow Jones index. The degree of uncertainty associated to the payoff of an asset being higher, so more valuable one would expect the possibility of government intervention to be, as investors would have more safeguards against those not contemplated cases where the asset performs extremely poorly.

With respect to (ii), the more pervasive is the presence of the asset in the economy,

the more one would expect it to contribute to systemic risk, making the possibility of an intervention more valuable too. After all, were the asset to represent only a small fraction of the market, an intervention would not make much of a difference, unless for the handful of agents invested in it.

Implications like those outlined above, resulting from the possibility of government intervention and being reflected in the behavior of investors and prices, constitute the focus of the paper. The main goal is to study how safeguards from the government interact with different characteristics of assets - in particular, uncertainty - as far as the determination of equilibrium prices is concerned. With that, those effects that are a byproduct of the tension between creating more incentives for risk taking and preventing the economy from getting into a worse situation - tension which is inherent in the intervention decision of the government - can be analysed in more solid grounds.

In the setup to be presented, the measure that the government looks at when deciding to intervene is the *total welfare of investors*, to be called *social welfare* - defined as the sum across agents of the return on their portfolios. Whenever that measure goes below a *critical level*, the government intervenes and social welfare is restored to that level<sup>2</sup>.

Taking the possibility of government intervention into account, investors are asked to form their portfolios. First, agents are *risk neutral*, with utility adventing only from the proceeds of the investment strategy. Second, investment can be made in two assets, a *riskless* and a *risky* one. The riskless asset pays no interest and only allows agents to transfer wealth from the initial to the final period of the economy, whereas the risky asset has a payoff which is perfectly correlated with the state of fundamentals.

The state of fundamentals is modeled as a uniform random variable, the realization of which the government is assumed to know. To capture the variation in the degree of uncertainty attached to different assets, investors are placed in one of three possible informational scenarios, namely (i) *imperfect information scenario*, where each investor receives a private signal of the realized state of fundamentals, (ii) *perfect information scenario*, where every agent knows what the state of fundamentals happens to be, and (iii) *common prior scenario*, where all the agents know is the probability distribution of the state of fundamentals.

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<sup>2</sup>The paper abstracts from *how* government intervention is implemented, e.g., a decrease in the interest rate or a bailout of a specific institution. All that matters is that intervention results in the social welfare being set at the critical level.

To explore the consequences of government intervention, two *frameworks* are studied: one where investors entertain the possibility of intervention - labeled *government intervention* - and another where investors rule out that possibility from the outset - *no government*. To see how the possibility of intervention affects different types of assets - as far as uncertainty is concerned - for each of the frameworks there are three possible *scenarios*, regarding the information of agents and labeled as such, following the above - *imperfect information*, *perfect information* and *common prior*<sup>3</sup>.

As indicated by the diagram of Figure 2, for each possible combination of framework and scenario, the equilibrium price is derived. Then, for a given framework, the prices that prevail under each of the scenarios are compared - a comparison *intra framework*, *inter scenario*. In a similar fashion, for a given scenario, the prices that result under each of the frameworks are related - a comparison *inter framework*, *intra scenario*. As far as the ordering of equilibrium prices is concerned, the former provides a way of analysing the effects of uncertainty - for a given level of government intervention - whereas the latter focus on the effects of the possibility of government intervention - for a given level of uncertainty.

In the setup considered, a prerequisite for an equilibrium to exist is the market clearing condition being satisfied, i.e., the supply being equated to the demand. At the equilibrium price, the mass of investors willing to buy matches the quantity of the asset available - any deviation from that is a sign that the price is either *too high* or *too low*. Since investors know that (i) the government steps in only if social welfare falls below a critical level and (ii) the social welfare is roughly given by the total supply of the asset multiplied by the difference between its payoff (revenue) and its price (cost), that very same price at which investors are offered the asset indicates *how likely* the government is to intervene. By forming beliefs of the likelihood of an intervention, investors can calculate the expected return on any portfolio, which is crucial since, with risk neutral preferences, all that matters is the expected return - as far as the decision of choosing a portfolio is concerned.

Regarding the portfolio decision of agents, they are allowed to invest as much as they want in the riskless but not in the risky asset, conditional on their budget constraint being satisfied. The reason is that, rather than exploring the impact on prices of the trading activity of a particular investor - particularity that could arise from an agent

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<sup>3</sup>From now on, *framework* refers always to the presence or not of the government, whereas *scenario* is related to the degree of uncertainty faced by investors.

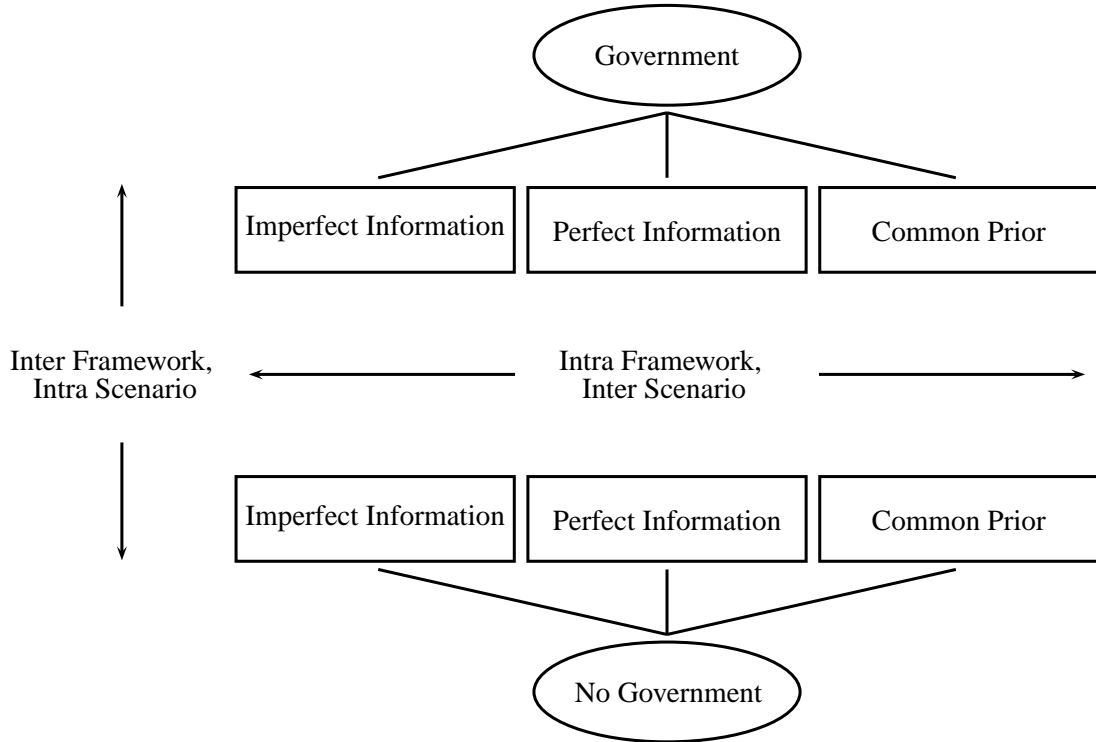


Figure 2: Effects from government intervention.

being wealthier or better informed than the others - by modeling the investment on the risky asset as a “buy” or “not buy” decision, one can focus on the effects of agents’ overall *participation* in the market, which can be as important a driver of prices as wealth or information<sup>4</sup>.

As it turns out - manifested in the expression of the equilibrium prices - the *downside risk* is one of the crucial factors determining the investment decision of agents. For, since utility derives only from *net* investment proceeds, agents need to contemplate how much of a relative loss a *worst case*-type of situation would entail, were an investment in the risky asset to be made. By relative loss it is meant the difference between (i) the total wealth that could be secured for consumption through a fully investment in the riskless asset and (ii) the level of consumption that would result in the aforementioned worst case situation - by definition, that where the government is required to intervene. This is precisely what a measure of the downside risk captures.

Since government intervention results in the state of fundamentals being set at the

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<sup>4</sup>This point is debatable but it is the view the paper takes on the issue, nonetheless. One could well argue that, for some types of assets, e.g., *illiquid* ones, trade is motivated primarily by information, so that, price-wise, the behavior of a well informed investor would be more relevant vis-a-vis the overall level of investment.



critical level, which is higher than what would prevail otherwise, and, being the state of fundamentals perfectly correlated with the payoff of the risky asset, intervention can be seen as an *insurance* - or, as a *safety net* - provided by the government. Intrinsicly associated with this insurance is, therefore, the downside risk: the higher the former, the lower the later. Different assets with different levels of downside risk can be interpreted as assets with different levels of government guarantee.

Not only those factors directly involved in the definition of the downside risk per unit of the asset - investors's *individual wealth*, government's *insurance* or *safety net* and *supply level* - but also others like the *state of fundamentals*, *range of uncertainty* and *transaction cost* matter for the determination of equilibrium prices. They appear in different ways and, as such, have different implications for the comparative statics of the equilibrium price, in accordance with the framework / scenario under question.

In terms of results, following Figure 2, a comparison of equilibrium prices *inter framework*, *intra scenario* reveals that (i) the possibility of government intervention has undoubtedly a positive effect on prices: no matter what the level of uncertainty, prices under the possibility of intervention are at least as high as those that prevail when government is absent, whereas a comparison of equilibrium prices *intra framework*, *inter scenario* shows that (ii) the possibility of government intervention does not alter the effect of uncertainty on prices: overall, with or without the possibility of intervention, prices in different informational scenarios obey a certain ordering, in accordance with the realized state of fundamentals. A result that holds under any framework / scenario specification is that (iii) no equilibrium can be sustained when investors are certain about government intervention.

Looking specifically to the framework with the presence of the government, the equilibrium derived in the imperfect information scenario shows that (iv) the equilibrium price increases with uncertainty for assets in low supply, but decreases otherwise, whereas that very same equilibrium price is (v) increasing in both the state of fundamentals and the government's safety net, while decreasing in the supply level, the individual wealth of investors and the transaction cost. In the perfect information scenario, the comparative statics are straightforward: (vi) the equilibrium price depends only on the state of fundamentals and the transaction cost, increasing in the former and decreasing in the later. Finally, in the common prior scenario (vii) the equilibrium price depends on the individual

wealth of agents: for sufficiently low / high levels, the wealth constraint is binding and the price that prevails equals that individual wealth level, whereas for intermediate levels the equilibrium price is increasing in both the supply level and the government's safety net, while decreasing in the individual wealth of agents and the transaction cost.

In the framework with no government, for the scenario with imperfect information, the effects of changes in the degree of uncertainty are also dependent on the supply level of the asset, as in the framework with the possibility of intervention: it is shown that (viii) the equilibrium price increases with uncertainty for assets in low supply, but decreases otherwise, whereas that very same equilibrium price is (ix) increasing in the state of fundamentals, while decreasing in the supply level and the transaction cost. In the perfect information scenario, the result is the same one obtained in the previous framework: (x) the equilibrium price depends only on the state of fundamentals and the transaction cost, increasing in the former and decreasing in the later. Finally, in the common prior case, (xi) the only variable that enters the expression of the equilibrium price is the transaction cost of the asset: investors are willing to pay the unconditional expected payoff of the risky asset minus the transaction cost that they are charged when the liquidation of the portfolio takes place.

## 1.1 Related Literature

If not directly studying the impact on prices, other papers also approach the implications adventing from government intervention in crises episodes<sup>5</sup>. Farhi and Tirole (2010) show that a policy whereby the government responds to crises decreasing the interest rate leads banks to choose a higher leverage ratio, with government intervention and maturity transformation being *strategic complements*. This implies that, with government intervention, the degree of maturity mismatch in the economy increases, leaving it more exposed to liquidity shocks.

In Diamond and Rajan (2011), however, the very same interest rate policy - an *undirected* intervention - turns out to be better than the alternatives, in particular a bailout of a specific institution - a *direct* intervention. The interest rate policy preserves the private commitment of banks with their depositors, a point stressed by Diamond and Rajan

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<sup>5</sup>The word government in the present work is used in a broad sense, being understood either as a country's Treasury or its Central Bank.

(2001). Relative to the natural equilibrium rate, since banks know that the interest rate will be lower at times of stress - when needs are high - in order to discourage the very same banks to make commitments that increase the need for intervention, the interest rate needs to be higher in normal times - when needs are low - which requires a *credibility in a new direction*, a term coined by the authors. Without this, a central bank might actually *increase* the likelihood of booms and busts.

Still related to the portfolio choice of financial institutions, Acharya, Shin, and Yorulmazer (2011) ask the question of how banks' ex ante choice of liquidity is affected by different government policies: (i) bailouts, (ii) unconditional liquidity support to surviving banks and (iii) liquidity support to surviving banks conditional on the level of liquid assets in their portfolios. The first two policies are shown to reduce banks' incentives to hold liquid assets, whereas those incentives increase with the third.

Acharya and Yorulmazer (2007) analyse the implications of government intervention policies that are designed to be implemented *only* in systemic crises - those that affect a significant portion of the banking industry. If that is the case, banks are induced to herd, i.e., correlate their portfolios, since only when they fail together they will be bailed out - otherwise they are just liquidated. This results in a *too-many-too-fail*-type of guarantee by the government, exacerbating the number of systemic banking crises. Interestingly, differently from a *too-big-too-fail* policy, also small institutions are susceptible to this effect.

Among others, Ennis and Keister (2009) study the ex ante effects of government intervention in the context of *bank runs*. The government policy that would rule out the run equilibrium in the Diamond and Dybvig (1983) setup, namely a deposit freeze, is shown to be ex post *inefficient*, i.e., once a run on a bank is under way, prohibiting withdrawals is not the best course of action for the government - the classic *time-inconsistency problem*, Kydland and Prescott (1977). Anticipating that, depositors have incentives to participate in a run once one takes place, implying that ex post efficient policies might have a destabilizing effect.

The presence of a *lender of last resort*, e.g. the IMF, is another instance where the private motives of agents might be distorted, eventually leading to *moral hazard*. In situations where a country is illiquid, Morris and Shin (2006) identify the circumstances under which an IMF bailout would convince investors to roll-over their maturing claims

and also the country being helped to engage in the costly adjustment effort. Similar results are obtained by Corsetti, Guimarães, and Roubini (2006), where the trade-off between bailouts and moral hazard is also central.

One paper that addresses the issue of investors' participation in the market is Allen and Gale (1994). In order to explain asset-price volatility, Allen and Gale develop a model based on *incomplete market participation* and *heterogenous liquidity preferences* of investors. According to their theory, a fixed setup cost might prevent some investors from trading in a market so that, when a liquidity shocks hits the economy, it cannot be absorbed without causing a large change in prices. In their model, limited market participation is caused by the heterogeneity in investors' liquidity preferences whereas in the present paper the driver is the heterogeneity in the information received: when investors have imperfect information, only those with a *favorable* outlook on the asset payoff will choose to participate in the market. Similarly to their model, however, is the result that different equilibria with different degrees of investors' participation can be accomodated, resulting in diverse equilibrium prices.

The structure of the paper is as follows: section 2 introduces the model, with the corresponding measure of social welfare used by the government to decide whether to intervene or not in the economy; section 3 focus on the framework where investors entertain the possibility of government intervention, with the following subsections dealing with the different informational scenarios: imperfect information, perfect information and common prior; analogously, section 4 refers to the framework where investors acknowledge the absence of the government; section 5 compares the equilibrium prices across frameworks (with or without the government) and across scenarios (imperfect, perfect and common prior information); section 6 concludes. The derivation of some of the results is contained in the appendix.

## 2 Model

The model is composed of a continuum of agents, represented by the unit mass interval,  $I = [0, 1]$ , facing a static decision problem. There are four dates,  $t = 0, 1, 2, 3$ :

- At  $t = 0$ , the state of fundamentals is drawn, with  $\tilde{\theta} \sim U[0, 1]$ ;
- At  $t = 1$ , agents decide whether to buy or not,  $X_i = 1$  or  $X_i = 0$ , respectively, a sin-

gle unit of an asset that has a payoff equal to the realized state of fundamentals, with the degree of information about the asset's payoff depending on the informational scenario:

- *Imperfect information scenario*: investors receive a signal about the realized state of fundamentals;
  - *Perfect information scenario*: investors are informed of the asset's payoff;
  - *Common prior scenario*: investors only know the probability distribution of the state of fundamentals, assumed to be common knowledge.
- After agents make their investment decisions, at  $t = 2$  the stage is set for the possibility of government intervention, which depends on the *framework*:
    - *Government framework*: the government anticipates the social welfare level (to be defined) that results from investors' strategies and decides whether or not to intervene;
    - *No government framework*: no intervention ensues, regardless of the social welfare level, with the third period being just a convention.
  - At  $t = 3$ , agents liquidate their portfolios and consume the proceeds from the corresponding investment strategies, which turns out to be dependent on the outcome of the previous date (intervention or not by the government).

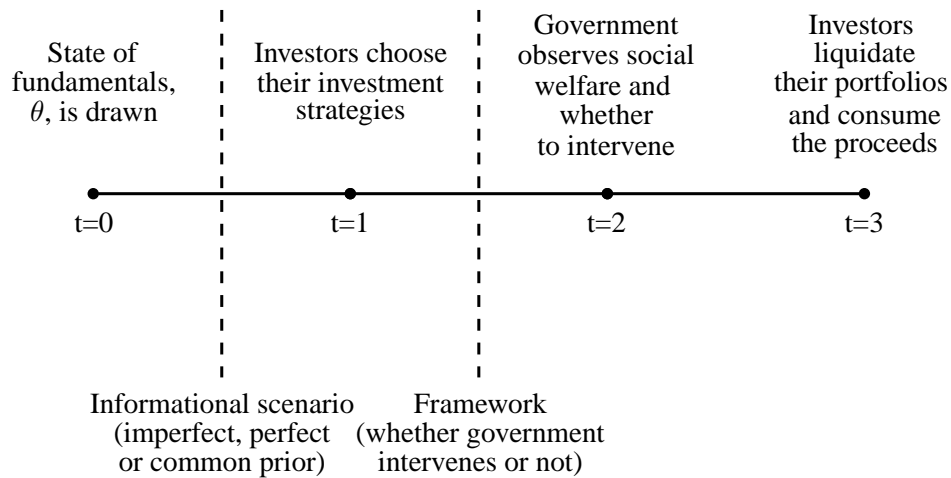


Figure 3: Timeline of events.

Figure 3 illustrates the timeline of events. The asset is in total supply of  $K$ , and every agent is initially endowed with wealth  $A$ . In case the price of the asset is  $p$ , buying is affordable if and only if  $A \geq p$ . An agent that buys the asset at the second period has to liquidate it in the final one, incurring a transaction cost,  $t$ . Without buying, an agent just carries over her initial endowment to the end of the economy. All the consumption occurs at the final date, determined by the investment strategy chosen. Agents are risk neutral and the utility from consumption is equal to the proceeds from the portfolio, written as

$$R_i(X_i, \theta, A, t) \equiv X_i(\theta - t) + A - X_i p = X_i(\theta - p - t) + A, \quad \forall i \in I. \quad (1)$$

If an agent chooses to buy,  $X_i = 1$ , she gets the payoff from the asset minus the transaction cost to be paid,  $\theta - t$ , plus whatever is left from the initial endowment after the asset was purchased,  $A - p$ . On the other hand, by not buying,  $X_i = 0$ , an agent keeps her total endowment,  $A$ , for later consumption.

In the framework with the presence of the government, intervention occurs whenever social welfare goes below a certain threshold,  $C$ . Social welfare,  $S$ , is defined as the sum across agents of the proceeds from their investment strategies, *before any government intervention whatsoever*,

$$S(\theta, A, t) \equiv \int_0^1 [X_i(\theta - p - t) + A] di. \quad (2)$$

The condition for intervention is

$$S(\theta, A, t) < C \quad (3)$$

and, if that is the case, the government is assumed to act<sup>6</sup> in such a way that results in the social welfare being restored to  $C$ . Market clearing condition,  $\int_0^1 X_i di = K$ , implies that, when there is government intervention, the state of affairs is equivalent to the one that would prevail in case the realization of  $\tilde{\theta}$  was the one implicitly given by

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<sup>6</sup>Government intervention could take the form of a reduction in the interest rate or a bailout of a particular agent, for example.

$$\begin{aligned}
S(\theta, A, t) &= C \\
\Leftrightarrow \int_0^1 [X_i(\theta - p - t) + A] di &= C \\
\Leftrightarrow (\theta - p - t) \int_0^1 X_i di + A \int_0^1 di &= C \\
\Leftrightarrow (\theta - p - t)K + A &= C \\
\Leftrightarrow \theta = \frac{C-A}{K} + p + t &\equiv \theta^*. \tag{4}
\end{aligned}$$

In other words, the condition for government intervention amounts to the realization of  $\tilde{\theta}$  being sufficiently low,  $\theta < \theta^*$ . The interesting case is  $A > C$  for, whenever the government steps in, the resulting state of affairs is such that agents cannot fully recover the amount invested: from (4), conditional on government intervention, the strategy of buying the asset yields  $\theta^* - p - t = (C - A)/K < 0$ . Not only that, if the case was  $A < C$ , investors' problem in face of the possibility of government intervention would be less relevant: whatever the strategy chosen, the resulting outcome would always be positive. Not only that,  $A < C$  would imply government intervention even when no one invests,  $X_i = 0, \forall i \in I$ , which is not plausible given the chosen measure of social welfare.

That investors lose “money” when there is intervention is something that one would expect: as common sense dictates, intervention is to be associated with bad states. As such, it is not reasonable to have investors making positive profits even at those states. Indeed, if an agent decides to invest,  $X_i = 1$ , and the realized  $\theta$  is one that demands government intervention, the proceeds from the investment strategy are, following (1),

$$\begin{aligned}
R_i(1, \theta^*, A, t) &= \overbrace{\frac{C-A}{K} + p + t}^{\theta^*} - p - t + A \\
&= \frac{C-A}{K} + A, \quad \forall i \in I. \tag{5}
\end{aligned}$$

Since  $A > C$  and any agent  $i \in I$  has the option of saving the entire wealth  $A$  for future consumption, anyone who had chosen  $X_i = 1$  instead of  $X_i = 0$  regrets having made such a choice, upon observing an intervention: while the former strategy gives utility  $(C - A)/K + A$ , the later would had delivered  $A$ ; with  $A > C$ , it follows that  $A > (C - A)/K + A$ . Intervention by the government is synonymous of a bad choice when that choice was to buy the asset.

Summarizing, the problem each agent faces is to build a portfolio, acknowledging that the resulting payoff depends not only on state of fundamentals of the economy but also on the decision of the government to intervene or not, which is in turn a function of social welfare. The framework where investors are aware of the possibility of government intervention is examined in turn.

### 3 Government Intervention

In the framework with the presence of the government, investors are aware of the possibility of intervention. Since government's action affects the payoff of the risky asset, when building their portfolios, agents need to determine the probability of that intervention happening. Beliefs are formed in accordance with the informational scenario at hand:

- Imperfect information: investors receive private signals about the realized state of fundamentals, inferring the probability of government intervention from the signal obtained;
- Perfect information: investors know what the state of fundamentals happens to be, immediately concluding if there will be or not intervention, according to the price at which the asset is being offered;
- Common prior: all that is known is the probability distribution of the state of fundamentals, being that the only artefact used by agents to calculate the probability of intervention.

Each of the above scenarios is examined in turn, starting with the imperfect information case of the next section.

#### 3.1 Imperfect Information About the Fundamentals

In the case of imperfect information about the state of fundamentals, each investor  $i$  receives a private signal  $\xi_i$  about the realized value of  $\tilde{\theta}$ , denoted by  $\theta$ . Signals are uniformly distributed around this realized value,  $\tilde{\xi} \sim \mathbb{U}[\theta - \tau, \theta + \tau]$ , with  $\tau > 0$ . Agents know that  $\theta$  is at most  $\tau$  units away from the signal received,  $\theta \in [\xi_i - \tau, \xi_i + \tau], \forall i \in I$ . Investors' problem is



$$\begin{aligned} \max_{X_i} \quad & \mathbb{E} \left[ X_i \left( \tilde{\theta} - p - t \right) + A \mid \xi_i \right] = X_i \left[ \mathbb{E} \left( \tilde{\theta} \mid \xi_i \right) - p - t \right] + A \\ \text{s.t.} \quad & A \geq p \quad \text{if } X_i = 1, \quad \forall i \in I. \end{aligned} \quad (6)$$

Agent  $i$ 's signal  $\xi_i$  conveys information not only about  $\theta$  but also about the likelihood of government intervention. When deciding whether to buy or not the asset, each agent asks what is the probability that  $\theta < \theta^*$ , conditional on  $\theta \in [\xi_i - \tau, \xi_i + \tau]$ . This probability in turn depends on the price of the asset, since  $\theta^*$  is a function of  $p$ :  $\theta^* = (C - A) / K + p + t$ . For a given  $p$ , the situation a particular investor faces falls into one of three possible cases: for  $\forall i \in I$ , either

$$(I) \quad \frac{C-A}{K} + p + t \leq \xi_i - \tau ;$$

$$(II) \quad \xi_i - \tau < \frac{C-A}{K} + p + t \leq \xi_i + \tau ; \text{ or}$$

$$(III) \quad \xi_i + \tau < \frac{C-A}{K} + p + t.$$

In (I), since  $\theta \in [\xi_i - \tau, \xi_i + \tau]$ , it is clear that  $(C - A) / K + p + t \leq \theta$ . Therefore, conditional on the agent facing a price  $p$  and having received a signal  $\xi_i$  such that (I) holds, she knows that, at the current price, there will be no government intervention. In that case the expectation of the asset payoff,  $\tilde{\theta}$ , is given by

$$\begin{aligned} \mathbb{E} \left( \tilde{\theta} \mid \xi_i \right) &= \frac{1}{2\tau} \int_{\xi_i - \tau}^{\xi_i + \tau} \theta d\theta \\ &= \xi_i. \end{aligned} \quad (7)$$

In range (II), investors entertain the possibility of government intervention and, conditional on that, they calculate the expected value of  $\tilde{\theta}$  as

$$\begin{aligned} \mathbb{E} \left( \tilde{\theta} \mid \xi_i \right) &= \frac{1}{2\tau} \left[ \int_{\xi_i - \tau}^{\frac{C-A}{K} + p + t} \theta^* d\theta + \int_{\frac{C-A}{K} + p + t}^{\xi_i + \tau} \theta d\theta \right] \\ &= \frac{1}{2\tau} \left\{ \left( \frac{C-A}{K} + p + t \right) \left[ \frac{1}{2} \left( \frac{C-A}{K} + p + t \right) - (\xi_i - \tau) \right] + \frac{1}{2} (\xi_i + \tau)^2 \right\}. \end{aligned} \quad (8)$$

Finally, facing (III) any agent  $i$  knows the government will step in, yielding

$$\begin{aligned} \mathbb{E} \left( \tilde{\theta} \mid \xi_i \right) &= \frac{1}{2\tau} \int_{\xi_i - \tau}^{\xi_i + \tau} \theta^* d\theta \\ &= \frac{C-A}{K} + p + t. \end{aligned} \quad (9)$$

Given all the three possible scenarios, the objective function in the investors' maximization problem,  $X_i \left[ \mathbb{E} \left( \tilde{\theta} \mid \xi_i \right) - p - t \right] + A$ , can be rewritten as

$$\begin{aligned}
& X_i \left\{ \mathbb{I}_{(I)} \xi_i + \mathbb{I}_{(II)} \frac{1}{2\tau} \left\{ \left( \frac{C-A}{K} + p + t \right) \left[ \frac{1}{2} \left( \frac{C-A}{K} + p + t \right) - (\xi_i - \tau) \right] + \frac{1}{2} (\xi_i + \tau)^2 \right\} \right. \\
& \left. + \mathbb{I}_{(III)} \left( \frac{C-A}{K} + p + t \right) - p - t \right\} + A \\
& \equiv U_i (X_i, \xi_i, A, p), \quad \forall i \in I,
\end{aligned} \tag{10}$$

where  $\mathbb{I}_{(I)}$  is the indicator function for case (I), namely  $\mathbb{I}_{(I)} = 1$  if  $(C - A) / K + p + t \leq \xi_i - \tau$  and zero otherwise. Similar for cases (II) and (III).

Since any investor must satisfy a budget constraint in deciding to buy,  $A \geq p$ , and given that the entire wealth can be secured for consumption in the final period - without buying, agents are sure to get utility  $A$  - the optimal strategy is to choose  $X_i = 1$  in case the following two conditions are satisfied:

$$\begin{cases} U_i (1, \xi_i, A, p) \geq A; \\ A \geq p. \end{cases} \tag{11}$$

Plugging in (11) the expression for  $U_i (\cdot)$  defined in (10) yields

$$\begin{aligned}
& \mathbb{I}_{(I)} \xi_i + \mathbb{I}_{(II)} \frac{1}{2\tau} \left\{ \left( \frac{C-A}{K} + p + t \right) \left[ \frac{1}{2} \left( \frac{C-A}{K} + p + t \right) - (\xi_i - \tau) \right] + \frac{1}{2} (\xi_i + \tau)^2 \right\} \\
& + \mathbb{I}_{(III)} \left( \frac{C-A}{K} + p + t \right) \geq p + t.
\end{aligned} \tag{12}$$

Hence, if it is feasible to buy the asset -  $A \geq p$  - condition (12) is the pivotal one for the buying decision. Among other things, the criterion for choosing  $X_i = 1$  is dependent on  $p$ . At the equilibrium price, the fraction of agents in the interval  $[0, 1]$  that satisfies (12) must be equal to  $K$ , the total supply. The price adjustment process - not modeled in the paper - can be thought of as a *price tâtonnement process*: at certain price levels demand is higher than supply, at others the converse is true: only at the equilibrium price demand equals supply, and from an out-of-equilibrium situation the price evolves until an equilibrium is reached.

It is argued in the sequence that an equilibrium (to be defined) can be sustained for different levels of government intervention, i.e., there are multiple equilibria associated with different levels of  $\theta^*$ . Since the only variable that is endogenously determined in the definition of  $\theta^*$  is  $p$ , the statement above is equivalent to saying that different price levels can be supported in equilibrium. By definition, the higher  $\theta^*$  the higher the equilibrium  $p$  is and - as the government

intervenes only if  $\theta < \theta^*$  - this implies the equilibrium price of the asset being higher the more likely government intervention is to occur.

Six intervals where  $\theta^*$  may lie can be identified, namely:

- (i)  $0 \leq \frac{C-A}{K} + p + t \leq \theta - 2\tau \Leftrightarrow 0 \leq \theta^* \leq \theta - 2\tau$ ;
- (ii)  $\theta - 2\tau < \frac{C-A}{K} + p + t \leq \theta - \tau \Leftrightarrow \theta - 2\tau < \theta^* \leq \theta - \tau$ ;
- (iii)  $\theta - \tau < \frac{C-A}{K} + p + t \leq \theta \Leftrightarrow \theta - \tau < \theta^* \leq \theta$ ;
- (iv)  $\theta < \frac{C-A}{K} + p + t \leq \theta + \tau \Leftrightarrow \theta < \theta^* \leq \theta + \tau$ ;
- (v)  $\theta + \tau < \frac{C-A}{K} + p + t \leq \theta + 2\tau \Leftrightarrow \theta + \tau < \theta^* \leq \theta + 2\tau$ ; and
- (vi)  $\theta + 2\tau < \frac{C-A}{K} + p + t \leq 1 \Leftrightarrow \theta + 2\tau < \theta^* \leq 1$ .

The implications of  $\theta^*$  belonging to each of the six ranges are discussed in turn. As a remark, appropriate conditions need to be imposed on the primitives of the model, in accordance with the interval where  $\theta^*$  is to be located in equilibrium<sup>7</sup>.

In ranges associated with a higher probability of government intervention, i.e., with a higher  $\theta^*$ , one would expect to be easier to obtain an equilibrium - after all those are the cases where investing in the asset is supposedly safer. However, it is also true for those cases that the price to be paid in order to acquire the asset,  $p$ , is higher, making the buying decision less attractive. It is the interaction of these two opposite effects that ultimately determines the overall likelihood of having an equilibrium in a particular interval.

Moving from interval (i) to (vi), agents' uncertainty changes from being related to the *possibility of government absence* to being related to the *possibility of government intervention*. From interval (i) to (iii), accurate signals indicate that the government *will not* intervene, whereas from (iv) to (vi), the more accurate the signal, the more one is certain that the government *will* intervene.

In intervals (i), (ii) and (iii), no government intervention ensues, with no agent believing that such an action would certainly occur. The mass of agents knowing exactly what the government behavior will be gets progressively *smaller*, as one goes from (i) to (iii). For instance, in (i) everyone knows the government will not intervene, in (ii) a fraction of the investors knows there will not be intervention and in (iii) relatively fewer agents can claim to be certain the government will not intervene - even though in equilibrium this is what happens.

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<sup>7</sup>The conditions for each of the six intervals are derived in a supplemental material to this paper and available upon request.

In intervals (iv), (v) and (vi), there *is* government intervention, with no agent being able to rule out such possibility. Differently from the previous intervals, however, the mass of agents who know exactly what the government behavior will be gets progressively *larger*, as one moves from (iv) to (vi). In (iv) some agents can claim to know for sure the government will intervene, in (v) a relatively larger fraction of the investors knows there will be intervention and in (vi) everyone knows the government will intervene, indeed.

In order to make the above argument formal, recall that intervention ensues if and only if  $\theta \in [0, \theta^*)$ , with agents receiving signals distributed as  $\tilde{\xi} \sim \mathbb{U}[\theta - \tau, \theta + \tau]$  and acknowledging that  $\theta \in [\xi_i - \tau, \xi_i + \tau]$ . Individually, the *range of uncertainty* of each agent  $i \in I$  is  $[\xi_i - \tau, \xi_i + \tau]$ . Taking the union of intervals where each investor contemplates the presence of  $\theta$  yields  $\bigcup_i [\xi_i - \tau, \xi_i + \tau] = [\theta - 2\tau, \theta + 2\tau]$ , called *range of group uncertainty*. Following that, in interval (i) no agent believes in government intervention, since  $[0, \theta^*) \cap [\theta - 2\tau, \theta + 2\tau] = \emptyset$ : everyone is aware of the fact the government will not step in. In (ii) and (iii), on the other hand,  $[0, \theta^*) \cap [\theta - 2\tau, \theta + 2\tau] \neq \emptyset$ , and, as  $\theta^*$  increases as one moves along the intervals, the fraction of agents unaware of the fact the government does not intervene becomes larger. In intervals (iv) and (v), no agent can rule out the possibility of government intervention:  $(\theta^*, 1] \cap [\theta - 2\tau, \theta + 2\tau] \neq \emptyset$ . Again using the fact that  $\theta^*$  increases with the intervals, the fraction of agents aware of the fact the government does intervene becomes larger, the extreme case being interval (vi), where everyone knows the government will act:  $(\theta^*, 1] \cap [\theta - 2\tau, \theta + 2\tau] = \emptyset$ .

The *degree of confusion* is defined as the fraction of investors - induced by  $\theta^*$  - which, *in equilibrium*, is uncertain about the behavior of the government. As a function of the critical level of government intervention, such a measure can be depicted as in Figure 4.

An equilibrium is defined as:

**Definition 1** *An equilibrium is a collection of decision rules,  $\mathbb{X} = \{\bar{X}_i \mid \bar{X}_i \in \{0, 1\}, \forall i \in I\}$ , and price  $p \in \mathbb{R}_{++}$ , such that:*

(i) *given price  $p$  and private signal  $\xi_i$ ,  $\bar{X}_i \in \arg \max \{U_i(X_i, \xi_i, A, p) \mid A \geq p \text{ if } X_i = 1\}$ ,  $\forall i \in I$ ; and*

(ii) *market clears:  $\int_0^1 \bar{X}_i di = K$ .*

For  $\theta^*$  lying in any of the six intervals previously listed, the method used to obtain an equilibrium is the following:

(i) Postulate that  $\theta^*$  lies in a particular interval among those from (i)-(vi);

(ii) Using the market clearing condition, determine the equilibrium price;

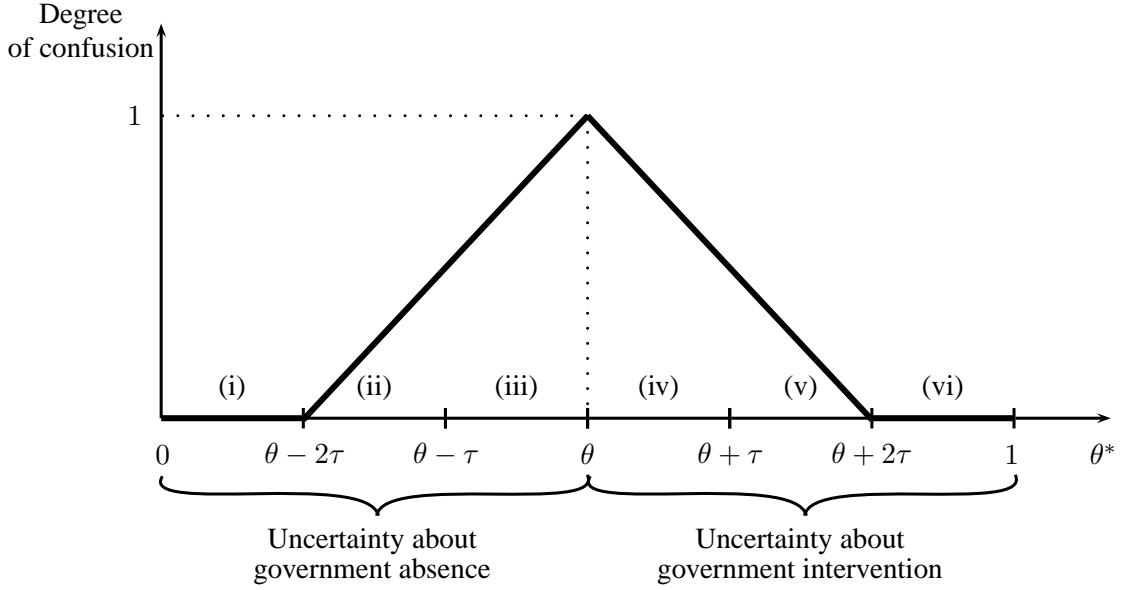


Figure 4: Level of uncertainty as a function of  $\theta^*$ .

- (iii) Impose conditions on the primitives of the model such that, at the equilibrium  $p$ ,  $\theta^*$  lies in the interval postulated in the first step and the conditions for an equilibrium given in Definition 1 are satisfied.

The aforementioned method is illustrated in Figure 5. Deriving the conditions such that the critical level of government intervention,  $\theta^*$ , can be located in any of the six intervals, two factors turn out to be important - as far as the functional form of the equilibrium price and the corresponding comparative statics are concerned:

- (i) The magnitude of  $\tau$  relative to the downside risk  $(A - C)/K$ ;
- (ii) The supply level of the asset,  $K$ .

Regarding  $\tau$ , the *high (low) uncertainty* case is the one where  $\tau > (<) (A - C)/K$ . Recall that  $\tau$  is the parameter defining the length of the range of uncertainty of each agent, and also the support of the distribution of signals: a higher  $\tau$  means both that investors contemplate a larger interval where the realized  $\theta$  might be located and that the signals are more spread out.

About  $K$ , the supply level, two cases are distinguished: one of *low supply*, with  $0 < K \leq 1/2$ , and one of *high supply*, where  $1/2 < K \leq 1$ . Given the low/high dichotomy in terms of uncertainty,  $\tau$ , and the supply level,  $K$ , the asset and its correspondingly equilibrium price can be characterized in two dimensions, given in Table 1.

Table 1 presents the functional form of the equilibrium price, for each possible combination of uncertainty and supply level. The comparative statics follow from the corresponding expressions. Two results emerge:

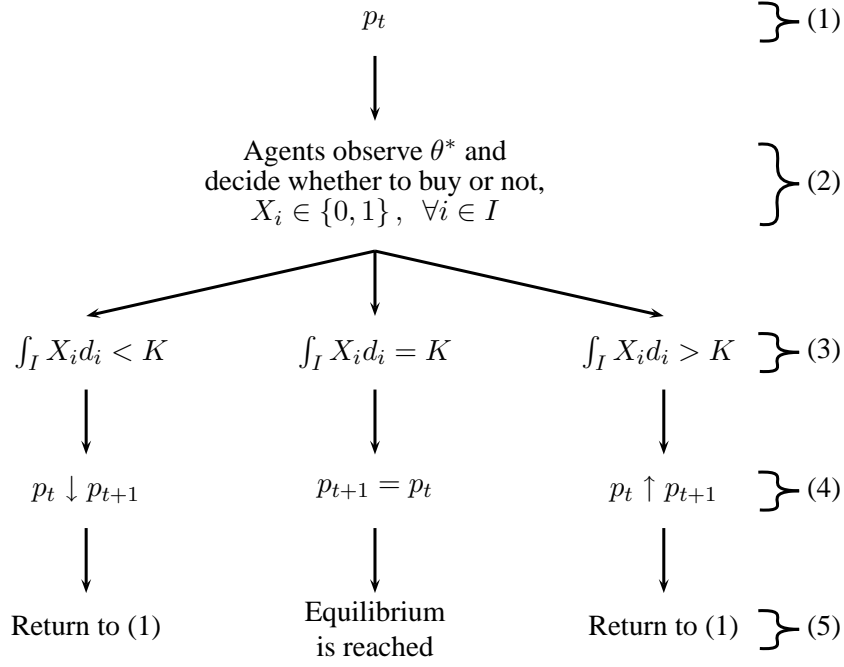


Figure 5: Equilibrium process.

Supply level, $K$	Uncertainty level, $\tau$	
	$\tau < (A - C) / K$	$\tau > (A - C) / K$
$0 < K \leq 1/2$	$P = \theta + 2\tau(1 - K) + (A - C) / K - \left\{ 2[\tau(A - C) / K]^{1/2} + t \right\}$	$P = \theta + \tau(1 - 2K) - t$
$1/2 < K \leq 1$	$P = \theta + 2\tau(1 - K) + (A - C) / K - \left\{ 2[\tau(A - C) / K]^{1/2} + t \right\}$	$P = \theta + \tau(1 - 2K) - t$

Table 1: Asset price characterization in terms of uncertainty and supply level.

- (i) For a given level of uncertainty, the equilibrium price (functional form) is uniform across assets in low and high supply;
- (ii) For a given level of supply, the comparative statics are uniform across assets of low and high uncertainty.

Figure 6 depicts the the equilibrium price and comparative statics according to the *downside-risk intervals*<sup>8</sup>, uncertainty and supply level. If the critical level of government intervention,  $\theta^*$ , is to lie in a particular range from (i)-(vi), the corresponding downside-risk  $(A - C) / K$  has to be within the appropriate interval in the figure. The first line corresponds to the low supply case, measuring the downside risk: if  $(A - C) / K$  is greater (smaller) than  $\tau$ , the asset is one of low (high) uncertainty. Analogously for the second line, which corresponds to the case of high supply.

Regardless of the uncertainty and supply level, the comparative statics of the equilibrium price,  $P$ , with respect to the state of fundamentals  $\theta$ , wealth  $A$ , insurance  $C$ , supply  $K$ , and

<sup>8</sup>These intervals are also derived in the supplemental material to the paper.

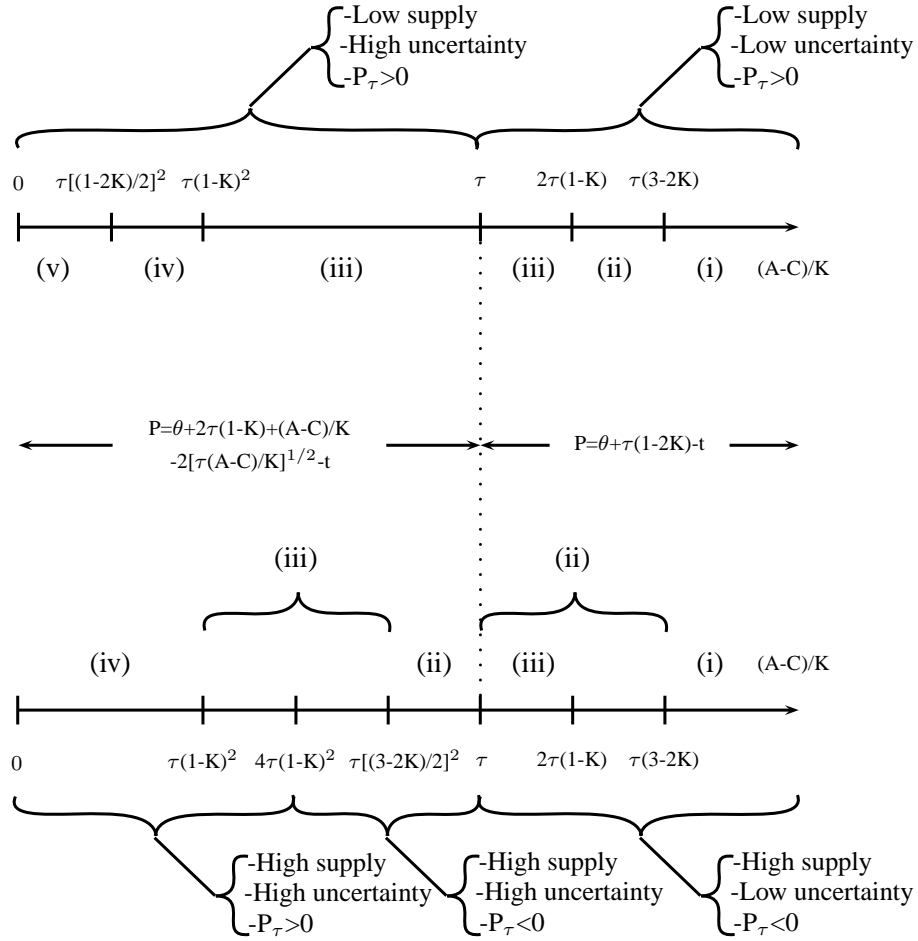


Figure 6: Downside-risk intervals and equilibrium price according to uncertainty and supply levels.

transaction cost  $t$ , are the same, namely:

- (i)  $P_\theta > 0$ ;
- (ii)  $P_A < 0$ ;
- (iii)  $P_C > 0$ ;
- (iv)  $P_K < 0$ ;
- (v)  $P_t < 0$ .

As one would expect, the equilibrium price is increasing in both the state of fundamentals and the insurance provided by the government - the safety net - whereas it is decreasing in the supply level and the transaction cost. Regarding the comparative statics with respect to wealth, the reason of the price decreasing is the enhanced attractiveness of the *saving for future consumption option* investors have: by not buying, a consumption level on par with the initial

wealth can be secured. The more investors save, the more the demand falls, depressing the equilibrium price of the asset. Another way of seeing it, recall that  $(A - C)/K$  is the downside risk, i.e., the loss that buying the asset would entail when  $\theta < \theta^*$ , vis-a-vis the option of saving the wealth for future consumption. Increasing  $A$  means the downside risk getting larger, and buying the asset riskier relative to not participating in the market<sup>9</sup>.

Regarding the comparative statics with respect to a broadening of the range of uncertainty, the effect on the equilibrium price of an increase in  $\tau$  depends *only* on the supply level of the asset - the reason why it was previously stated that, for a given level of supply, the comparative statics are uniform across assets of low and high uncertainty. If the asset is in *low supply*, the effect on the price of an increase in  $\tau$  is positive, whereas if the asset is in *high supply*, that effect is negative. All in all, government intervention makes an increase in the level of uncertainty to generate extra potential gains without the commensurate effect on the side of potential losses - after all, investors know the state of affairs will be never below a certain level, which by definition is the critical level of government intervention. Alone, this effect would undoubtedly lead to a price increase.

There is, however, a second effect to be considered resulting from an increase in the range of uncertainty. For, recall from (5) that, whenever there is government intervention, any investor  $i \in I$  would had been better-off had she saved her entire wealth for future consumption,  $X_i = 0$ , rather than choosing to buy. Therefore, changes that lead to an increase in the likelihood of government intervention *relative* to the likelihood of government absence make buying the asset,  $X_i = 1$ , less of an attractive choice.

To make the argument formal, notice from Figure 7 that, for an agent  $i \in I$  with signal  $\xi_i$ , the probability of government intervention is given by

$$Pr(\text{Intervention} \mid \xi_i) = \frac{\theta^* - (\xi_i - \tau)}{2\tau}, \quad (13)$$

whereas the probability of government absence is

$$Pr(\text{Absence} \mid \xi_i) = \frac{(\xi_i + \tau) - \theta^*}{2\tau}. \quad (14)$$

Defining a function  $\Delta$  of the difference between the two, namely

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<sup>9</sup>Just to reinforce the point, agents are heterogenous only with respect to the information received, which implies that the decision of not buying results from having a low signal, not from lacking resources. Therefore, increasing the wealth of agents will not induce more agents to participate in the market, which would possibly increase demand and, consequently, the price.



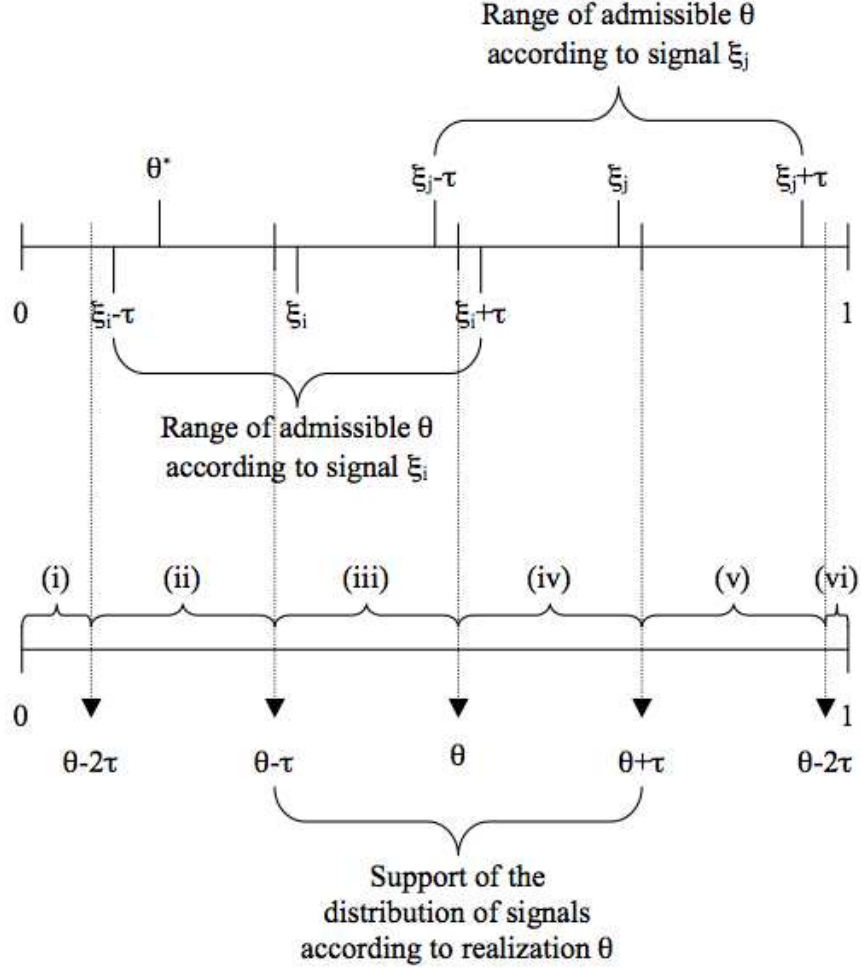


Figure 7: Support of  $\tilde{\theta}$ , distribution of signals, range uncertainty and location of  $\theta^*$ .

$$\begin{aligned}
\Delta_i(\theta^*, \tau; \xi_i) &\equiv \frac{\theta^* - (\xi_i - \tau)}{2\tau} - \frac{(\xi_i + \tau) - \theta^*}{2\tau} \\
&= \frac{\theta^* - \xi_i}{\tau}, \quad \forall i \in I,
\end{aligned} \tag{15}$$

and taking the derivative with respect to  $\tau$  yields

$$\begin{aligned}
\frac{\partial}{\partial \tau} \Delta(\theta^*, \tau; \xi_i) &= \frac{\xi_i - \theta^*}{\tau^2} \\
&= \begin{cases} > 0 & \text{if } \xi_i > \theta^*, \\ < 0 & \text{if } \xi_i < \theta^*. \end{cases}
\end{aligned} \tag{16}$$

Since the critical level of government intervention,  $\theta^*$ , is decreasing in the supply level,  $K$ , the additional probability of intervention that results from an increase in the range of uncertainty -  $\partial\Delta/\partial\tau$  above - is higher when the supply is high than when it is low. The intuition is that,

the more the asset permeates the economy - the case of high supply - the more social welfare is exposed to changes in  $\tau$ . Since social welfare is the measure upon which the intervention decision is taken, the likelihood of the government stepping in ends up increasing with the range of uncertainty.

It is the interaction of these two effects that determines the overall impact of changes in  $\tau$ : first, the benefit of having an extra exposure to potential gains without having to incur the risk of additional losses, second the increase in the probability of government intervention that makes buying less of an attractive choice relative to saving the wealth for future consumption. In the low supply case, the first effect is more important, as the downside risk per unit of the asset,  $(A - C)/K$ , is large. In the case of high supply, the risk is spread among a larger number of investors, with an increase in uncertainty not adding much to the prospects of buying the asset - in fact, more uncertainty ends up being detrimental. This is the reason why the equilibrium price *increases* with uncertainty, if the asset is in low supply, and *decreases* otherwise, i.e., if the supply happens to be high.

Making a parallel between supply and liquidity level, it could be said that assets in low supply would correspond to assets of *low liquidity*, whereas assets in high supply would be ones of *high liquidity*. From this perspective, the above result could be translated into saying that an increase in the range of uncertainty is beneficial for investors long in assets of low liquidity and detrimental for those invested in assets of high liquidity: as it was just argued, the price of the asset increases with uncertainty in the first case and decreases in the second. From a self-selection point of view, that would be reasonable: investors who opt for assets of low liquidity reveal their preferences for risky investments and, accordingly, an increase in the level of uncertainty would be seen positively. Those who choose investments of high liquidity tend to choose safer investments, making an increase in uncertainty less attractive<sup>10</sup>.

Returning to Figure 1, a close inspection shows that the functional form of the equilibrium price is the same across assets in low and high supply - conditional on the level of uncertainty. As it turns out, the equilibria in the *low uncertainty* case involve no government intervention. Shown in Figure 6, the equilibrium price in this case does not depend on the safety net provided by the government,  $C$ , leading one to believe that, after all, the possibility of intervention is not important if the asset's payoff is characterized by low uncertainty. This is misleading, however: among those intervals where in equilibrium there is no intervention, i.e., (i)-(iii), only in the first one - the one where the critical level of government intervention is at its lowest - the signals

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<sup>10</sup>This argument is not pushed further since agents are risk neutral and, therefore, are not supposed to have preferences towards more or less risk. This could be explored were the agents endowed with different preferences.

received by the agents are fully informative of the government's behavior. For the others, even though some investors still know the government safety net will be absent, some others cannot reach the same conclusion, which affects the way they calculate the expected payoff of buying the asset and, therefore, their investment decision.

With high uncertainty, the equilibrium price is clearly dependent on the level of insurance, as Figure 6 indicates. Differently from the low uncertainty case, however, the government safety net appears in the equilibrium price not only in those intervals where there is intervention but also in those where the government ends up not stepping in. A high level of uncertainty seems to be crucial if changes in the safety net,  $C$ , are to have an impact in the equilibrium price. Accordingly, one could well argue that the possibility of intervention has more of an impact in *turbulent environments* - those characterized by a high level of uncertainty. As discussed previously, however, this is *not* to say that the possibility of government intervention is unimportant in situations of low uncertainty, it is just that - with high uncertainty - the presence of the government is important enough to leave its footprints in the equilibrium price.

As one can deduce from Figure 6, an equilibrium cannot be supported in interval (vi), where the critical level of government intervention,  $\theta^*$ , is at its highest. The reason is that, with such a  $\theta^*$ , agents infer they are paying a high price to invest, after all insurance provided by the government is costly. A high price makes the strategy of buying more likely to perform badly: only if the asset performs extremely well can the investment be salvaged. Facing a high price, some investors prefer to abstain from buying the asset, which in extreme can lead to the market clearing condition not being satisfied. Unless the price decreases sufficiently enough, no equilibrium can be supported, as in the process depicted in Figure 5.

This non-existence of equilibrium could be related to the burst of a *bubble*, indicating the impossibility of prices going beyond a certain level. Just as when a bubble bursts there is not enough demand to keep prices increasing, so is the non-existence of equilibrium caused by a lack of investors willing to buy the asset, as few want to commit to such a costly investment - implicitly in those instances where the probability of intervention is high<sup>11</sup>.

### 3.2 Perfect Information

The perfect information case is trivial but mentioned here, nonetheless. In this case, investors know the state of fundamentals,  $\theta$ , so that the critical level of government intervention,  $\theta^*$ , leads to one of two possible outcomes:

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<sup>11</sup>As the model is static and a specific definition of a bubble is not provided, this remark is just to indicate a point that could be further explored in an extension of the paper.

(i)  $0 \leq \frac{C-A}{K} + p + t \leq \theta \Leftrightarrow 0 \leq \theta^* \leq \theta$  - no government intervention;

(ii)  $\theta < \frac{C-A}{K} + p + t \leq 1 \Leftrightarrow \theta < \theta^* \leq 1$  - government intervention.

Investors' maximization problem is

$$\begin{aligned} \max_{X_i} \quad & X_i (\theta - p - t) + A \\ \text{s.t.} \quad & A \geq p \quad \text{if } X_i = 1. \end{aligned} \tag{17}$$

Therefore, in case agents can afford to buy the asset,  $A \geq p$ , they choose  $X_i = 1$  only if  $\theta - p - t + A \geq A \Leftrightarrow \theta \geq p + t$ . If that is the case,  $A > C$  implies that

$$[p + t, 1] \cap [(C - A) / K + p + t, 1] = [p + t, 1], \tag{18}$$

whereas

$$[p + t, 1] \cap [0, (C - A) / K + p + t] = \emptyset, \tag{19}$$

which means an equilibrium can be supported only in the range where there is no government intervention. Hence, defining  $p^{PI}$  as the equilibrium price in the perfect information case, in equilibrium  $\theta \geq p^{PI} + t \Leftrightarrow p^{PI} \leq \theta - t$ . With the assumption that  $0 \leq K \leq 1$ , i.e., the supply of the asset is never greater than potential demand, competition would bid the price up, to the point where  $p^{PI} = \theta - t$ , which would constitute the maximum price investors would be willing to pay knowing the government will not act.

Following the expression of the equilibrium price, the comparative statics are straightforward: the price is increasing in the state of fundamentals and decreasing in the transaction cost, as one would expect. There is no downside risk to be considered by the investors since they know what the payoff of the asset will be. Because of that, the wealth level  $A$ , government insurance  $C$  and supply  $K$  do not appear in the equilibrium price, not mattering for the comparative statics.

With perfect information, either all investors choose to buy or everyone refrains from doing so - the heterogeneity in information from the previous scenario is gone. In case all investors choose to buy the asset, if the supply is less than one - i.e., supply is less than demand - some mechanism must be available so that only a fraction of the agents get selected to make the investment. As discussed before, if the wealth constraint is not binding, those who are not selected would be willing to make a higher offer in order to participate in the market, which would increase the price up to the point where no one is willing to pay a price higher than the one prevailing. In this sense, there is a combined effect on the equilibrium price coming from

the interaction of the supply and wealth levels, effect that is absent if one is to look only to the expression of the equilibrium price.

### 3.3 No Information: Common Prior About the Fundamentals

The common prior scenario refers to the case where agents have no information other than the distribution of  $\tilde{\theta}$ , which is assumed to be common knowledge and given by  $\tilde{\theta} \sim \mathbb{U}[0, 1]$ . Investors' problem is

$$\begin{aligned} \max_{X_i} \quad & \mathbb{E} \left[ X_i (\tilde{\theta} - p - t) + A \right] = X_i \left[ \mathbb{E} (\tilde{\theta}) - p - t \right] + A \\ \text{s.t.} \quad & A \geq p \quad \text{if } X_i = 1, \end{aligned} \quad (20)$$

and, as usual, they acknowledge that intervention is in place only if  $\theta < \theta^*$ , a condition which is equivalent to  $\theta < (C - A) / K + p + t$ . Also, agents know that government intervention makes the prevailing situation to be equivalent to one where the state of fundamentals equals  $\theta = \theta^* = (C - A) / K + p + t$ . Hence,

$$\begin{aligned} \mathbb{E} (\tilde{\theta}) &= \int_0^1 \theta d\theta = \int_0^{\frac{C-A}{K} + p + t} \theta^* d\theta + \int_{\frac{C-A}{K} + p + t}^1 \theta d\theta \\ &= \frac{1}{2} \left[ 1 + \left( \frac{C-A}{K} + p + t \right)^2 \right]. \end{aligned} \quad (21)$$

In case agents can afford to buy the asset,  $A \geq p$ , they choose  $X_i = 1$  only if

$$\begin{aligned} & \frac{1}{2} \left[ 1 + \left( \frac{C-A}{K} + p + t \right)^2 \right] - p - t + A \geq A \\ \Leftrightarrow & \frac{1}{2} \left[ 1 + \left( \frac{C-A}{K} + p + t \right)^2 \right] - p - t \geq 0. \end{aligned} \quad (22)$$

The left-hand side of the above inequality has two roots, namely

$$p' = \frac{A-C}{K} - t + 1 - \sqrt{2 \left( \frac{A-C}{K} \right)}, \quad (23)$$

$$p'' = \frac{A-C}{K} - t + 1 + \sqrt{2 \left( \frac{A-C}{K} \right)}. \quad (24)$$

For any  $p \leq A$  such that either  $p \leq p'$  or  $p'' \leq p$ , condition (22) is satisfied. With the assumption that  $0 \leq K \leq 1$  - supply is never greater than potential demand - competition would bid the price up. Denoting  $p^{CP}$  as the equilibrium price in the common prior scenario, the following outcomes might emerge, according to the wealth level  $A$ :

- (i)  $0 < A \leq p' \Rightarrow p^{CP} = A$ ;
- (ii)  $0 < p' < A < p'' \Rightarrow p^{CP} = p'$ ;
- (iii)  $0 < p' < p'' \leq A \Rightarrow p^{CP} = A$ .

If supply is strictly less than potential demand,  $0 \leq K < 1$ , the agents allowed to buy the asset get selected randomly from the population of investors<sup>12</sup>.

Notice that, if  $p^{CP} = A$ ,  $\theta^* = (C - A)/K + A + t$  and, since  $\theta^* \in [0, 1]$ , it must be that  $(A - C)/K < A + t$ . Analogously, if  $p^{CP} = p'$ , a feasible  $\theta^*$  requires  $(A - C)/K < 1/2$ . Therefore, an equilibrium with the possibility of government intervention - i.e., one with a feasible  $\theta^*$  - can be supported only if the downside risk,  $(A - C)/K$ , is not too high or, in other words, if the support from the government in a crisis is sufficiently large<sup>13</sup>.

From the possible outcomes specified above, a wealth level  $A$  either sufficiently small or sufficiently high implies the equilibrium price being  $p^{CP} = A$ , with corresponding trivial comparative statics. With an intermediate wealth level, it follows that

$$p^{CP} = \frac{A - C}{K} - t + 1 - \sqrt{2 \left( \frac{A - C}{K} \right)} \equiv P(K, t, A, C). \quad (25)$$

With the condition  $(A - C)/K \leq 1/2$  in place, the comparative statics are:

- (i)  $P_K = \frac{1}{K} \left[ \sqrt{\frac{(A-C)/K}{2}} - \frac{A-C}{K} \right] > 0$ ;
- (ii)  $P_t = -1 < 0$ ;
- (iii)  $P_A = \frac{1}{K} \left[ 1 - \frac{1}{\sqrt{2(A-C)/K}} \right] < 0$ ; and
- (iv)  $P_C = \frac{1}{K} \left[ \frac{1}{\sqrt{2(A-C)/K}} - 1 \right] > 0$ .

With respect to  $K$ , there are two effects to be considered. The first one is the usual positive supply shock - price decrease. The second - and, as it turns out, the relevant one - is the decrease in the *individual* downside risk. The downside risk associated with a change in social welfare

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<sup>12</sup>As in the perfect information framework, there is no heterogeneity whatsoever among the agents when all they know is the distribution of  $\tilde{\theta}$ , implying the equilibrium to be one where either none or everyone is willing to buy the asset. Since the mass of investors is equal to one, if the supply  $K$  is less than that there must exist a device such that only a fraction of the investors will be selected to buy - in case them all want to do so.

<sup>13</sup>Otherwise, i.e., if  $\theta^* < 0$ , every agent believes that the government will be absent, since it is known that intervention happens only if  $\theta < \theta^*$  and  $\theta \in [0, 1]$ . That being the case would imply  $\mathbb{E}(\tilde{\theta}) = 1/2$ , automatically changing the analysis above.

upon an intervention is still the same -  $A - C$  - but that risk per investor -  $(A - C)/K$  - decreases when  $K$  increases. Individually, therefore, each investor bears less risk when the supply of the asset goes up. That makes the buying decision more attractive, increasing the demand and, consequently, the price<sup>14</sup>.

Regarding  $A$ , one must bear in mind that the comparative statics at hand hold for a wealth level  $p' < A < p''$ . Since paying any price  $p \in (p', p'')$  yields a negative expected utility, a marginal increase in wealth will not increase the demand for the asset. Nonetheless, there is a secondary effect that comes from changes in  $A$  that does affect the price: an increase in wealth makes the option of saving for future consumption more attractive relative to buying, inducing agents to demand a price cut if they are expected to invest in the asset. This is what ultimately happens and is reflected in  $P_A < 0$ .

Finally, the comparative statics with respect to  $t$  and  $C$  are as expected: an increase in the transaction cost decreases the price of the asset whereas if the insurance offered by the government increases -  $C$  goes up - so does the equilibrium price.

### 3.4 Analysis of the Equilibrium Price Across Different Informational Scenarios

After deriving the expression of the equilibrium price - and respective comparative statics - for each informational scenario, the focus now is on the comparison of those prices - the *intra framework, inter scenario* analysis of Figure 2. The question one asks is how the interaction of the degree of information with the possibility of intervention affects the equilibrium outcome.

Recall first that, in the imperfect information scenario, the magnitude of  $\tau$  with respect to the downside risk - i.e., the level of uncertainty each agent faces when making the decision to invest - was one of the determinants of the equilibrium price. Second, for the perfect information and common prior scenarios, in equilibrium either all or none of the investors choose to buy the asset. Following suit, our analysis henceforth will distinguish between scenarios of low and high uncertainty, for each of them the supply level is assumed to be unitary.

With unitary supply, for the low uncertainty case the equilibrium prices are:

(i) Imperfect information:  $p^{IP} = \theta - \tau - t$ ;

(ii) Perfect information:  $p^{PI} = \theta - t$ ;

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<sup>14</sup>This effect can be seen as a *negative externality* investors cause on each other: from their own perspectives, buying the asset becomes more attractive since less risky, but that in turn increases demand and, ultimately, the price everyone else has to pay upon deciding to invest.

(iii) Common prior<sup>15</sup>:  $p^{CP} = A$  or  $p^{CP} = A - C - t + 1 - \sqrt{2(A - C)}$ .

Since  $\tau > 0$ , it is clear that  $p^{PI} > p^{IP}$ . Also, as the asset is affordable to investors only if wealth is higher than price, regardless of the state of fundamentals the ordering of prices is  $p^{CP} > p^{PI} > p^{IP}$ , in case  $p^{CP} = A$ .

More interesting is the case where the wealth constraint is not binding and the price in the common prior framework is given by  $p^{CP} = A - C - t + 1 - \sqrt{2(A - C)}$ . Following expression (25), it was pointed out that, in order to be consistent with the possibility of government intervention, the downside risk must be such that  $0 \leq A - C \leq 1/2$ . With this assumption in place, it is shown in the appendix - Proposition 3 - that, for a *sufficiently high* realization of  $\theta$ , the ordering of equilibrium prices is

$$p^{PI} > p^{IP} > p^{CP}, \quad (26)$$

whereas for an *intermediate level*,

$$p^{PI} > p^{CP} > p^{IP}, \quad (27)$$

with the following prevailing if the state of fundamentals happens to be *sufficiently low*,

$$p^{CP} > p^{PI} > p^{IP}. \quad (28)$$

Figure 8 summarizes the ordering of equilibrium prices for the low uncertainty case, according to the state of fundamentals. For any level of  $\theta$ , the equilibrium price with perfect information,  $p^{PI}$ , is higher than the one that prevails in the imperfect information framework,  $p^{IP}$ , reflecting the cost of information - more information commands a higher price. If investors only have knowledge of the distribution of  $\tilde{\theta}$  - the common prior scenario - the expected state of fundamentals they envisage is *higher* than  $\theta$  when that realization is low and *lower* than that when that realization is high, which is the reason why  $p^{CP} > p^{PI} > p^{IP}$  for a sufficiently low  $\theta$  and  $p^{PI} > p^{IP} > p^{CP}$  for a sufficiently high.

Turning to the high uncertainty case, the equilibrium prices across the different scenarios, with a unitary supply, are:

(i) Imperfect information:  $p^{IP} = \theta + A - C - t - 2\sqrt{\tau(A - C)}$ ;

(ii) Perfect information:  $p^{PI} = \theta - t$ ;

(iii) Common prior:  $p^{CP} = A$  or  $p^{CP} = A - C - t + 1 - \sqrt{2(A - C)}$ .

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<sup>15</sup>In the common prior case,  $p^{CP} = A$  if the wealth constraint is binding.



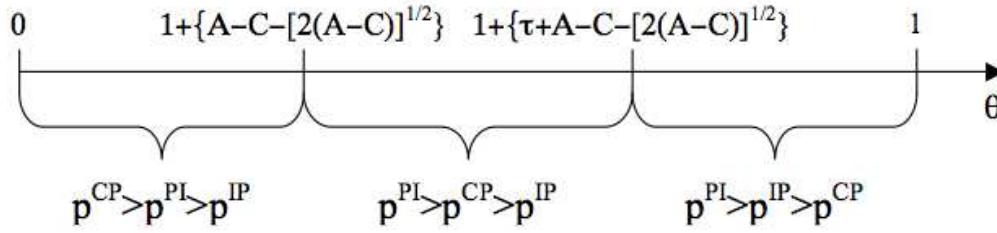


Figure 8: Equilibrium price ordering across different informational scenarios for the low uncertainty case.

As Proposition 4 in the appendix shows, all in all the ordering of equilibrium prices in the high uncertainty scenario is similar to the one obtained for the low uncertainty one. As before, the equilibrium price when investors have perfect information is always higher than the one prevailing when all they observe is a private signal about the state of fundamentals. The price differential is, again, just a consequence of the cost of information: better (perfect) information commands a higher price relative to worse (imperfect) information.

Regarding the equilibrium price in the common prior scenario, for a *sufficiently low* realization of the state of fundamentals, the unconditional expectation of  $\tilde{\theta}$  is *higher* relative to that in the other scenarios, and so is the equilibrium price. By the same token, that unconditional expectation is *lower* relative to the others for a *sufficiently high* level of fundamentals, and so lower is the equilibrium price. Figure 9 summarizes the ordering of prices for the high uncertainty case<sup>16</sup>.

## 4 No Government Intervention

In the setup with no government, investors acknowledge the impossibility of an intervention, regardless of the state of fundamentals coming to the fore: by assumption, that is ruled out from the outset. The insurance or safety net they would otherwise rely on, as in the framework of the previous section, is now nonexistent. Agents therefore must take this new paradigm into account when calculating the expected payoff of buying the asset.

The informational constraints, however, are still there: the scenario might still be one of imperfect, perfect or common prior information, in the same fashion of the previous analysis.

<sup>16</sup>As Proposition 4 in the appendix shows,  $\tau$  being greater or less than  $1/2$  turns to be important for the ordering of equilibrium prices in the high uncertainty scenario: the ordering in the first line of Figure 9 corresponds to the case where  $\tau < 1/2$ , whereas in the third it is considered  $\tau > 1/2$ . Being simply a technicality, this point is ignored in the text.

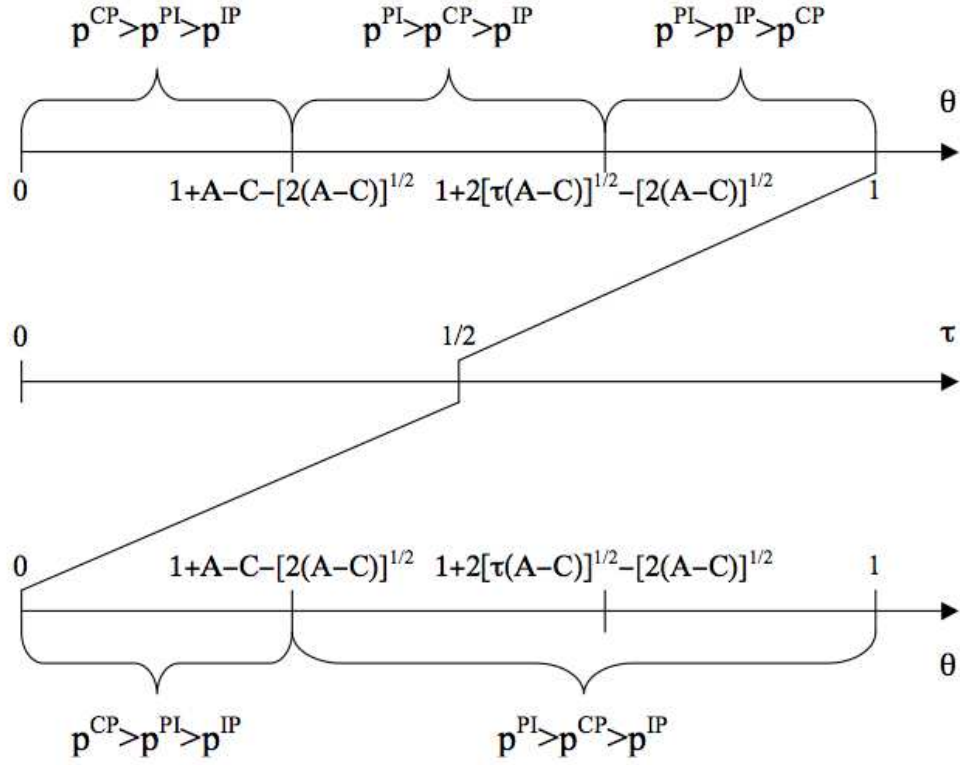


Figure 9: Equilibrium price ordering across different informational scenarios for the high uncertainty case.

The imperfect information case of the next section is discussed first.

#### 4.1 Imperfect Information About the Fundamentals

With no government and investors receiving a private signal about the state of fundamentals - by definition, the imperfect information scenario - one knows from (7) that, for any  $i \in I$ ,  $\mathbb{E}(\tilde{\theta} | \xi_i) = \xi_i$ , and, from (12), that agents opt to buy only if  $\xi_i \geq p + t$  - in case buying the asset is affordable,  $A \geq p$ . The market clearing condition is

$$\begin{aligned} \frac{1}{2\tau} \int_{p+t}^{\theta+\tau} d\xi = K &\Leftrightarrow \theta + \tau - p - t = 2\tau K \\ &\Leftrightarrow p = \theta + \tau(1 - 2K) - t \equiv p^{NG}. \end{aligned}$$

Proposition 5 in the appendix establishes the necessary conditions under which  $p^{NG}$  can be supported as an equilibrium price. The comparative statics are derived from the expression of  $p^{NG}$ . In that regard,  $\theta$ ,  $K$  and  $t$  are related to the equilibrium price as one would expect:  $p^{NG}$  is increasing in the state of fundamentals and decreasing in both the supply level and the transaction cost.

Similarly to the comparative statics for the imperfect information scenario with the possibility of intervention, here the effect of changes in  $\tau$  - the parameter that commands the range of uncertainty each agent faces - depends on the supply level of the asset,  $K$ . For, it is clear from the expression of  $p^{NG}$  that the price increases in  $\tau$  if  $K < 1/2$ , and decreases otherwise.

With risk-neutral agents, one would at first believe that changing  $\tau$  would not cause any effect on the equilibrium price - after all, with such preferences, investors' concerns regard only the expected value of  $\tilde{\theta}$ , and an increase in the range of uncertainty would constitute nothing more than a *mean-preserving spread*.

However, the market clearing condition - being a restriction on the *proportion* of investors buying the asset - makes the supply level pivotal in determining the effect on the equilibrium price of changes in  $\tau$ . The market clearing condition implies that the proportion of investors buying the asset is

$$\frac{\theta + \tau - p - t}{2\tau} = K. \quad (29)$$

Changing  $\tau$  has an effect on both the numerator and the denominator of the ratio in the left-hand side of relation (29), while the right-hand side is fixed. If  $K > 1/2$ , changes in the numerator must be overall *greater* than those in the denominator. Since the price enters the expression with a negative sign, a decrease in it must follow, leading to  $P_{\tau}^{NG} < 0$ . With a low supply level,  $K < 1/2$ , the converse applies and, therefore, the equilibrium price increases with  $\tau$ .

As in the framework with the possibility of government intervention, if an analogy between supply level and liquidity is to be made, the above could be translated as the following: an increase in the uncertainty level of assets in low supply has a positive effect on prices, whereas for assets in low supply that effect is negative. As argued before, from a self-selection point of view this would be reasonable: those investing in assets of low liquidity - by revealing a preference towards risk - would appreciate an increase in  $\tau$ , causing a corresponding increase in the equilibrium price, the opposite being the case for those investing in assets of high liquidity. Since investors are risk-neutral and, therefore, do not exhibit any preference for more or less risk, this point is not elaborated further.

## 4.2 Perfect Information About the Fundamentals

The perfect information scenario, as in the framework with the presence of the government, is trivial but mentioned nonetheless. The investors' maximization problem is

$$\begin{aligned} \max_{X_i} \quad & X_i (\theta - p - t) + A \\ \text{s.t.} \quad & A \geq p \quad \text{if } X_i = 1. \end{aligned} \tag{30}$$

Therefore, in case agents can afford to buy the asset,  $A \geq p$ , they choose  $X_i = 1$  only if

$$\begin{aligned} \theta - p - t + A &\geq A \\ \Leftrightarrow \theta &\geq p + t. \end{aligned} \tag{31}$$

Denoting the equilibrium price by  $p^{NGP}$ , it is the case that

$$\begin{aligned} \theta &\geq p^{NGP} + t \\ \Leftrightarrow p^{NGP} &\leq \theta - t. \end{aligned} \tag{32}$$

For a supply level as large as potential demand<sup>17</sup>,  $0 < K \leq 1$ , competition for the asset bids the price up, to the point where  $p^{NGP} = \theta - t$ : this constitutes the maximum price investors would be willing to pay given that government will not intervene.

From  $p^{NGP} = \theta - t$ , the comparative statics are trivial: the price is increasing in the state of fundamentals and decreasing in the transaction cost. There is no effect from the government safety net and uncertainty: first, by assumption the government is absent and, second, there is no uncertainty, given that the realization of  $\tilde{\theta}$  is known - it is the perfect information scenario after all.

### 4.3 No Information: Common Prior About the Fundamentals

Finally, the focus turns to the case where agents do not receive any signal, being aware only of the distribution of  $\tilde{\theta}$ , given by  $\tilde{\theta} \sim \mathcal{U}[0, 1]$  and assumed to be common knowledge. Investors' problem is

$$\begin{aligned} \max_{X_i} \quad & \mathbb{E} \left[ X_i (\tilde{\theta} - p - t) + A \right] = X_i \left[ \mathbb{E} (\tilde{\theta}) - p - t \right] + A \\ \text{s.t.} \quad & A \geq p \quad \text{if } X_i = 1. \end{aligned} \tag{33}$$

Taking into account the impossibility of intervention, it follows that

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<sup>17</sup>As remarked in other instances, if supply is strictly less than demand,  $0 < K < 1$ , it is assumed that investors are selected randomly to buy the asset.

$$\begin{aligned}\mathbb{E}(\tilde{\theta}) &= \int_0^1 \theta d\theta \\ &= \frac{1}{2}.\end{aligned}\tag{34}$$

Therefore, in case agents can afford to buy the asset,  $A \geq p$ , they choose  $X_i = 1$  only if

$$\begin{aligned}\frac{1}{2} - p - t + A &\geq A \\ \Leftrightarrow \frac{1}{2} - p - t &\geq 0.\end{aligned}\tag{35}$$

The left-hand side of the above inequality has one root, namely

$$p' = \frac{1}{2} - t.$$

For any  $p \leq A$  such that  $p \leq p'$ , condition (35) is satisfied. If, as in the perfect information case, it is assumed that  $0 < K \leq 1$ , i.e., supply is never greater than potential demand, competition forces the price to its maximum. Hence, denoting by  $p^{NGC}$  the equilibrium price, the possible outcomes are

- (i)  $0 < A \leq p' \Rightarrow p^{NGC} = A$ ;
- (ii)  $0 < p' < A \Rightarrow p^{NGC} = p'$ .

With the equilibrium price being either  $p^{NGC} = A$  or  $p^{NGC} = 1/2 - t$ , the comparative statics are trivial. For the first case, where the wealth constraint is binding - in the sense that it is consumed in its entirety when investors buy the asset - an increase in wealth leads clearly to an increase in the equilibrium price. For the second, all that matters is the expected value of the state of fundamentals -  $1/2$ , which is fixed - and the transaction cost to be paid in the final period. In this way, only changes in  $t$  have an effect on the equilibrium price - in particular, a negative one.

## 4.4 Analysis

With equilibrium price expressions for the different scenarios, namely  $p^{NG}$ ,  $p^{NGP}$  and  $p^{NGC}$  - imperfect, perfect and common prior information, respectively - a comparison across prices can be made: the so called *intra framework*, *inter scenario* analysis of Figure 2, in the same fashion as the one done in the framework with the presence of the government. The objective is to better understand how the complete absence of government support interact with different levels of payoff uncertainty in determining equilibrium prices.

To begin with, since with perfect and common prior information the equilibrium outcome is such that either none or all investors buy the asset, it is assumed henceforth a unitary supply level. With that in place, the expressions of equilibrium prices are:

- (i) Imperfect information:  $p^{NG} = \theta - \tau - t$ ;
- (ii) Perfect information:  $p^{NGP} = \theta - t$ ;
- (iii) Common prior:  $p^{NGC} = A$  or  $p^{NGC} = 1/2 - t$ .

If the wealth constraint binds in the common prior case - implying  $p^{NGC} = A$  - it follows that  $p^{NGC} > \max \{p^{NG}, p^{NGP}\}$ , since affordability requires the price of the asset being smaller than wealth. That  $\tau > 0$  in turn implies  $p^{NGP} > p^{NG}$  and, accordingly, the ordering of prices is  $p^{NGC} > p^{NGP} > p^{NG}$ .

The wealth constraint not binding in turn implies  $p^{NGC} = 1/2 - t$  and, in this case, Proposition 7 in the appendix shows that, for a sufficiently low realization of the state of fundamentals,  $0 \leq \theta < 1/2$ , the ordering of equilibrium prices is

$$p^{NGC} > p^{NGP} > p^{NG}, \quad (36)$$

whereas for a sufficiently high,  $1/2 < \theta \leq 1$ , the ordering is given first by

$$p^{NGP} > p^{NG} > p^{NGC}, \quad (37)$$

- if  $1/2 + \tau < \theta \leq 1$  - and second by

$$p^{NGP} > p^{NGC} > p^{NG}, \quad (38)$$

- in case  $1/2 < \theta < 1/2 + \tau$ .

Figure 10 summarizes the orderings given above. Regardless of the realization of  $\tilde{\theta}$ , it is always the case that  $p^{NGP} > p^{NG}$ , i.e., the equilibrium price in the perfect information scenario is always higher than the one prevailing when investors have imperfect information: *better* (perfect) information, after all, commands a higher price than *worse* (imperfect) information.

For a sufficiently low realization of the state of fundamentals,  $0 \leq \theta < 1/2$ , the unconditional expected value assigned by investors to the asset's payoff,  $1/2$ , is higher than the value investors would expect in both the perfect and imperfect information scenarios, and accordingly higher is the price they accept to pay,  $p^{NGC}$ , compared to what would be agreeable in the other scenarios. This argument is reversed for sufficiently high state of fundamentals, in particular for  $1/2 + \tau < \theta < 1$ , in which case  $p^{NGC}$  is the lowest of the equilibrium prices.

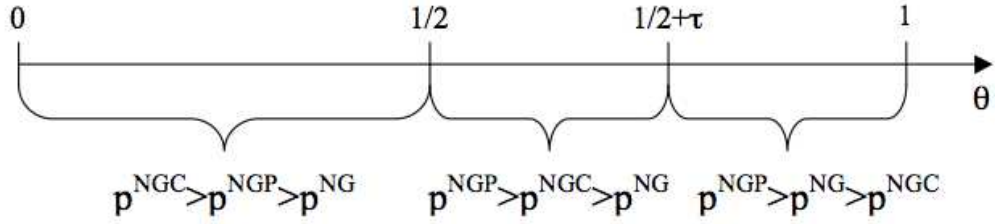


Figure 10: Equilibrium price ordering across different informational scenarios without the possibility of government intervention.

## 5 Government vs No Government Prices

From the analysis in the two frameworks - *with* and *without* the participation of the government - across the different informational scenarios - *imperfect*, *perfect* and *common prior information* - the conclusion is that, overall, prices respect the following order:

- For a *high* realization of the state of fundamentals, the perfect information price dominates, whereas the common prior price is dominated;
- For a *moderate* realization of the state of fundamentals, the perfect information price still dominates, whereas the imperfect information price is the one dominated;
- For a *low* realization of the state of fundamentals, the common prior price dominates, while the imperfect information price is dominated.

This conclusion can be drawn from a comparison of Figure 8, 9 and 10. The possibility of intervention, therefore, does not seem to alter significantly the ordering of equilibrium prices when a comparison is made across the different informational scenarios. The possibility of intervention reveals to be more substantial when one performs the *inter framework*, *intra scenario* analysis of Figure 2, which is done in turn.

### 5.1 Imperfect Information

In the imperfect information scenario, comparing the equilibrium outcome that emerges with the possibility of government intervention to the one that prevails when the government is absent, it follows that the later can be viewed as a special case of the former, the special case being the one where the critical level of government intervention is so low that - no matter what the state of fundamentals happens to be - every investor knows there will not be any intervention.

Recalling that the higher the critical level of government intervention, the higher the equilibrium price, the above translates into saying that only at those prices deemed to be lowest in the framework with government participation can an equilibrium be supported in a framework where that very same government is absent. In other words, all the equilibria with higher prices that could be supported were the agents to entertain the possibility of an intervention cease to exist when that intervention is ruled out from the outset. It is precisely the possibility of government intervention, therefore, that allows higher prices to emerge and be supported as equilibria<sup>18</sup>.

## 5.2 Perfect Information

In the perfect information scenario, the equilibrium price is the same in both frameworks, with and without the participation of the government. Recall that, in the setup where the government is allowed to participate, an equilibrium can be sustained only when investors pay a price that is conducive to an outcome where the government ends up *not* intervening. In other words, the only type of equilibrium in the perfect information scenario, *with* the participation of the government, is the one where the government does not intervene - and this is the reason why the equilibrium price is identical to that in the setup without the possibility of intervention.

The reason why an equilibrium cannot be supported when government intervention is certain is the following. Since the government steps in only when the realized state of fundamentals falls *below* a critical level - with that critical level being an increasing function of the price of the asset - when investors are certain of government intervention, odds are that the critical level is being set too high and, accordingly, too high is the price being asked. Another way of seeing it, when investors pay too high of a price, the level of fundamentals required to make the investment profitable is correspondingly high, which makes it more likely that investors will suffer losses and, hence, more likely that the government will intervene, which is just what a high critical level implies. The price being too high pushes investors away from investing in the asset, causing a failure of the market clearing condition and preventing an equilibrium to emerge.

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<sup>18</sup>That higher prices can emerge in the presence of the government but not otherwise could be interpreted as the case of the government inducing a *bubble*. This path is not pursued, however, since what is behind those higher prices is a factor which, if not related intrinsically to the fundamentals of the asset, still affects the investors' payoff, nevertheless: that is the insurance provided by the government.



### 5.3 Common Prior

When investors have only knowledge of the distribution of the state of fundamentals - the common prior scenario - the equilibrium price in the framework *with* the possibility of government intervention is at least as high as the one that results in the framework *without* the participation of the government. Proposition 8 in the appendix shows this result formally.

The reason is straightforward: since investors do not receive any information regarding the level of fundamentals, all they calculate is the *unconditional* expectation of  $\tilde{\theta}$ . In the framework without the presence of the government, that expectation is taken over the whole support of the distribution of  $\tilde{\theta}$ , whereas in the alternative framework, the support is truncated at the critical level of intervention - after all, investors know the payoff of the asset will not be lower than the critical level. This truncation makes the expected value of the state of fundamentals higher in the framework *with* the government than in the framework *without*, with the same conclusion being carried to the equilibrium prices.

### 5.4 Informational Scenarios Combined: the Main Result

Combining the conclusions above - obtained after a comparison across frameworks of equilibrium prices under each informational scenario - yields the main result, stated in the following

**Theorem 2** *Regardless of the informational scenario faced by investors being one of imperfect, perfect, or common prior information, the resulting equilibrium price is at least as high in the framework with the presence of the government relative to the one where the government's safety net is absent.*

## 6 Concluding Remarks

The paper studies the effects on prices of the possibility of government intervention in situations of distress. This is contrasted with a framework where investors acknowledge the nonexistence of a safety net that could otherwise attenuate potential losses. In order to understand the consequences in a diverse class of investments, different informational scenarios are used as proxies for the heterogeneity in assets' characteristics.

In terms of modelling assumptions, first, investors are taken to be risk neutral, allowing one to concentrate on those instances where the possibility of intervention and the degree of information are the only drivers behind agents' decision, rather than preferences towards less or more risk that could advent from having the utility function specified in another way. Second,

upon deciding to invest, agents can only buy a single unit of a risky asset, with the leftovers from the wealth they are all endowed with being saved for future consumption, as if invested in a riskless asset paying no interest. This turns the investment decision into a choice of whether or not to participate in a market - here being called stock market - which can be as important a factor of impact on prices as the trading activities of a large investor.

Upon deriving the equilibrium price for each combination of framework and scenario - i.e., a choice of setup with or without government's presence, the information agents are endowed with being either imperfect, perfect or common prior - it emerges that government's presence does not affect the ordering of prices when a comparison is made across scenarios: invariably, according to the state of fundamentals, the relation among prices derived under the assumption of imperfect, perfect and common prior information is the same.

However - and this is one of the main results obtained - the equilibrium prices under the framework with the government are at least as high as the ones prevailing when a safety net is ruled out from the outset. One must bear in mind that the government's presence is not synonymous of an intervention taking place but only that the government will step in if social welfare - the sum across agents of the return on their investments - is to cross a certain threshold in case no action is taken. As it turns out, this possibility alone is enough to make prices to be higher in the government's framework compared to the alternative, regardless of the informational scenario at hand.

Providing the conditions for higher prices, however, does not allow the government's safety net to sustain just any price: given the measure of social welfare, the higher the price, the higher the likelihood of an intervention: after all, investments are less likely to be profitable when the price paid for them, i.e., their cost, is high. Naturally, intervention - or, as one might call it, insurance - by the government does not come for free: even though it alleviates losses, were the investors to know an intervention would ensue they would rather had preferred to abstain from participating in the market, putting all their wealth into the riskless asset. This implies that an equilibrium cannot be sustained in those instances where prices are too high and, therefore, a government intervention is imminent.

In terms of comparative statics, some nontrivial results come to the fore. First, since losses are limited in the presence of the government's safety net, one might be induced to think that increases in uncertainty make buying the asset more attractive, with a corresponding positive effect on prices. Not only that, however, turns to be important, as increases in uncertainty also lead investors to put more weight on the side of an intervention: as discussed in the previous paragraph, in this case the best decision would be to save the entire wealth for future consumption, instead of choosing to buy the asset. Ultimately, it is the supply level that happens to

be pivotal in determining which of these two effects is more important: if the supply is low, market clearing implies few agents participating in the market and, accordingly, social welfare would not be significantly exposed to large swings in the asset's payoff adventing from changes in uncertainty - the equilibrium price ends up increasing. With a high supply, on the other hand, social welfare is more susceptible to change even after marginal increases in uncertainty, and so is the likelihood of an intervention - the equilibrium price ends up decreasing.

Second, an increase in the wealth level does not necessarily lead to an increase in the equilibrium price, as one would reasonably expect. The reason is that heterogeneity in the model comes not from the distribution of wealth but, rather, from the distribution of signals. Every agent being endowed with the same wealth, the marginal buyer is by definition the one who, conditional on the signal received, is indifferent between buying or not the asset. In other words, an increase in wealth does not have the effect of increasing the equilibrium price by changing the marginal buyer: that is still defined solely by the distribution of signals. In fact, increasing wealth might even decrease the asset's equilibrium price, since by not investing agents can secure their entire endowments for future consumption - saving might become a better option.

Regarding the functional form of the equilibrium prices, one factor that turns to be important is the unitary downside risk associated with the risky investment. Upon the decision of buying the asset, this measure gives how much of a loss one would face in a worst case scenario - i.e., with government intervention - relative to the option of saving the entire wealth for future consumption. By definition, the downside risk is closely related to the government safety net: the higher the former, the lower the latter. It is the relation of the parameter defining the investor's degree of uncertainty and the downside risk that defines the functional form of the equilibrium price: in the low uncertainty case, the safety net does not appear in the equilibrium price, regardless of the framework; with high uncertainty, on the other hand, government insurance becomes explicit even in those cases where in equilibrium an intervention does not take place.

The main motivation of the paper, as it can be grasped from the introduction, is the 2007-2009 financial crisis that hit the world economy. One of the debates back in the period - which still seems to persist as an open question - refers to how can a government salvage its economy without exposing itself to the problem of moral hazard. The too-big-to-fail policy creates incentives to institutions to behave recklessly: being aware they cannot be simply let go, they allow themselves to indulge in all sorts of investments. At the same time, since a big chunk of the systemic risk an economy faces is represented by the failure of those very same institutions, they cannot be allowed to go under water indeed: the government has to step in to prevent a collapse with more serious consequences.

As it is, the model cannot address questions that deal with the strategic choices of financial

institutions entertaining the possibility of a bailout in times of distress. Several papers already do this, among them the few cited in the related literature section. Rather, the aim is to study how the government, by means of the pursued policies, might end up sowing the seeds of a future crisis that is its intention to avoid in the first place. The effect on prices of government's action seems to be a good place to start: with the possibility of intervention commanding higher prices, if the asset's payoff cannot keep up with the increased cost of the investment, it is more than natural to expect losses to ensue, which would cause a negative shock in the demand and, consequently, a decline in prices. In extreme cases, these mechanics could generate the boom-bust patterns that are often associated with (financial) crises.

Few remarks about extensions and next steps are:

- The way the government's safety net induces higher prices in the model is by attracting investors that otherwise would prefer not to buy - since the asset is in fixed supply, the only way the market can accommodate new participants, i.e., for the market to clear, is through a price increase. To see how this process would evolve requires a dynamic model, which is not the case here;
- Wealth, e.g. availability of credit, seems to be important in fueling those very same bubbles that go hand-in-hand with financial crises and, still, the model is silent about that. Introducing heterogeneity in the endowment of investors would allow one to study the effects on prices of changes in the distribution of wealth<sup>19</sup>;
- The social welfare measure adopted includes, in fact, only the welfare of investors. The model abstracts from the welfare of the government itself, without providing any clue of what the optimal level of intervention should be;
- From the results obtained, following are some testable hypotheses that could be verified empirically<sup>20</sup>:
  - In comparing assets that otherwise would be deemed with the same characteristics, those with an implicitly government guarantee command a price higher than those without;
  - In cases where it is credible to count on the government's safety net, increasing the degree of uncertainty leads to a price increase for assets in low supply and a price

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<sup>19</sup>Dealing with two distributions - one for wealth and another for the signals - might be tricky so in a first step the assumption could be that wealthier agents are also better informed.

<sup>20</sup>A recent empirical paper related to the themes discussed here is Kelly, Lustig, and Van Nieuwerburgh (2011).

decrease for those in high supply;

- Situations of high prices that make the government extremely susceptible to intervene are followed by price disruptions.

# Appendix

## A Comparison of Equilibrium Prices across Different Frameworks with Government Intervention

For the low uncertainty case, the equilibrium price in the common prior scenario is given by  $p^{CP} = A - C - t + 1 - \sqrt{2(A - C)}$  - if the wealth constraint is not binding. To be consistent with the possibility of government intervention, the downside risk must be such that<sup>21</sup>  $0 \leq A - C \leq 1/2$ . With this assumption in place, the following holds

**Proposition 3** *For a sufficiently high realization of the state of fundamentals,*

$$1 > \theta > 1 + \left[ \tau + A - C - \sqrt{2(A - C)} \right], \quad (39)$$

*the ordering of equilibrium prices is*

$$p^{PI} > p^{IP} > p^{CP}; \quad (40)$$

*for an intermediate realization,*

$$1 + \left[ A - C - \sqrt{2(A - C)} \right] < \theta < 1 + \left[ \tau + A - C - \sqrt{2(A - C)} \right], \quad (41)$$

*the ordering is*

$$p^{PI} > p^{CP} > p^{IP}; \quad (42)$$

*finally, for*

$$0 < \theta < 1 + \left[ A - C - \sqrt{2(A - C)} \right], \quad (43)$$

*it holds that*

$$p^{CP} > p^{PI} > p^{IP}, \quad (44)$$

*all the above being true provided the level of uncertainty is low,  $\tau < A - C$ , and the downside risk being such that  $0 \leq A - C \leq 1/2$ , with the equilibrium prices across the different scenarios given by*

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<sup>21</sup>The original assumption to be made, following the discussion after expression (25), is  $0 \leq (A - C) / K \leq 1/2$ . However, recall we are considering the supply level,  $K$ , to be unitary.

(i) *Imperfect information:*  $p^{IP} = \theta - \tau - t$ ;

(ii) *Perfect information:*  $p^{PI} = \theta - t$ ; and

(iii) *Common prior:*  $p^{CP} = A - C - t + 1 - \sqrt{2(A - C)}$ .

**Proof.** We start with  $p^{PI} > p^{IP} > p^{CP}$ . We have

$$\begin{aligned} p^{IP} &> p^{CP} \\ \Leftrightarrow \theta &> 1 + \left[ \tau + A - C - \sqrt{2(A - C)} \right]. \end{aligned} \quad (45)$$

Also,

$$\tau + A - C < 2(A - C) < \sqrt{2(A - C)}, \quad (46)$$

where the first inequality follows from the low uncertainty assumption,  $\tau < A - C$ , and the second from  $0 \leq A - C \leq 1/2$ . Therefore,  $\tau + A - C - \sqrt{2(A - C)} < 0$ , which in turn implies that  $1 + \left[ \tau + A - C - \sqrt{2(A - C)} \right] < 1$ . As  $\theta \in [0, 1]$ , for  $\theta$  sufficiently high, i.e.,  $\theta > 1 + \left[ \tau + A - C - \sqrt{2(A - C)} \right]$ , condition (45) is satisfied, which is equivalent to  $p^{IP} > p^{CP}$ . Finally, since  $\tau > 0$ , it holds trivially that  $\theta - t > \theta - \tau - t$ , i.e.,  $p^{PI} > p^{IP}$ , which completes the proof of the ordering given in (40).

For  $p^{PI} > p^{CP} > p^{IP}$ , first notice that, from (45),  $\theta < 1 + \left[ \tau + A - C - \sqrt{2(A - C)} \right]$  implies  $p^{IP} < p^{CP}$ . Since from  $\tau > 0$  it is trivially true that  $p^{PI} > p^{IP}$ , all that is left is to compare  $p^{PI}$  to  $p^{CP}$ . We have

$$\begin{aligned} p^{PI} &> p^{CP} \\ \Leftrightarrow \theta &> A - C + 1 - \sqrt{2(A - C)}, \end{aligned}$$

which implies that, for

$$1 + \left[ A - C - \sqrt{2(A - C)} \right] < \theta < 1 + \left[ \tau + A - C - \sqrt{2(A - C)} \right], \quad (47)$$

it follows that  $p^{PI} > p^{CP} > p^{IP}$ , as in (42), whereas for

$$0 < \theta < 1 + \left[ A - C - \sqrt{2(A - C)} \right], \quad (48)$$

the ordering of prices is  $p^{CP} > p^{PI} > p^{IP}$ , as in (44), completing the proof. ■

**Proposition 4** *In the high uncertainty scenario,  $\tau > A - C$ , and for a downside-risk such that  $0 \leq A - C \leq 1/2$ , the following holds:*

(i) *If the wealth constraint is binding, the ordering of prices is*

$$p^{CP} > p^{PI} > p^{IP}; \quad (49)$$

(ii) *If the wealth constraint is not binding,*

(a) *For  $\tau > 1/2$ , a sufficiently high realization of the state of fundamentals,*

$$1 > \theta > 1 + A - C - \sqrt{2(A - C)}, \quad (50)$$

*implies the ordering of equilibrium prices to be*

$$p^{PI} > p^{CP} > p^{IP}, \quad (51)$$

*whereas for a sufficiently low realization,*

$$0 < \theta < 1 + A - C - \sqrt{2(A - C)}, \quad (52)$$

*the ordering is reversed to*

$$p^{CP} > p^{PI} > p^{IP}; \quad (53)$$

(b) *For  $\tau < 1/2$ , a sufficiently high realization of the state of fundamentals,*

$$1 > \theta > 1 + 2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}, \quad (54)$$

*implies the ordering of equilibrium prices to be*

$$p^{PI} > p^{IP} > p^{CP}, \quad (55)$$

*whereas for an intermediate realization,*

$$1 + A - C - \sqrt{2(A - C)} < \theta < 1 + 2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}, \quad (56)$$

*the ordering is*



$$p^{PI} > p^{CP} > p^{IP}, \quad (57)$$

and finally, in a sufficiently low realization of the state of fundamentals,

$$0 < \theta < 1 + A - C - \sqrt{2(A - C)}, \quad (58)$$

the ordering that holds is

$$p^{CP} > p^{PI} > p^{IP}, \quad (59)$$

with the equilibrium prices across the different scenarios given by

(i) Imperfect information:  $p^{IP} = \theta + A - C - t - 2\sqrt{\tau(A - C)}$ ;

(ii) Perfect information:  $p^{PI} = \theta - t$ ; and

(iii) Common prior<sup>22</sup>:  $p^{CP} = A$  or  $p^{CP} = A - C - t + 1 - \sqrt{2(A - C)}$ .

**Proof.** Since feasibility requires the wealth to be greater than the price - in case an investment is to be made - it holds trivially that  $p^{CP} > \max\{p^{PI}, p^{IP}\}$ , whenever  $p^{CP} = A$ . Comparing  $p^{PI}$  to  $p^{IP}$  we have

$$\begin{aligned} p^{PI} > p^{IP} \\ \Leftrightarrow \tau > \frac{1}{4}(A - C). \end{aligned} \quad (60)$$

In the high uncertainty case,  $\tau > A - C$ , therefore  $\tau > (A - C)/4$ , implying condition (60) to be satisfied, which is in turn equivalent to  $p^{PI} > p^{IP}$ . Therefore, the ordering of equilibrium prices -in case the wealth constraint is binding - is  $p^{CP} > p^{PI} > p^{IP}$ .

If the wealth constraint is not binding, the price in the common prior case is  $p^{CP} = A - C - t + 1 - \sqrt{2(A - C)}$ . Since we know from the above that  $p^{PI} > p^{IP}$ , all is left is to compare  $p^{CP}$  to  $p^{IP}$  and  $p^{PI}$ , if need be. First,

$$\begin{aligned} p^{IP} > p^{CP} \\ \Leftrightarrow \theta > 1 + \left[2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}\right]. \end{aligned} \quad (61)$$

If  $\tau > 1/2$ , then  $2\sqrt{\tau(A - C)} - \sqrt{2(A - C)} > 0$ , hence  $1 + \left[2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}\right] > 1$ . As  $\theta \in [0, 1]$ , it follows that  $\theta < 1 + \left[2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}\right]$  which, from (61), is

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<sup>22</sup>In the common prior case,  $p^{CP} = A$  if the wealth constraint is binding.

equivalent to  $p^{CP} > p^{IP}$ . Since it was previously established that  $p^{PI} > p^{IP}$ , we can write  $p^{IP} < \min\{p^{PI}, p^{CP}\}$ . Comparing  $p^{PI}$  to  $p^{CP}$ , the equivalent condition is

$$\begin{aligned} p^{PI} &> p^{CP} \\ \Leftrightarrow \theta &> 1 + A - C - \sqrt{2(A - C)}. \end{aligned} \quad (62)$$

The assumption  $0 \leq A - C \leq 1/2$  implies that the right-hand side of (62) is decreasing in  $A - C$ , with  $1 + A - C - \sqrt{2(A - C)} \in [0, 1]$  as a result. Since  $\theta \in [0, 1]$ , (62) can be split in two cases: for a *sufficiently high*  $\theta$ ,

$$1 > \theta > 1 + A - C - \sqrt{2(A - C)}, \quad (63)$$

it follows that (62) is satisfied, which is equivalent to  $p^{PI} > p^{CP}$ , whereas for a *sufficiently low*  $\theta$ ,

$$0 < \theta < 1 + A - C - \sqrt{2(A - C)}, \quad (64)$$

the ordering is reversed to  $p^{CP} > p^{PI}$ .

In summary, combining the above with the previous result that  $p^{IP} < \min\{p^{PI}, p^{CP}\}$ , the final ordering of prices, for a sufficiently high level of fundamentals, is  $p^{PI} > p^{CP} > p^{IP}$ , whereas for a sufficiently low that ordering is  $p^{CP} > p^{PI} > p^{IP}$ , with both prevailing provided that  $\tau > 1/2$  and the wealth constraint is not binding.

We continue with the case where the wealth constraint is not binding, but focusing now on the scenario where  $\tau < 1/2$ . Starting again from (61), we first notice that

$$2\sqrt{\tau(A - C)} - \sqrt{2(A - C)} > -1, \quad (65)$$

since the left-hand side is monotone in  $\tau$  and, for  $\tau \rightarrow 0$ ,  $2\sqrt{\tau(A - C)} - \sqrt{2(A - C)} \rightarrow -\sqrt{2(A - C)}$ , with  $-\sqrt{2(A - C)} > -1$  being implied by the assumption that  $0 \leq A - C \leq 1/2$ . Therefore, if  $\tau > 1/2$ ,  $1 + [2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}] \in [0, 1]$  and, accordingly, two cases are possible: for a *sufficiently high*  $\theta$ ,

$$1 > \theta > 1 + [2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}], \quad (66)$$

condition (61) is satisfied, which is equivalent to  $p^{IP} > p^{CP}$ , whereas for a *sufficiently low* realization,

$$0 < \theta < 1 + [2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}], \quad (67)$$

the ordering is reversed to  $p^{CP} > p^{IP}$ .

Since in the high uncertainty case condition (60) is trivially satisfied - implying  $p^{PI} > p^{IP}$  - for  $\tau < 1/2$  and a sufficiently high realization of  $\theta$ , the price ordering is  $p^{PI} > p^{IP} > p^{CP}$ , whereas for a sufficiently low level of fundamentals we have both  $p^{CP} > p^{IP}$  and  $p^{PI} > p^{IP}$ , meaning that to get the final ordering we still need to compare  $p^{CP}$  to  $p^{PI}$ .

For that, proceeding exactly like in (62), for  $1 > \theta > 1 + A - C - \sqrt{2(A - C)}$ , it follows that  $p^{PI} > p^{CP}$ , whereas if  $0 < \theta < 1 + A - C - \sqrt{2(A - C)}$ , the ordering is  $p^{CP} > p^{PI}$ . In the high uncertainty case we are analysing,  $\tau > A - C$  and, in particular,  $\tau > (A - C)/4$ , which in turn implies that

$$1 + A - C - \sqrt{2(A - C)} < 1 + 2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}, \quad (68)$$

and, given the above, the ordering of prices for a state of fundamentals such that

$$0 < \theta < 1 + A - C - \sqrt{2(A - C)} \quad (69)$$

is  $p^{CP} > p^{PI} > p^{IP}$ , whereas if

$$1 + A - C - \sqrt{2(A - C)} < \theta < 1 + 2\sqrt{\tau(A - C)} - \sqrt{2(A - C)}, \quad (70)$$

the ordering that holds is  $p^{PI} > p^{CP} > p^{IP}$ , completing the proof. ■

## B Equilibrium Conditions in the Imperfect Information Case without Government Intervention

In the case of no government intervention, from (7) it is known that, for any  $i \in I$ ,  $\mathbb{E}(\tilde{\theta} | \xi_i) = \xi_i$ , and from (12) that agents opt to buy only if  $\xi_i \geq p + t$ , in case they can afford to invest in the asset, i.e.,  $A \geq p$ . The market clearing condition is given by

$$\begin{aligned} \frac{1}{2\tau} \int_{p+t}^{\theta+\tau} d\xi = K &\Leftrightarrow \theta + \tau - p - t = 2\tau K \\ &\Leftrightarrow p = \theta + \tau(1 - 2K) - t \equiv p^{NG}. \end{aligned}$$

To support  $p^{NG}$  as the equilibrium price, the following conditions must be satisfied

1.  $\theta - \tau \leq p^{NG} + t \leq \theta + \tau$  (integral condition);
2.  $p^{NG} > 0$  (price positiveness); and

3.  $A \geq p^{NG}$  (feasibility).

For the integral condition,

$$\begin{aligned} \theta - \tau &\leq p^{NG} + t \\ \Leftrightarrow K &\leq 1, \end{aligned} \tag{71}$$

and

$$\begin{aligned} p^{NG} + t &\leq \theta + \tau \\ \Leftrightarrow 0 &\leq K, \end{aligned} \tag{72}$$

and therefore for  $0 \leq K \leq 1$  that condition is satisfied.

For price positiveness

$$\begin{aligned} p^{NG} &> 0 \\ \Leftrightarrow \theta &> -\tau(1 - 2K) + t, \end{aligned} \tag{73}$$

and finally, for feasibility we need

$$\begin{aligned} A &\geq p^{NG} \\ \Leftrightarrow A - \tau(1 - 2K) + t &\geq \theta. \end{aligned} \tag{74}$$

Combining the above yields the following

**Proposition 5** *For  $0 \leq K \leq 1$ , we have that, for any*

$$\theta \in (-\tau(1 - 2K) + t, A - \tau(1 - 2K) + t] \cap [0, 1], \tag{75}$$

*the following holds:*

1.  $\theta - \tau \leq p^{NG} + t \leq \theta + \tau$  (integral condition);
2.  $p^{NG} > 0$  (price positiveness); and
3.  $A \geq p^{NG}$  (feasibility),

where  $p^{NG}$  is the equilibrium price, given by

$$p^{NG} = \theta + \tau(1 - 2K) - t. \quad (76)$$

As a corollary we have

**Corollary 6** *Supposing true the conditions in the proposition above, the equilibrium is characterized by*

1.  $\bar{X}_i = 1, \forall i \in I : \xi_i \geq \theta + \tau(1 - 2K)$ , and  $\bar{X}_i = 0$  otherwise;
2.  $p^{NG} = \theta + \tau(1 - 2K) - t$ .

## C Comparison of Equilibrium Prices across Different Frameworks without Government Intervention

**Proposition 7** *Given a unitary supply level,  $K = 1$ , the following ordering of equilibrium prices prevails across the different informational scenarios - without the participation of the government:*

(i) *If the wealth constraint is binding, the ordering of prices is*

$$p^{NGC} > p^{NGP} > p^{NG}; \quad (77)$$

(ii) *If the wealth constraint is not binding,*

(a) *A sufficiently low realization of the state of fundamentals,*

$$0 \leq \theta < 1/2, \quad (78)$$

*implies the ordering of equilibrium prices to be*

$$p^{NGC} > p^{NGP} > p^{NG}, \quad (79)$$

(b) *A sufficiently high realization,*

$$1/2 < \theta \leq 1, \quad (80)$$

*implies that ordering to be*

$$p^{NGP} > p^{NG} > p^{NGC} \quad (81)$$

if  $1/2 + \tau < \theta \leq 1$ , and

$$p^{NGP} > p^{NGC} > p^{NG}, \quad (82)$$

in case  $1/2 < \theta < 1/2 + \tau$ ,

with the equilibrium prices across the different scenarios given by

(i) Imperfect information:  $p^{NG} = \theta - \tau - t$ ;

(ii) Perfect information:  $p^{NGP} = \theta - t$ ; and

(iii) Common prior<sup>23</sup>:  $p^{NGC} = A$  or  $p^{NGC} = 1/2 - t$ .

**Proof.** If the wealth constraint is binding, in the common prior case the equilibrium price is  $p^{NGC} = A$  and - since feasibility requires the wealth being greater than the price in any scenario - it follows that  $p^{NGC} > \max\{p^{NG}, p^{NGP}\}$ . Given that  $\tau > 0$ ,  $p^{NGP} > p^{NG}$  and, therefore, the ordering of equilibrium prices is  $p^{NGC} > p^{NGP} > p^{NG}$ .

If the wealth constraint does not bind, the equilibrium price in the common prior case is  $p^{NGC} = 1/2 - t$  and, since it is still true that  $p^{NGP} > p^{NG}$ , it remains to compare  $p^{NGC}$  to  $p^{NGP}$  and to  $p^{NG}$ , if need be. For that,

$$\begin{aligned} p^{NGP} > p^{NGC} \\ \Leftrightarrow \theta > 1/2 \end{aligned} \quad (83)$$

and, therefore, according to condition (83), if  $0 \leq \theta < 1/2$ , then  $p^{NGP} < p^{NGC}$ , with the final price ordering being  $p^{NGC} > p^{NGP} > p^{NG}$ . Otherwise, i.e., if  $1/2 < \theta \leq 1$ , then  $p^{NGP} > p^{NGC}$  and, since  $p^{NGP} > p^{NG}$ , it follows that  $p^{NGP} > \max\{p^{NG}, p^{NGC}\}$ , so that to obtain the final ordering of prices it is still needed to compare  $p^{NG}$  to  $p^{NGC}$ . In that regard,

$$\begin{aligned} p^{NG} > p^{NGC} \\ \Leftrightarrow \theta > 1/2 + \tau, \end{aligned} \quad (84)$$

so that, following condition (84),  $p^{NGP} > p^{NG} > p^{NGC}$  in case the level of fundamentals is such that  $1/2 + \tau < \theta \leq 1$ , with  $p^{NGP} > p^{NGC} > p^{NG}$  whenever  $1/2 < \theta < 1/2 + \tau$ . ■

<sup>23</sup>In the common prior case,  $p^{NGC} = A$  if the wealth constraint is binding.

## D Government vs No Government Prices in the Common Prior Scenario

**Proposition 8** *In the common prior scenario, the equilibrium price that prevails in the framework with the possibility of government intervention,  $p^{CP}$ , is at least as high as the one that prevails in the framework without,  $p^{NGC}$ , with the equilibrium prices being given by*

(i) *Government:  $p^{CP} = A$  or  $p^{CP} = \frac{A-C}{K} - t + 1 - \sqrt{2\left(\frac{A-C}{K}\right)}$ ; and*

(ii) *No Government:  $p^{NGC} = A$  or  $p^{NGC} = 1/2 - t$ .*

**Proof.** Recall that, when there is the possibility of government intervention, for the case of common priors about the fundamentals, the equilibrium price is either

$$p^{CP} = A, \tag{85}$$

if  $A \in \left(0, (A - C)/K - t + 1 - \sqrt{2(A - C)/K}\right)$ , else

$$p^{CP} = \frac{A - C}{K} - t + 1 - \sqrt{2\left(\frac{A - C}{K}\right)}, \tag{86}$$

with  $(A - C)/K < 1/2$ . For the case where government is absent, we have the equilibrium price being either

$$p^{NGC} = A, \tag{87}$$

if  $A \in (0, 1/2 - t)$ ,

else

$$p^{NGC} = 1/2 - t. \tag{88}$$

We have, for  $0 < (A - C)/K < 1/2$ ,

$$\frac{A - C}{K} - t + 1 - \sqrt{2\left(\frac{A - C}{K}\right)} > \frac{1}{2} - t, \tag{89}$$

which implies that,

- (i) For  $0 < A < 1/2 - t$ , the two equilibrium prices are the same,  $p^{CP} = p^{NGC} = A$ ;
- (ii) For  $1/2 - t < A < (A - C)/K - t + 1 - \sqrt{2(A - C)/K}$ , the equilibrium price in the case where there is the possibility of government intervention,  $p^{CP} = A$ , is higher than the one when government is absent,  $p^{NGC} = 1/2 - t$ ; and

(iii) For  $(A - C)/K - t + 1 - \sqrt{2(A - C)/K} < A$ , the equilibrium price with no government is  $p^{NGC} = 1/2 - t$ , whereas with government,  $p^{CP} = (A - C)/K - t + 1 - \sqrt{2(A - C)/K}$  and, from (89), the later is greater than the former.

Hence, unless the wealth of investors is low enough so that  $p^{CP} = p^{NGC} = A$ , the equilibrium price when agents entertain the possibility of government intervention is undoubtedly higher than the one that prevails when government is absent. ■



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