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Government Induced Bubbles

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Abstract

We build a model of bubble inflation based on Morris and Shin (1998), with investors deciding whether or not to buy an asset that entails the risk of a collapse in prices, in which case only government intervention could make them avoid a substantial loss. The more investors decide to buy, the more the bubble inflates, and government intervention takes place only when the collapse in prices is sufficiently large. In the benchmark scenario of common knowledge, self-fulfilling beliefs lead to multiplicity of equilibria. Using a global games approach, the introduction of a small noise in the signal received by the speculators yields a unique equilibrium, with intervention occuring only if the state of fundamentals happens to be higher than a particular threshold. In a comparative static exercise, it is shown that the government is more likely to step in and bubbles be large the less liquid the asset and the higher the aggregate wealth of investors.

Keywords: Bubbles; financial crises; global games; unique equilibrium.

JEL Classification: E32; E44; G01.

1 Introduction

Bubble is the term used in the economic literature to define situations where the price at which an asset is transacted does not correspond to the one that would be dictated by its *fundamentals*, i.e., the factors that determine its payoff - the so-called *fundamental price*. In case there is one, the size of a bubble could be obtained by comparing the fundamental price to the price that prevails in the market[1].

To assert that there is a bubble in a particular asset or market is not an easy task, though. For the fundamental price depends on the model adopted and, in case there is a discrepancy between the observed price and the fundamental one, it could be argued that the model being used is misspecified or, indeed, not correct. Moreover, one usually has to recur to irrationality assumptions on the part of investors to justify an asset being transacted at a price which is not the fundamental - itself an assumption hard to call for if one whishes to maintain a minimum level of structure in the model. Finally, there is a problem in justifying the existence of bubbles if one believes that there are no *arbitrage* opportunities in the market, since this implies that investors would react to any misspricing, pushing the price of the asset back to its fundamental value.

The goal of the present paper is not to make a case or not for the existence of bubbles but rather to study a situation that could lead to their emergence - the possibility of government intervention in bust episodes. In particular, the situation we try to depict is the following: investors face the possibility of buying an asset, whose payoff depends only on the state of fundamentals of the economy; the proportion of investors buying the asset makes the observed price to be different from the fundamental one, as if by force of *demand pressure*; given the existence of a bubble, the state of fundamentals determines if there will be a bust - *no* if the fundamentals are *strong*, *yes* if they are *weak*; in case of a bust, the government has the opportunity to intervene and absorb part of the drop in prices, trading off the cost of intevention and the cost of letting the bubble burst, with the later being a function of the magnitude of the *crash* - the difference between the observed and the fundamental price.

In case there is a bust and the government decides to intervene, it does so by *lowering* the interest rate, which increases the fundamental value of the asset and hence reduces the

[1] For a literature review of bubbles, see Brunnermeier (2007).

size of the crash[2]. We think of a situation where, if the crash is too large, by just letting the bubble burst the government risks the economy dipping into a recession, which is potentially more costly than what would result from the adoption of a more lax monetary policy - inflation, for example. This cost comparison is what ultimately drives the action of the government in our setup.

The investors in turn know that, if the bubble does not burst, the capital gain from the appreciation in prices, regardless of its magnitude, always covers the cost of buying, given by the price paid plus a transaction fee that is meant to represent a *liquidity premium*[3]. However, in the case of a bust, the same buying decision is justified only when there is government intervention, otherwise staying out of the market would had been the best decision.

In the sequence, it is shown that, if the investors were to have *perfect information*, buying (not buying) would be a *dominant strategy* for a sufficiently high (low) realization of the state of fundamentals, with multiple equilibria at intermediate levels. This later case of multiplicity of equilibria damages the exercise, as it does not allow one to perform a comparative static analysis - based only on economic reasoning, it is not possible to justify the prevalence of one equilibrium over another when the primitives of the model change. In cases like that, nothing can be said if one is interested in studying the problem from a *policy* perspective, as we are.

As the literature on *global games* shows, however, we can avoid this multiplicity of equilibria by approximating the original game with one of *imperfect information*, where the state of fundamentals is observed by everyone but the government with a small error, rather than being common knowledge. In the limit as the error goes to zero, the game of imperfect information resembles the original one and, by characterizing the *unique* strategies' profile of investors and government that result in equilibrium, the possibility of a comparative static analysis is restored.

We view the assumption of investors receiving a *signal* rather than observing the true realization of the state of fundamentals as a more realistic one, since some degree of noise is always expected to be present in any information dealing with the economy, given the

[2]The fundamental value is given by the expected discounted value of the asset's payoff: decreasing the interest rate decreases the discount factor and hence increases the fundamental price.

[3]Liquidity is defined as the easiness with which one unit of the asset can be transacted, without substantial price changes.

complex nature of the later. The fact that the government keeps its capacity of observing the true state can be viewed as an *information differential* it has over the investors which, even though debatable, we view not unreasonable of an assumption to have. In this way, framing the original situation as a global game, with investors facing *strategic uncertainty*, is justified.

To yield a unique equilibrium, the global game's approach bites when it requires the situation being modeled to satisfy some properties, the particularly important one being *strategic complementarity* on the actions of investors. Strategic complementarity means that the (expected) payoff of adopting one specific action is increasing in the others adopting that very same action. In our model, it boils down to saying that when all the others are buying (not buying), one particular investor is better off buying (not buying) as well.

The strategic complementarity property, not surprisingly, is satisfied in our model, by the following reason: first, investors know that the government, when deciding whether or not to intervene upon a bust episode, trades off the cost of intervention and the cost of the crash itself; second, by buying the asset when all the others also do so, an investor realizes a larger capital gain if the bubble does not burst and, in case it does, she increases the likelihood of government intervention, which is precisely what she wants in this type of situation. This reasoning is reversed in case the buying decision is taken considering that no one else buys.

The unique equilibrium that we characterize is of the *threshold* type, i.e., there is a particular level such that, in a bust episode, if the realization of the state is higher (lower) than that, the government always (never) intervenes. Investors in turn always (never) buy the asset when the signal received is sufficiently high (low). By assumption, the bubble does not burst when the level of fundamentals is sufficiently high and, since that implies the fundamentals being sound, it is not harmful for the government to tolerate a bubble - the economy is in a good shape anyway. This is our justificative for the absence of government intervention for those cases where the government acknowledges that there is a bubble but it does not burst[4].

In the comparative static analysis we explore how the threshold equilibrium is altered when the primitives of the model are changed. For instance, we study the consequences

[4]This is a by-product of the fact that the model does not consider the cost of the economy becoming *overheated*.

of changes in the transaction cost (liquidity) associated with the asset; the aggregate wealth of investors; the critical level of fundamentals that determines if the bubble bursts or not and; the cost of government intervention. By looking to how the equilibrium threshold varies when the primitives of the models change we are able to see under which circumstances government intervention is more likely to occur.

According to the results obtained, government intervention is increasing in the transaction cost of the asset and in the aggregate wealth of investors, while decreasing in the frequency of bubble burts and the cost of intervention. Given that investors have more incentives to buy the asset when it is more likely that the government will intervene, and the higher the proportion of investors buying the asset the more inflated the bubble gets, we can rephrase the last statement saying that bubble inflation is increasing in the transaction cost of the asset and in the aggregate wealth of the investors, while decreasing in the frequency of bubble bursts and the cost of intervention.

Related to the results, first, the bubble getting more inflated the less liquid the asset has a parallel with the latest bubble in the U.S. real state market, whose collapse in the beginning of 2007 subsequently triggered a financial crisis. The real state market is notoriously characterized by assets of low liquidity, which is analogous to a high transaction cost in our model[5]. Second, the positive relationship between the size of the bubble and the aggregate wealth of investors agrees with the usual story according to which an excess of credit helps in fuelling bubbles, by creating an artificial demand that ultimately pushes prices up. Third, increasing the frequency of bubble bursts and the cost of government intervention has more straightforward implications: both make buying the asset less of an attractive choice - capital gains are more difficult to realize and the government is less likely to be there if needs be - which makes bubbles to be smaller, in turn diminishing even more the likelihood of government intervention.

In case the assumptions of the model are in place, the results mentioned above imply

[5]This is not to say that a bubble emerged in the housing market just because of the low liquidity of that particular market. In our model, the less liquid the asset the more costly it is to sell it at the final period, which makes the collapse to be even more pronounced in case of a bust. Since the government intervenes only in large crashes, this implies that there will be more government intervention in less liquid assets, making this type of market more conducive to receive investments and consequently easening the conditions for bubbles to appear. The last bubble episode in the U.S. economy being in the housing market seems to be related to the abundance of credit for investment in that particular market, and not to the fact that bailout guarantees were more explicit there than anywhere else.

the following testable implications: bubbles should occur more often in *less* liquid assets; the more credit is available to investors, or the wealthier they are, the more prone should be the economy to bubbles; the more optimistic the investors are about the economy, i.e., the less likely they think a crash is to happen, the more likely it should be the occurrence of bubbles; bubbles should be observed less often in markets where the cost for the government to intervene is smaller and; bubbles should be observed less often in economies with weak fundamentals.

1.1 Related Literature

The paper is mainly related to the literature on bubbles and global games, and it is motivated by the latest financial crisis of 2007-09. The idea we explore is how the government's anticipated policy during a crisis episode might create conditions for a bubble to emerge. In the sequence we mention some papers that touch upon this theme.

The financial crisis of 2008-09, as documented in Brunnermeier (2009) and Greenlaw, Hatzius, Kashyap, and Shin (2008), was triggered by the collapse of what was arguably viewed as a bubble in the housing market. Different factors leading to bubbles and boom and bust episodes have been studied by several authors. We cite a few.

In Geanakoplos (1997) and Geanakoplos (2003), it is developed the *leverage cycle theory*, according to which boom and bust episodes are due to the leverage conditions available to investors. When the economy is doing well, there is a loosening in credit conditions that leads to a higher leverage of investors' portfolios, producing a boom. When the fundamentals of the economy deteriorate, the very same investors, for different reasons (e.g., margin requirements), are forced to deleverage their positions, making assets to change hands from people who appreciate them more to people who appreciate them less, which depresses prices. Therefore, times when leverage is allowed to increase lead to booms (and potentially bubbles), with busts happening when investors are required to reduce their exposition to risk due to a worsening in the economy's fundamentals.

Along similar lines, Mendoza (2010) builds an equilibrium business cycle model with a collateral constraint, introducing in the model a *Fisherian* debt-deflation mechanism: a sufficiently high leverage ratio leads to the fire-sale of assets in case the economy is hit by a negative shock, which causes prices to fall and consequently tightens the collateral constraint even more, leading to further fire-sales and so on. It is again the drivers of the leverage ratio of investors that are deemed responsible for episodes of boom and bust.

Brunnermeier and Sannikov (2010) develop a macroeconomic model with a financial sector and, rather than just analyzing the behavior of the system around the steady state, provide the full dynamics of the economy, showing how it oscilates between episodes of high and low volatility. They explore the interplay between the economy's financial and real sector, focusing on the externality the former imposes on the later. It is by neglecting this externality that the financial sector gets too leveraged, leading to an amplification of negative shocks.

The connection between *market liquidity* and *funding liquidity* is the relevant issue in Brunnermeier and Pedersen (2009). In their model, market liquidity refers to how easily an asset can be transacted, with funding liquidity being related to the availability of funding for trading by speculators. The authors show under which circumstances the economy might face *liquidity spirals*, the process by which a loss suffered by speculators leads to funding problems, increasing margins and subsequently decreasing the market liquidity of assets, depressing prices and making the losses even more pronounced, which in turn feedbacks into further decreases in funding liquidity. It is in this sense that the authors claim that margins can be destabilizing. The model offers several new testable implications demanding further empirical investigation.

Arguably, one important factor triggering boom and bust episodes is the *coordination motive* of investors, in the following sense: if there is a perception that a good investment opportunity is available on the market, by coordinating to buy altogether investors might produce a boom; if the sentiment is that an asset currently held does not have positive prospects anymore, investors might want to sell it, which might cause prices to collapse if done jointly. Importantly, the perception of a good investment opportunity depends not only on the research made by the investor herself but also on what she thinks the others think of the asset, i.e., her *beliefs about the beliefs of the others*, after all it is more realistic to assume that agents have different information sets.

One consequence of investors following strategies that depend on their beliefs is that, in case there is a mechanism device by which actions might be coordinated, multiple equilibria can be supported. This is forcefully exemplified in the seminal work of Diamond and Dybvig (1983) on bank runs, where both a "good" and a "bad" equilibrium are supported: the bad one, for example, happens when investors anticipate that others will panick and run to the bank, leading them to choose to run as well, materializing the bank run that originally was only a hypothesis in investors' minds. Several other economic situations can be framed in a similar setup, as exposed in Cooper (1999).

Morris and Shin (2000), however, argue that this indeterminacy of beliefs leading to multiple equilibria is not grounded in economic terms but it is rather a consequence of two modeling assumptions usually made: *common knowledge* of fundamentals and agents being assumed, in equilibrium, to be certain about each other's behavior. A public signal about the fundamentals, once assumed to be common knowledge, for instance, might constitute the mechanism device that allows agents to perfectly coordinate their beliefs and actions, in a way that, as the authors put it, invites to a multiplicity of equilibria.

The *global games* approach, a term coined in Carlsson and van Damme (1993), provides a way around this problem. The basic idea is to add a small noise to the signals to be received by the agents, breaking down the assumption of common knowledge. Without that assumption, agents are not sure about what each other knows, i.e., the information each other has. This uncertainty ends up restricting the set of possible actions to a *unique* optimal one. Morris and Shin (2003) exposes the theory behind the global games approach and discuss some applications.

Several papers use the global games approach to the study of financial crises of some sort. Morris and Shin (2009) compares the portion of credit risk that is due to insolvency, i.e., when the payoff of an asset falls short of the obligation to be paid, to that resulting from illiquidity, i.e., when default in turn is due to a lack of funding that otherwise would allow the borrower to carry on the project, which can occur despite the investment being profitable. This gives rise to situations where an illiquid but otherwise solvent institution defaults (aka Bear Sterns). He and Xiong (2010) develop a similar idea but in a dynamic context, showing that, by spreading the maturity of debt obligations over time, borrowers create a dynamic coordination problem to lenders, potentially leading to preemptive runs. Guimarães (2006) studies currency attacks, in a framework where agents have to decide when to act and, due to market frictions, bear the risk of being caught by a devaluation. In Abreu and Brunnermeier (2003), agents become aware sequentially of the existence of a bubble, and at any point in time there is uncertainty regarding the fraction of speculators who ackowledge it. Speculators then might prefer to ride the bubble and realize one extra period of capital gains, instead of cashing in the profits immediately, bearing the risk of a bust in between. This leads bubbles to persist, regardless of the presence of arbitrageurs.

The paper we draw heavily on is Morris and Shin (1998), where a static model of currency attack is developed. In their model, speculators obtain a positive gain in case they attack the currency and the government is forced to abandon the peg, which happens whenever the cost of defending is larger than the cost of letting the currency float. The cost of mantaining the peg is decreasing in the state of fundamentals and increasing in the number of speculators, as economic intuition would dictate. In the benchmark case where the state of fundamentals is common knowledge, a multiplicity of equilibria arises, due to the self-fulfilling nature of beliefs, whereas if the speculators observe the state with a small noise, this multiplicity reduces to a unique equilibrium, with speculators attacking if and only if the signal received is sufficiently small - that indicates, after all, that the government is in a weak position to defend the peg.

Among other things, the unique equilibrium obtained in Morris and Shin (1998) is interesting because it allows one to perform a comparative static analysis and, in our context, to study which factors are more conducive to the emergence of bubbles. This is the main motivation behind us following Morris and Shin's approach. In particular, as in their model speculators attack the currency based on (i) their perception of the government's cost of defending the peg and (ii) the fraction of others attacking, in ours agents buy an asset and fuel a bubble in accordance with (i) what they perceive to be the cost for the government to intervene and (ii) the fraction of other speculators who also want to invest.

One of the main debates throughout the financial crisis of 2007-09 was regarding the participation of the government in the bailout of instituions that suffered prominent losses, whose eventual collapse would supposedly represent a serious threat to the stability of the entire financial system. One view is that, with the implicit government support of the so-called "too-big-to-fail" institutions, incentives are created so that more risks are taken, since profits are privately gained and losses are public shared[6]. Although not exactly with the same flavor, the assumption of government intervention absorbing crashes is made exactly trying to capture this type of situation, in order to see if the possibility of government participation during crises would result in speculators buying the asset more often, making the economy more prone to bubbles. As it turns out, this is indeed what

[6] The bailout money, after all, usually comes from tax payers.

happens in our model.

The structure of the paper is as follows: in the next section we detail the model, first analyzing the benchmark case of perfect information about the state of fundamentals; following, we assume that speculators only receive a signal about the state of the economy and, with that assumption in place, the unique equilibrium is derived, with an exercise in comparative static analysis being performed subsequently; finally, the last section digresses on some of the testable implications of the model.

2 Model

Consider a 1-good, 3-period economy, $t = 0, 1, 2$, populated by a unit mass of agents, to be called *speculators*, represented by the set $I = [0, 1]$[7]. At the initial date, $t = 0$, each speculator can take one of two actions, *buy or not* the asset, that's to say, each $i \in I$ can either choose $X_0^i = 1$ (buy) or $X_0^i = 0$ (not buy). If a speculator decides to buy the asset she pays its initial price, p_0, plus a fee that represents a transaction cost, t, to be incurred at $t = 2$ when the asset is sold. If she decides not she incurs no cost. Each speculator can buy at most one unit of the asset.

After the speculators make their decisions, in period $t = 1$, the *interim stage*, the price of the asset adjusts to $p_1 \equiv g(\theta, r, \alpha)$, where θ is a variable that represents the state of fundamentals of the economy, r is the prevailing interest rate and α is the fraction of speculators who buy the asset. We think of $g(\cdot)$ as the *observed price* of the asset at the interim stage, with the following assumption[8]:

Assumption 1 *$g_\theta > 0$, $g_r < 0$ and $g_\alpha > 0$.*

The assumption above states that the observed price is increasing in the fundamentals of the economy, decreasing in the interest rate and increasing in the fraction of speculators who buy the asset. If we think of the asset as being an investment whose performance is better the better are the fundamentals of the economy, it is natural to assume that $g(\cdot)$ is increasing in its first argument. Because the price represents the present value of future payoffs, an increase in the interest rate should decrease it. The price being positively

[7] We use the words good and asset interchangeably.

[8] The subscript represents the partial derivative of the function with respect to that argument.

dependent on the fraction of speculators buying the asset represents the effect of demand pressure.

The observed price, given by the function $g(\theta, r, \alpha)$, must not necessarily be equal to the *fundamental price* of the asset, which we denote by $f(\theta, r)$. The important difference is that, in our interpretation, the fundamental price should not be a function of the fraction of investors buying the asset, since the asset itself represents the present value of a future payoff that is independent of the number of buyers. With the observed price being different from the fundamental one, we define a *bubble* by:

$$b(\theta, \alpha) \equiv g(\theta, r, \alpha) - f(\theta, r). \tag{1}$$

We concentrate on *positive bubbles*, and we also assume that, when the fundamentals improve, the observed price increases more than the fundamental value, which could be seen as a result of an increase in the confidence of the speculators, whose actions have an effect on the observed price but not in the fundamental value. In other words, we have:

Assumption 2 *For any θ, r and α, the functions $f(\cdot)$ and $g(\cdot)$ are such that $g(\theta, r, \alpha) \geq f(\theta, r)$. Also, $b_\theta > 0$.*

Analogously to the assumption made for the function representing the observed price, we impose that:

Assumption 3 $f_\theta \geq 0$, $f_r < 0$.

At the interim stage, if the fundamentals of the economy are strong enough (strong economy), that is to say, if $\theta > L$, for a specific L, the bubble does not burst and the price of the asset in period $t = 2$ equals the price at the interim stage, $g(\theta, r, \alpha)$. If the bubble does burst (weak economy), $\theta \leq L$, the price in the final period depends on the *government's market intervention decision*. We model the government behavior assuming that it faces a cost equal to the size of the bubble in case it bursts and the government does not interverne, causing the price to drop from $g(\theta, r, \alpha)$ to the fundamental one, $p_2 = f(\theta, r)$. Otherwise, by intervening the government pays the cost c to set the interest rate at $r^* < r$, making the price to drop to $p_2 = f(\theta, r^*) > f(\theta, r)$ instead. Therefore, the investors who buy the asset always benefit when the government decides to intervene in a bust episode, and the government does so whenever $b(\theta, \alpha) \geq c$.

We can think of the above specification in the following way: if the economy is strong enough, the bubble has no significant cost for the government, after all the fundamentals are sound and the fact that there is a positive bubble represents no significant threat. On the other hand, that is to say, if the fundamentals are relatively weak and there is a bubble, $\theta \leq L$, it bursts and the drop in the price level can be sufficiently large, $b(\theta, \alpha) \geq c$, such that, if it realizes, the economy faces the risk of dipping into a serious recession, which would cause a large cost for the government. If that is the case, the government prefers to intervene in the market, lowering the interest rate and therefore absorbing some of the impact of the bust. The parameter c can be viewed as a cost associated with the adoption of a laxer monetary policy, like inflation, after all the government reduces the interest rate when it intervenes in the market.

Given the speculators' decisions and the government's action, the payoff (return) to a speculator at period $t = 2$ is:

$$R^i\left(X_0^i, \theta, r\right) \equiv X_0^i\left(p_2 - p_0 - t\right) = \begin{cases} X_0^i\left[g\left(\theta, r, \alpha\right) - p_0 - t\right] & \text{if no bust} \\ X_0^i\left[f\left(\theta, r\right) - p_0 - t\right] & \text{if bust \& w/o govt} \\ X_0^i\left[f\left(\theta, r^*\right) - p_0 - t\right] & \text{if bust \& w/ govt} \end{cases} \quad (2)$$

where *no bust* represents the case when the bubble does not burst, $\theta > L$, and, in a bust episode, $\theta \leq L$, *w/o govt* means that the government does not intervene, with no change in the interest rate, while if the government intervenes, *w/ govt*, it sets the interest rate at r^*.

With the specification above, we proceed to analyze the equilibrium that would result in the economy under two different scenarios: first when the speculators have perfect information about the state of fundamentals, θ, and then assuming they receive only a signal x about it, while the government can still observe the true value. As we will see in the sequel, with perfect information, for a certain range of parameters, there is multiplicity of equilibria, whereas in the case of a small level of uncertainty about the fundamentals the resulting equilibrium is unique, allowing us to perform comparative static analysis.

2.1 Perfect information about the fundamentals

We start assuming that, in the economy specified above, the speculators have perfect information about the parameter θ, representing the fundamentals of the economy. The parameter L is also assumed to be common knowledge. We can think of the following scenario:

1. Nature draws θ according to a uniform distribution, $\theta \sim \mathcal{U}[0,1]$
2. Government and speculators observe θ
3. Speculators choose to buy or not the asset
4. Government observes the realized proportion of speculators who buy the asset, α, and decides to intervene or not
5. Game ends (investors sell the asset).

An *equilibrium* is a profile of *strategies* for the speculators and the government such that, in equilibrium, none of them has an incentive to deviate. A strategy for a speculator is a rule that assigns a specific acion for each possible realization of the fundamentals, θ, while for the government is the decision of intervening or not in the market for all possible combinations of the fundamentals and the fraction of speculators who buy the asset, θ and α. We can solve for the equilibrium in this model by proceeding backwards. First we determine the optimal strategy of the government, given the fundamentals and the action of the speculators, and, based on that, we determine the optimal strategy of the speculators themselves. Following Morris and Shin (1998) we make some additional assumptions in order to make the problem economically interesting:

Assumption 4 *The functions representing the observed price and the fundamental value, $g(\theta, r, \alpha)$ and $f(\theta, r)$, respectively, are such that the induced bubble $b(\theta, \alpha)$ satisfies the following:*

- $b(L, 0) < c$
- $b(0, 1) < c$
- $b(L, 1) > c.$

An illustration of assumption 4 is given in the first graph of figure 1. The first above, $b(L, 0) < c$, implies that, when the speculators do not buy the asset, that is to say, for $\alpha = 0$, the cost for the government to intervene in the market is always larger than the cost of the bubble bust itself, either if, conditional on a bust, the economy is at its worst or best state of fundamentals, $\theta = 0$ or $\theta = L$, respectively[9]. We could say that one way a bust episode can severely impair the economy, and hence be very costly for the government, is by affecting the confidence of investors who ultimately suffer the losses caused by the price drop. If the fraction of investors in the market is negligible, or if the market in question does not represent a significant fraction of the economy, with $\alpha = 0$ being a proxy for both situations, then the above says that the government does not intervene.

[Figure 1 around here.]

The second assumption made above is that, in the worst state of fundamentals given a bubble burst, $\theta = 0$, the bubble is never large enough to justify government intervention, even if all the speculators are long on the asset, $\alpha = 1$, which corresponds to $b(0,1) < c$. This tries to capture the idea that, if the fundamentals are too poor, a bubble never gets too large just as a byproduct of the action of speculators going long on the asset, that is to say, if the economy is too weak the speculators cannot have any hope to inflate the price of the good just by putting demand pressure on it.

The last one, $b(L,1) > c$, represents the fact that, in a bust episode, even when the fundamentals are at their best, $\theta = L$, if there is too much demand pressure, $\alpha = 1$, the bubble burst becomes too large to be tolerated, after all the economy in this scenario is weak, $\theta \leq L$. The government then intervenes in the market.

Following Morris and Shin (1998), let us denote by $\underline{\theta}$ the value of θ which solves, for $\theta \leq L$, the equation $b(\theta,1) = c$. In other words, $\underline{\theta}$ is the value of θ at which the government is indifferent between intervening or not in the market when all the speculators are long on the asset, that is, $\alpha = 1$. We assume the following:

Assumption 5 *The functions representing the observed price and the fundamental value, $g(\theta,r,\alpha)$ and $f(\theta,r)$, together with the initial price and transaction cost of the asset, p_0 and t, satisfy:*

- $f(\theta,r) < p_0 + t$
- $f(\underline{\theta},r^*) = p_0 + t$

[9] Remember that $b_\theta > 0$ and, therefore, $b(L,0) < c$ implies that $b(0,0) < c$.

- $g(L, r, \alpha) \geq p_0 + t$.

Hence, if there is no government intervention, the fundamental value in a bust episode is always less than the total cost for the speculator to be long on the asset. On the other hand, the speculator always benefit from government intervention when $\theta \in [\underline{\theta}, L]$, since in that range $f(\underline{\theta}, r^*) > p_0 + t$. The last above means that, in case the bubble does not burst, the observed price is always larger than the total cost of buying the asset. An illustration of assumption 5 is given in the second graph of figure 1.

With the above assumptions in place, for the case when θ is common knowledge we have:

- $\theta \in [0, \underline{\theta}]$: the cost of intervention is larger than just letting the bubble burst, since we know that $b_\theta > 0$ and $b(\underline{\theta}, 1) = c$. No matter what the government's decision is, the total cost for an speculator to buy the asset is always larger than the maximum return that she could obtain. Therefore, in this range the government does not intervene and is a dominant strategy for each speculator to not buy the asset, which makes "not buy" (speculators) and "not intervene" (government) to be an equilibrium;

- $\theta \in (\underline{\theta}, L]$: the cost of intervention is less than the bubble burst cost, provided that a sufficiently large number of speculators buy the asset. In particular, if none of the speculators buy the asset, the bubble cost is less than the cost of intervention, as mentioned before, and hence the government will not intervene in the market, which in turn justifies the decision of not buy of the speculators in the first place, making "not buy" (speculators) and "not intervene" (government) to be an equilibrium. On the other hand, since by assumption $f(\theta, r^*) > p_0 + t$ for any $\theta \in (\underline{\theta}, L]$, that is, a speculator will make a positive profit if the government is to intervene when θ lies in this specific range, and the fact that in this case the government always intervene since $b(\theta, 1) > c$, "buy" (speculators) and "intervene" (government) is also an equilibrium. Analogously to the nomenclature used by Morris and Shin (1998), we name this range *ripe for intervention*;

- $\theta \in (L, 1]$: the fundamentals are sound and the bubble neither bust nor represents a threat for the economy, requiring no government intervention. Since $g(\theta, r, \alpha) > p_0 + t$ for any level of the fundamentals in this range, it is a dominant strategy for the speculators to buy the asset, making the pair "buy" (speculators) and "not intervene" (government) to be an equilibrium.

Summarizing, if the fundamentals are too poor, $\theta \in [0, \underline{\theta}]$, or sufficiently good, $\theta \in (L, 1]$, the government does never interverne, "not buy" being a dominant strategy for the speculators in the first case and "buy" in the second. Figure 1 provides an illustration of the resulting situations according to the state of fundamentals realized, θ.

Following Morris and Shin (1998), the interesting range of the fundamentals is the ripe for intervention region. If we suppose that the government makes its intervention decision based only on the trade-off between the cost of intervention, c, and the cost of no intervention when the bubble burst, $b(\theta, \alpha)$, then the fact that the speculators have perfect information regarding the realization of θ gives rise to the standard case of multiple equilibria due to the self-fulfilling nature of the speculators' beliefs, as discussed before. One major problem is that, given this multiplicity of equilibria, no prediction can be made as to whether the investor will end up buying the asset (and hence inflating the bubble) or not. As we will see next, this situation changes when the speculators face a small amount of uncertainty concerning the fundamentals, with each state of fundamentals giving rise to a unique equilibrium.

2.2 Imperfect information

We now relax the assumption of speculators having perfect information about the state of fundamentals in the economy, θ. In particular, we assume the following structure:

1. Nature draws θ according to a uniform distribution, $\theta \sim \mathcal{U}[0, 1]$
2. Government observes θ and each speculator receives a private signal x, $x \sim \mathcal{U}[\theta - \epsilon, \theta + \epsilon]$, for some small ϵ
3. Speculators choose to buy or not the asset
4. Government observes the realized proportion of speculators who buy the asset, α, and decides to intervene or not
5. Game ends (investors sell the asset).

As in the case with perfect information, we solve for the equilibrium by analyzing first the behavior of the government and then determining the reaction function of the investors. We know that the government intervenes in a bust episode only if $b(\theta, \alpha) \geq c$. Following Morris and Shin (1998), consider the critical proportion of speculators needed to trigger the government to intervene when the state of fundamentals is θ, and let $a(\theta)$ denote this critical mass. In other words, for given θ, $a(\theta)$ is the value of α that solves $b(\theta, \alpha) = c$.

As in Morris and Shin (1998), for a given profile of strategies of the speculators, we denote by $\pi(x)$ the proportion of speculators who buy the asset when the value of the signal received is x[10]. Also, let $s(\theta, \pi)$ be the proportion of speculators who end up buying the asset when the state of fundamentals is θ. Since $x \sim \mathcal{U}[\theta - \epsilon, \theta + \epsilon]$ we have:

$$s(\theta, \pi) = \frac{1}{2\epsilon} \int_{\theta-\epsilon}^{\theta+\epsilon} \pi(x)\, dx. \tag{3}$$

Let $A(\pi)$ be the event where the government intervenes if the speculators follow strategy π, that is:

$$\begin{aligned} A(\pi) &\equiv \{\theta |\, s(\theta, \pi) \geq a(\theta)\} \qquad (4) \\ &= \left\{\theta |\, \frac{1}{2\epsilon} \int_{\theta-\epsilon}^{\theta+\epsilon} \pi(x)\, dx \geq a(\theta)\right\}. \qquad (5) \end{aligned}$$

We express the payoff to a speculator of buying the asset at state θ, $X_0^i = 1$, when aggregate buying activity is π, as:

$$h(\theta, \pi) \equiv \begin{cases} g(\theta, r, \alpha) - p_0 - t & \text{if no bust} \\ f(\theta, r) - p_0 - t & \text{if bust \& w/o govt} \\ f(\theta, r^*) - p_0 - t & \text{if bust \& w/ govt.} \end{cases} \tag{6}$$

However, as mentioned before, the speculators do not observe the true state of fundamentals, θ, and hence they calculate the expected payoff to buying the asset conditional on the signal x, given by:

$$\begin{aligned} u(x, \pi) &\equiv \frac{1}{2\epsilon} \int_{x-\epsilon}^{x+\epsilon} h(\theta, \pi)\, d\theta \qquad (7) \\ &= \frac{1}{2\epsilon} \left[\int_{[L, x+\epsilon]} h(\theta, \pi)\, d\theta + \int_{[x-\epsilon, L] \cap A(\pi)^c} h(\theta, \pi)\, d\theta + \int_{[x-\epsilon, L] \cap A(\pi)} h(\theta, \pi)\, d\theta \right] \qquad (8) \end{aligned}$$

[10]Since there is a unit mass of speculator, this can be interpreted as the probability that a speculator chooses to buy the asset when she receives signal x.

where the second line follows from splitting the domain of integration in the first integral into three mutually exclusive sets, namely the range for which the bubble does not burst, $[L, x+\epsilon]$, and the range in which it does, $[x-\epsilon, L]$, which in turn we split in cases when the government intervenes or not, $A(\pi)$ and $A(\pi)^c$, respectively. Using the definition of $h(\theta, \pi)$ we can write:

$$\begin{aligned} u(x,\pi) &= \frac{1}{2\epsilon}\left[\int_{[L,x+\epsilon]} g(\theta, r, s(\theta,\pi))\, d\theta + \int_{[x-\epsilon,L]\cap A(\pi)^c} f(\theta, r)\, d\theta \right. \\ &\quad \left. + \int_{[x-\epsilon,L]\cap A(\pi)} f(\theta, r^*)\, d\theta \right] - p_0 - t \end{aligned} \tag{9}$$

Since a speculator can guarantee a payoff of zero by refraining from buying the asset, that is, by choosing $X_0^i = 0$, the rational decision conditional on signal x depends on whether $u(x,\pi)$ is positive or negative. Thus, if the government follows its unique optimal strategy, π is an equilibrium of the first period game if $\pi(x) = 1$ whenever $u(x,\pi) > 0$ and $\pi(x) = 0$ whenever $u(x,\pi) \leq 0$.

We now enunciate and prove the main result of this section, the analogous version of theorem 1 in Morris and Shin (1998), the unique equilibrium outcome that emerges in the game with imperfect information:

Theorem 6 *There is a unique θ^* such that, in any equilibrium of the game with imperfect information, conditional on a bubble bust episode the government intervenes if and only if $\theta^* \leq \theta \leq L$.*

To prove the theorem, we proceed in three steps: first we show that the buying decision of the speculators satisfies the property of *strategic complementarity*, that is, one speculator is always better off buying the asset whenever the others are doing the same; second, we prove that the function $u(x,\pi)$ is continuous and monotonic in its first argument; finally, we establish that the equilibrium π is given by a step function, with a unique x^* such that the speculators buy the asset whenever the signal they receive is such that $x \geq x^*$. With these three results is a short step to prove the theorem above. The structure of the proofs follow Morris and Shin (1998). We begin establishing the following:

Lemma 7 *If $\pi(x) \geq \pi'(x)$ for any x, then $u(x,\pi) \geq u(x,\pi')$, for any x.*

Proof. Since $\pi(x) \geq \pi'(x)$, we have $s(\theta,\pi) \geq s(\theta,\pi')$, for any θ, from the definition of s given by (3). Thus, from (4) we have:

$$A(\pi) \supseteq A(\pi') \Rightarrow A(\pi)^c \subseteq A(\pi')^c . \tag{10}$$

In other words, the event in which the government interves in the market, setting $r = r^*$, that is, lowering the interest rate, is strictly larger under π. This and assumptions 1 and 3 yields:

$$\begin{aligned} u(x,\pi) &= \frac{1}{2\epsilon}\left[\int_{[L,x+\epsilon]} g(\theta, r, s(\theta,\pi))\, d\theta + \int_{[x-\epsilon,L]\cap A(\pi)^c} f(\theta, r)\, d\theta \right. \\ &\quad \left. + \int_{[x-\epsilon,L]\cap A(\pi)} f(\theta, r^*)\, d\theta \right] - p_0 - t \\ &\geq \frac{1}{2\epsilon}\left[\int_{[L,x+\epsilon]} g(\theta, r, s(\theta,\pi'))\, d\theta + \int_{[x-\epsilon,L]\cap A(\pi')^c} f(\theta, r)\, d\theta \right. \\ &\quad \left. + \int_{[x-\epsilon,L]\cap A(\pi')} f(\theta, r^*)\, d\theta \right] - p_0 - t \\ &= u(x,\pi'), \end{aligned}$$

which proves the lemma. ■

For the next step, consider the strategy profile where every speculator buys the asset if and only if the message x is more than some fixed number k. Then, aggregate buying activity π will be given by the indicator function I_k, defined as:

$$I_k = \begin{cases} 1 & \text{if } x \geq k \\ 0 & \text{if } x < k. \end{cases} \tag{11}$$

When speculators follow this simple rule of action, the expected payoff to buying the asset satisfies the following property:

Lemma 8 *$u(k, I_k)$ is continuous and strictly increasing in k.*

Proof. As in Morris and Shin (1998), consider the function $s(\theta, I_k)$, the proportion of speculators who buy the asset at θ when the aggregate buying activity is given by the step function I_k. Since x is uniformly distributed over $[\theta-\epsilon, \theta+\epsilon]$ we have:

$$s(\theta, I_k) = \begin{cases} 0 & \text{if } k \geq \theta+\epsilon \Leftrightarrow \theta \leq k-\epsilon \\ \frac{1}{2} + \frac{(\theta-k)}{2\epsilon} & \text{if } k-\epsilon \leq \theta \leq k+\epsilon \\ 1 & \text{if } k \leq \theta-\epsilon \Leftrightarrow \theta \geq k+\epsilon \end{cases}$$

where the second line follows from calculating the integral $[1/(2\epsilon)] \int_k^{\theta+\epsilon} d\theta$.

If aggregate short sales are given by I_k, there is a unique θ (which depends on k) where the mass of speculators buying equals the mass of speculators necessary to cause the government to intervene in the market, that is, where $s(\theta, I_k) = a(\theta)$ [11]. Write $\psi(k)$ for the amount that θ must be short of k for this to be true. In other words, $\psi(k)$ is the unique value of ψ solving $s(k-\psi, I_k) = a(k-\psi)$.

Since the government intervenes in the market if and only if θ lies in the interval $[k-\psi(k), L]$, the payoff function $u(k, I_k)$ is given by:

$$\begin{aligned} u(k, I_k) &= \frac{1}{2\epsilon}\left\{\int_L^{k+\epsilon} g(\theta, r, s(\theta, I_k))\, d\theta + \int_{k-\epsilon}^{k-\psi(k)} f(\theta, r)\, d\theta \right. \\ &\left. + \int_{k-\psi(k)}^{L} f(\theta, r^*)\, d\theta \right\} - p_0 - t. \end{aligned}$$

We have:

$$\begin{aligned} & \frac{d}{dk}\left\{\int_L^{k+\epsilon} g(\theta, r, s(\theta, I_k))\, d\theta + \int_{k-\epsilon}^{k-\psi(k)} f(\theta, r)\, d\theta + \int_{k-\psi(k)}^{L} f(\theta, r^*)\, d\theta\right\} \\ = \; & g\left(k+\epsilon, r, \underbrace{s(k+\epsilon, I_k)}_{=1}\right) + \int_L^{k+\epsilon} g_\alpha(\theta, r, s(\theta, I_k))(-1/2\epsilon)\, d\theta \\ & + f(k-\psi(k), r)\left(1-\psi'(k)\right) - f(k-\epsilon, r) - f(k-\psi(k), r^*)\left(1-\psi'(k)\right) \\ = \; & g(k+\epsilon, r, 1) - f(k-\epsilon, r) + \left[f(k-\psi(k), r) - f(k-\psi(k), r^*)\right]\left(1-\psi'(k)\right) \\ & - \frac{1}{2\epsilon}\int_L^{k+\epsilon} g_\alpha(\theta, r, s(\theta, I_k))\, d\theta \qquad (12) \end{aligned}$$

where we have used that fact that, for $k-\epsilon \le \theta \le k+\epsilon$, $s(\theta, I_k) = 1/2 + (\theta - k)/2\epsilon$, which implies that, for $\theta = k - \psi(k)$, we can write $1/2 - \psi(k)/2\epsilon = a(k-\psi(k))$. By totally differentiating this last expression we arrive at:

$$\psi'(k) = \frac{-a'}{1/2\epsilon - a'} \ge 0 \qquad (13)$$

with the last inequality following from the fact that $a_\theta < 0$ (see footnote 11). From the above we can also see that for ϵ sufficiently small we have both $\psi(k) \approx 0$[12] and $\psi'(k) \approx 0$, hence $1 - \psi'(k) \ge 0$.

Returning to expression (12), for sufficiently small ϵ we have $k + \epsilon < L$ which, combined with $\psi(k) \approx 0$ and $\psi'(k) \approx 0$, allows us to claim that:

[11] Remember that, in a bust episode, for a given θ, $a(\theta)$ is the value of α such that $b(\theta, \alpha) = c$; the assumption that $b_\theta > 0$ implies that $a' < 0$ and hence there is a unique θ at which $s(\theta, I_k) = a(\theta)$.

[12] The fact that $\psi(k) \ge 0$ is implicitly given by $1/2 - \psi(k)/2\epsilon = a(k-\psi(k))$, with $a(\theta) \ge 0$, implies that for sufficiently small ϵ we must necessarily have $\psi(k) \approx 0$.

$$\underbrace{g\left(k+\epsilon,r,1\right)-f\left(k,r^{*}\right)}_{>0}+\underbrace{f\left(k,r\right)-f\left(k-\epsilon,r\right)}_{>0}>0 \tag{14}$$

leading us to conclude that $u(k, I_k)$ is monotonically increasing in k.

Finally, $u(k, I_k)$ is continuous since is an integral in which the limits of integration are themselves continuous in k. This concludes the proof of the lemma. ■

Before we proceed to the last result necessary to prove theorem 6, we discuss the intuition behind lemma 8. Recall that when buying activity π is given by the indicator function I_k, $\pi(x) = 1$ if $x \geq k$ and zero when $x < k$. The speculators therefore buy the asset only when their signal indicate that the state of fundamentals is sufficiently high. Otherwise they risk to be caught in a bust episode where the government does not intervene, which happens when the state of fundamentals is too poor, yielding a negative payoff [13]. Hence, being more restrictive in the investment decision, that is to say, increasing k, guarantees that speculators buy only at relatively better states, where bubbles are less likely to burst and, if that happens, the crash (bubble) will be large, demanding in turn government intervention and hence increasing the payoff to speculators who buy.

Put another way, when the fundamentals of the economy are stronger, the payoff to buying the asset is higher for a speculator on the margin from not buying to buying the asset. Analogously to the discussion in Morris and Shin (1998), such a property would be a reasonable feature of any model of bubbles where the bubble cannot be strong (large) enough if the fundamentals are weak and the government intervenes in the market only in cases of large busts. We now establish the last result necessary to prove the theorem above:

Lemma 9 *There is a unique x^* such that, in any equilibrium of the game with imperfect information of the fundamentals, a speculator with signal x buys the asset if and only if $x > x^*$.*

Proof. Again following Morris and Shin (1998), to prove the above we begin by establishing that there is a unique value of k at which

$$u(k, I_k) = 0. \tag{15}$$

From lemma 8, we know that $u(k, I_k)$ is continuous and strictly increasing in k. If we can show that it is positive for large values of k and negative for small ones, then we can guarantee that $u(k, I_k) = 0$ for some k.

When k is sufficiently large (i.e., $k \geq L + \epsilon$), the speculator knows that the bubble will not burst, since such a message is consistent only with a realization of θ in the interval $[L, 1]$.

[13] See assumptions 4 and 5.

Since the payoff to buying the asset is always positive when the bubble does not burst[14], we have $u(k, I_k) > 0$. Similarly, when k is sufficiently small (i.e., $k \leq \underline{\theta} - \epsilon$), the marginal speculator with message k knows that such a message is consistent with a realization of θ only in the interval $[0, \underline{\theta}]$, region where, as argued before, the cost of intervention for the government is always larger that the one it incurs by just letting the bubble burst, even if all the speculators are on the buy side, that is, $\alpha = 1$, which yields $u(k, I_k) < 0$. Therefore, there is a unique value of k for which $u(k, I_k) = 0$. We define the value x^* as the unique solution to $u(k, I_k) = 0$.

Consider now any equilibrium of the game and denote by $\pi(x)$ the proportion of speculators who buy the asset given message x. Define the numbers $\underline{x}$ and $\overline{x}$ as:

$$\underline{x} \equiv \inf \{ x | \pi(x) > 0 \} \quad (16)$$

$$\overline{x} \equiv \sup \{ x | \pi(x) < 1 \}. \quad (17)$$

We then have:

$$\overline{x} \geq \sup \{ x | 0 < \pi(x) < 1 \} \geq \inf \{ x | 0 < \pi(x) < 1 \} \geq \underline{x}, \quad (18)$$

where the first inequality follows from the fact that $\{ x | 0 < \pi(x) < 1 \} \subseteq \{ x | \pi(x) < 1 \}$ and the last from $\{ x | 0 < \pi(x) < 1 \} \subseteq \{ x | \pi(x) > 0 \}$. Therefore,

$$\underline{x} \leq \overline{x}. \quad (19)$$

When $\pi(x) < 1$, there are some speculators who are not buying the asset. This is consistent with an equilibrium behavior only if the payoff to not buying is at least as high as the payoff to buying given message x. By continuity, this is also true at $\overline{x}$. In other words,

$$u(\overline{x}, \pi) \leq 0. \quad (20)$$

Now, consider the payoff $u(\overline{x}, I_{\overline{x}})$. Clearly, $I_{\overline{x}} \leq \pi$[15], so that lemma 7 (strategic complementarity) and (20) above imply that $u(\overline{x}, I_{\overline{x}}) \leq u(\overline{x}, \pi) \leq 0$. Thus, $u\left(\underline{x}, I_{\underline{x}}\right) \leq 0$. Since we know from lemma 8 that $u(k, I_k)$ is increasing in k and x^* is the unique value of k which solves $u(k, I_k) = 0$, we have:

$$x^* \geq \overline{x}. \quad (21)$$

[14]See assumption 5.

[15]Since the indicator function is just a particular case of π and by the definition of $\overline{x}$.

Following the same line of reasoning, when $\pi(x) > 0$, there are at least some speculators buying the asset. This is consistent with an equilibrium behavior only if the payoff to buying is at least as high as the payoff to not buying given message x. By continuity, this must also be true at $\underline{x}$. In other words,

$$u(\underline{x}, \pi) \geq 0. \tag{22}$$

Now, consider the payoff $u\left(\underline{x}, I_{\underline{x}}\right)$. Clearly, $I_{\underline{x}} \geq \pi$ so that lemma 7 and (22) imply that $u\left(\underline{x}, I_{\underline{x}}\right) \geq u(\underline{x}, \pi) \geq 0$. Thus, $u\left(\underline{x}, I_{\underline{x}}\right) \geq 0$. Since we know from lemma 8 that $u(k, I_k)$ is increasing in k and x^* is the unique value of k which solves $u(k, I_k) = 0$, we have:

$$\underline{x} \geq x^*. \tag{23}$$

Thus, from (21) and (23), we have $\underline{x} \geq x^* \geq \overline{x}$. However, from (19) this implies

$$\underline{x} = x^* = \overline{x}. \tag{24}$$

Thus, the equilibrium π is given by the step function I_{x^*}, which is what lemma 9 states, completing the proof. ■

Following Morris and Shin (1998), it is a short step to the proof of the main result, theorem 6. Given that the equilibrium π is given by the step function I_{x^*}, the aggregate buying activity at the state θ is given by:

$$s(\theta, I_{x^*}) = \begin{cases} 0 & \text{if } \theta \leq x^* - \epsilon \\ \frac{1}{2} + \frac{(\theta - x^*)}{2\epsilon} & \text{if } x^* - \epsilon \leq \theta \leq x^* + \epsilon \\ 1 & \text{if } \theta \geq x^* + \epsilon. \end{cases} \tag{25}$$

Aggregate buying activity $s(\theta, I_{x^*})$ is increasing in θ, as it is clear from the above, when its value is strictly between 0 and 1, while $a(\theta)$ is decreasing in θ, as argued before[16]. Figure 2 below illustrates the derivation of the cutoff point at which the equilibrium buying activity is equal to the level that prompts government intervention.

[Figure 2 around here.]

We know that $x^* > \underline{\theta} - \epsilon$, otherwise not buying the asset is a strictly better action, contradicting the fact that x^* is a switching point. Thus, $s(\theta, I_{x^*})$ and $a(\theta)$ cross precisely once. Define θ^* to be the value of θ at which these two curves cross. Then, $s(\theta, I_{x^*}) \geq a(\theta)$ if and only if $\theta \geq \theta^*$, so that the government intervenes in the market if and only if $\theta \geq \theta^*$, which is the claim of the theorem above.

[16]See footnote 11.

3 Comparative statics and policy implications

3.1 Changes in the information structure

From the previous section, we saw that, when there is no noise, there are multiple equilibria throughout the "ripe for intervention" region of fundamentals. However, with positive noise, there is a unique equilibrium with critical value θ^*. The value of θ^* in the limit as ϵ tends to zero has the following characterization:

Theorem 10 *In the limit as ϵ tends to zero, θ^* is given by*

$$\phi_0'\left(-\theta^*\right)\left[f\left(\phi_0\left(-\theta^*\right), r\right) - f\left(\phi_0\left(-\theta^*\right), r^*\right)\right] + f\left(\theta^*, r\right) + f\left(\theta^*, r^*\right) = 2\left(p_0 + t\right). \tag{26}$$

Proof. Consider the switching point x^*, which is the solution to the equation $u\left(x^*, I_{x^*}\right) = 0$. We know that

$$\begin{aligned} u\left(x^*, I_{x^*}\right) &= \frac{1}{2\epsilon}\left\{\int_{[L, x^*+\epsilon]} g\left(\theta, r, s\left(\theta, I_{x^*}\right)\right) d\theta + \int_{[x^*-\epsilon, L]\cap A(I_{x^*})^c} f\left(\theta, r\right) d\theta \right. \\ &\left. + \int_{[x^*-\epsilon, L]\cap A(I_{x^*})} f\left(\theta, r^*\right) d\theta \right\} - p_0 - t \end{aligned}$$

and from expression (25)

$$\begin{aligned} A\left(I_{x^*}\right) &= \left\{\theta \middle| \, s\left(\theta, I_{x^*}\right) \geq a\left(\theta\right)\right\} \\ &= \left\{\theta \middle| \, \frac{1}{2} + \frac{\theta - x^*}{2\epsilon} \geq a\left(\theta\right)\right\}. \end{aligned}$$

Now, write

$$\begin{aligned} & \frac{1}{2} + \frac{\theta - x^*}{2\epsilon} \geq a\left(\theta\right) \\ \Leftrightarrow \quad & \epsilon + \theta - x^* \geq 2\epsilon a\left(\theta\right) \\ \Leftrightarrow \quad & \epsilon - x^* \geq \underbrace{-\theta + 2\epsilon a\left(\theta\right)}_{\equiv \Phi(\theta, \epsilon)} \end{aligned}$$

and notice that, from

$$\Phi_\theta\left(\theta, \epsilon\right) = -1 + 2\epsilon \overbrace{a'\left(\theta\right)}^{<0} < 0, \tag{27}$$

we conclude that $\Phi(\cdot,\epsilon)$ admits an inverse function, for fixed ϵ. Denote this inverse function by ϕ_ϵ. Now

$$\begin{aligned} & \epsilon - x^* \geq \Phi(\theta, \epsilon) \\ \Leftrightarrow \quad & \theta \geq \phi_\epsilon(\epsilon - x^*), \end{aligned}$$

and from this we can finally rewrite $A(I_{x^*})$ above as

$$A(I_{x^*}) = \{\theta | \, \theta \geq \phi_\epsilon(\epsilon - x^*)\}. \tag{28}$$

Following (28), the domains of integration in the expression for $u(x^*, I_{x^*})$ can be expressed as

$$\begin{aligned} [x^* - \epsilon, L] \cap A(I_{x^*})^c &= \{\theta | \, x^* - \epsilon < \theta < \phi_\epsilon(\epsilon - x^*)\} \\ [x^* - \epsilon, L] \cap A(I_{x^*}) &= \{\theta | \, \phi_\epsilon(\epsilon - x^*) < \theta < L\}. \end{aligned}$$

Using these and the fact that for ϵ sufficiently small we have $x^* + \epsilon < L$, $u(x^*, I_{x^*})$ can be written as

$$u(x^*, I_{x^*}) = \frac{1}{2\epsilon}\left\{\int_{\{\theta | x^* - \epsilon < \theta < \phi_\epsilon(\epsilon - x^*)\}} f(\theta, r)\, d\theta + \int_{\{\theta | \phi_\epsilon(\epsilon - x^*) < \theta < L\}} f(\theta, r^*)\, d\theta\right\} - p_0 - t. \tag{29}$$

Define

$$F(\epsilon) \equiv \int_{\{\theta | x^* - \epsilon < \theta < \phi_\epsilon(\epsilon - x^*)\}} f(\theta, r)\, d\theta + \int_{\{\theta | \phi_\epsilon(\epsilon - x^*) < \theta < L\}} f(\theta, r^*)\, d\theta. \tag{30}$$

The condition $u(x^*, I_{x^*}) = 0$ can be expressed as

$$\frac{F(\epsilon)}{2\epsilon} - p_0 - t = 0. \tag{31}$$

Because $F(\epsilon) \to 0$ as $\epsilon \to 0$[17], by using L'Hospital's rule

$$\begin{aligned} \lim_{\epsilon \to 0} \frac{F(\epsilon)}{2\epsilon} = \frac{F'(0)}{2} &= \frac{f(\phi_0(-x^*), r)\, \phi_0'(-x^*) + f(x^*, r) + f(x^*, r^*) - f(\phi_0(-x^*), r^*)\, \phi_0'(-x^*)}{2} \\ &= \frac{\phi_0'(-x^*)\left[f(\phi_0(-x^*), r) - f(\phi_0(-x^*), r^*)\right]}{2} + \frac{f(x^*, r) + f(x^*, r^*)}{2}, \end{aligned}$$

[17]Notice that the sets $\{\theta | \, x^* - \epsilon < \theta < \phi_\epsilon(\epsilon - x^*)\}$ and $\{\theta | \, \phi_\epsilon(\epsilon - x^*) < \theta < L\}$ can be equivalently written as $\{\theta | \, x^* - \epsilon < \theta < 2\epsilon(a(\theta) - 1/2) + x^*\}$ and $\{\theta | \, 2\epsilon(a(\theta) - 1/2) + x^* < \theta < x^* + \epsilon\}$, respectivaly, in the last using the fact that, for small ϵ, $x^* + \epsilon < L$. From this it is clear that both collapse to the empty set as $\epsilon \to 0$, hence $F(\epsilon) \to 0$.

where $\phi_0(\cdot) = \phi_\epsilon(\cdot)|_{\epsilon=0}$. Thus, in the limit as $\epsilon \to 0$, from expression (31) we have:

$$\phi_0'(-x^*)[f(\phi_0(-x^*), r) - f(\phi_0(-x^*), r^*)] + f(x^*, r) + f(x^*, r^*) = 2(p_0 + t). \tag{32}$$

Finally, we note that x^* converges to θ^* when ϵ tends to zero, since in the limit $s(\theta, I_{x^*}) = I_{x^*}$.

■

3.2 Changes in transaction costs

Theorem 10 provides a characterization of the cutoff point θ^*, the one that determines the states at which the government intervenes in the market. In this section we analyze how the cutoff point θ^* changes when t, the variable representing the transaction cost of the asset, changes. Totally differentiating the expression in theorem 10 yields:

$$\begin{aligned}
& -\phi_0''(-\theta^*)\frac{d\theta^*}{dt}[f(\phi_0(-\theta^*), r) - f(\phi_0(-\theta^*), r^*)] \\
+ \quad & \phi_0'(-\theta^*)\left[f_\theta(\phi_0(-\theta^*), r)\phi_0'(-\theta^*)\left(-\frac{d\theta^*}{dt}\right) - f_\theta(\phi_0(-\theta^*), r^*)\phi_0'(-\theta^*)\left(-\frac{d\theta^*}{dt}\right)\right] = 2 \\
\Leftrightarrow \quad & \frac{d\theta^*}{dt}[-\phi_0''(-\theta^*)][f(\phi_0(-\theta^*), r) - f(\phi_0(-\theta^*), r^*)] \\
+ \quad & \frac{d\theta^*}{dt}\left[-\phi_0'(-\theta^*)^2\right][f_\theta(\phi_0(-\theta^*), r^*) - f_\theta(\phi_0(-\theta^*), r)] = 2 \\
\Leftrightarrow \quad & \frac{d\theta^*}{dt} \\
= \quad & \frac{2}{[-\phi_0''(-\theta^*)][f(\phi_0(-\theta^*), r) - f(\phi_0(-\theta^*), r^*)] + \left[-\phi_0'(\theta^*)^2\right][f_\theta(\phi_0(-\theta^*), r^*) - f_\theta(\phi_0(-\theta^*), r)]}
\end{aligned}$$

Now, remember that, by definition, ϕ_ϵ is the inverse function of $\Phi(\cdot, \epsilon)$, for fixed ϵ, with $\Phi(\theta, \epsilon) = -\theta + 2\epsilon a(\theta)$. Therefore, $\Phi_\theta < 0$, which implies that $\phi_0' < 0$. The sign of $d\theta^*/dt$ is also dependent on the sign of ϕ_0'' and how f_1 changes for different levels of the interest rate. We impose the following assumption:

Assumption 11 *The bubble $b(\theta, \alpha)$ is a convex function of θ. Also, $f_\theta f_r = 0$.*

In the first part of assumption 11 we impose that the rate at which the bubble is inflated increases as the fundamentals improve[18]. Using the definition of $a(\theta)$, this implies that $a'' < 0$, which in turn implies that $\phi_0'' < 0$. The last statement in the assumption means

[18] We can interpret this assumption as an effect of *overconfidence*: as the fundamentals improve the speculators get more and more optimistic, leading to a buy decision that ultimately makes the bubble to inflate faster the higher the state of fundamentals.

that the rate at which the fundamental value changes as the state of fundamentals improve is independent of the prevailing interest rate, yielding $[f_\theta(\phi_0(-\theta^*), r^*) - f_\theta(\phi_0(-\theta^*), r)] = 0$. With this we then have:

$$\frac{d\theta^*}{dt} = \frac{2}{\underbrace{[-\phi_0''(-\theta^*)]}_{>0}\underbrace{[f(\phi_0(-\theta^*), r) - f(\phi_0(-\theta^*), r^*)]}_{<0}} < 0. \tag{33}$$

Therefore, if assumption 11 is in place, we conclude that increasing transaction costs decreases θ^*, hence increasing the range of government intervention in a bust episode. If we interpret t as a measure of the liquidity of the asset, the higher t the less liquid it is, in equilibrium the government intervenes *more* in the market the *less* liquid the asset. This implies that, for sufficiently large x, the signal received by a speculator regarding the state of the fundamentals, investment is more likely to be made on illiquid than liquid assets, which in turn implies that the government's policy fuels bubbles particularly in less liquidity markets. In a way, the last bubble episode occurring in the housing market, characterized by illiquid assets, offers empirical support for this result.

3.3 Changes in aggregate wealth

Following again Morris and Shin (1998), we consider now how the equilibrium cutoff point θ^* changes when *aggregate wealth* varies. As Morris and Shin (1998) mention, the international flow of so-called "hot money", or also a more lax monetary policy that increases *funding liquidity*, would constitute factors in determining the level of aggregate wealth, as well as changes in the number of speculators themselves. Changing aggregate wealth would cause an impact on the function $a(\theta)$, the critical proportion of speculators that would require government intervention in the case of a bust episode. When the aggregate wealth of the speculators increases, the critical proportion of speculators falls, since the government's decision to intervene is based on the absolute *level* of buying activity, and not the *number* of speculators buying the asset.

As we can see in figure 2, a downward shift in the function $a(\theta)$ decreases the equilibrium θ^* and hence has the effect of enlarging the set of states at which the government intervenes in the market in a bust episode, that is, the event $A(\pi) = \{\theta | s(\theta, r) \geq a(\theta)\}$ gets larger when the function $a(\cdot)$ gets smaller.

In this way, since in the case of a bubble burst the payoff to speculators who buy the asset is larger exactly when there is government intervention, the enlargement of the event $A(\pi)$ has the effect of increasing the (expected) payoff to buying the asset, at any value of the signal received, x. Therefore, with an increase in aggregate wealth augmenting the range of fundamentals for which the government intervenes, the speculators have more incentives to buy the asset, increasing the incidence of bubbles[19].

3.4 Changes in the threshold level of bubble burst, L

A change in L, if it is an increase, makes the bubble more likely to burst, that is, it turns buying the asset into less of an attractive investment from the viewpoint of the speculators, since they benefit more when they buy and the bubble does not burst. The speculators buying less causes bubbles to inflate less, demanding then also less government intervention in bust episodes. In this way, an increase in L should make θ^* larger, which is what happens if we follow figure 2. Analogously, a decrease in L implies that bubbles are less likely to burst, attracting more investors and hence becoming more inflated which requires, at a relatively lower θ, government intervention. Hence, decreasing L should decrease θ^*, which can also be seen in figure 2.

Changes in L can indirectly be interpreted as changes in "market sentiment", in the following way: if speculators believe that the true threshold level that separates bubble bursts from other events is given by L' rather than L, with $L' < L$, they will be more likely to buy the asset, since from figure 2 we can see that if L was to be decreased this would lead to a non-increasing change in x^* and, for sufficiently small ϵ, to a non-increasing change in θ^* as well. That means that, at worst, the investors do not change the likelihood of buying the asset when they think that the true threshold level is L', with a higher chance that they will be actually buying the asset more often, that is, adopt the buying decision for a larger range of signals, x. The reason for calling this "market sentiment" is that, if the belief of investors is that the threshold level is $L' < L$, they think that a bubble is *less* likely to burst than it actually is, since that happens whenever $\theta < L$. The investors could then be said of being *overconfident*, and here we link this with the concept of market sentiment[20].

[19]This point is made by some authors who point to *global imbalances* as one of the main factors triggering the start of bubbles.

[20]Shiller (2005) discusses the effects of market sentiment on the inflation of bubbles. There, it is argued that, usually, bubbles start to be inflated at periods the population associate with the beggining of so-

3.5 Changes in the threshold level of government intervention, c

Changes in c, that is, changing the cost of government intervention, is equivalent to shifts in $a(\theta)$, the critical proportion of speculators such that, when they buy, government action is required, in a bust episode. If c increases, that is, if it is more costly for the government to intervene, the bubble will have to be larger to justify such action, hence $a(\theta)$ should shift to the right, increasing θ^* and hence making the government to intervene less. With lower government intervention and for fixed L, it is less attractive for speculators to buy the asset, which then makes bubbles to be less inflated. An analogous argument imply that a decrease in c leads to a decrease in θ^*, which is intuitive since a decrease in the cost of intervention should make a bubble burst relatively more costly, making a better option for the government to intervene.

4 Concluding Remarks

In this paper we build a model of the onset of bubbles, where government intervention in bust episodes, lowering the interest rate and hence reducing the size of crashes and losses to speculators, plays a key role. The intervention of the government in crises creates more incentives for speculators to buy the asset, resulting in a demand pressure type of effect that inflates bubbles. Using the global games approach, as in Morris and Shin (1998), we are able to derive a unique equilibrium where the government intervenes if and only if the state of fundamentals in the economy is larger than a particular level, which induces the speculators to also adopt a strategy of the thereshold type relative to the signals they receive.

Following an exercise in comparative static analysis, we obtain that government intervention is higher the less liquid the asset and the higher the aggregate wealth of investors. Given that investors are always better off when there is government intervention and that government intervention depends on the size of the bubble when there is a crash, with the size of the bubble increasing in the fraction of speculators who opt to be long in the asset, we conclude that bubbles are more prone to happen in illiquid assets and

called *new eras*, often depicted in a rosy way, indicating that prosperous times came for good, to the benefit of everyone.

when credit afloat. These two results are in a way related to the last bubble bust in the american housing market, a market characterized by assets of low liquidit and where credit was abundant due to both global imbalances and the apetite of investors for huge gains, symbolized in the trading activities of complex derivative instruments. It would be interesting not only to relate our findings to the last financial crisis but rather perform a rigorous empirical analysis to check if such claims are indeed supported by the data.

The model we present is static but dynamic factors would need to be considered if more robust results were to be obtained. For instance, if we were to model the government as an institution targeting to control the volatility of the business cycle, with a higher volatility implying a higher cost for the government, in a dynamic model government intervention would happen *both* when price increases or decreases too much, with the government raising and lowering the interest rate in such episodes, respectively. In our model, the government does not "control" the volatility of the business cycle, since the crash that matters is the *intra period crash* rather than the *inter period crash*, that is to say, the cost of intervention for the government is a function of the difference between the price at the interim stage and the one that would result after the crash, not the difference between the intial price at $t = 0$ and the final price at $t = 2$, which is what actually determines the volatility of the market. In a dynamic setting, our intuition tells that the frequency of bubbles should be smaller, and the government would control both *positive* and *negative* bubbles. We leave such issues for future research.

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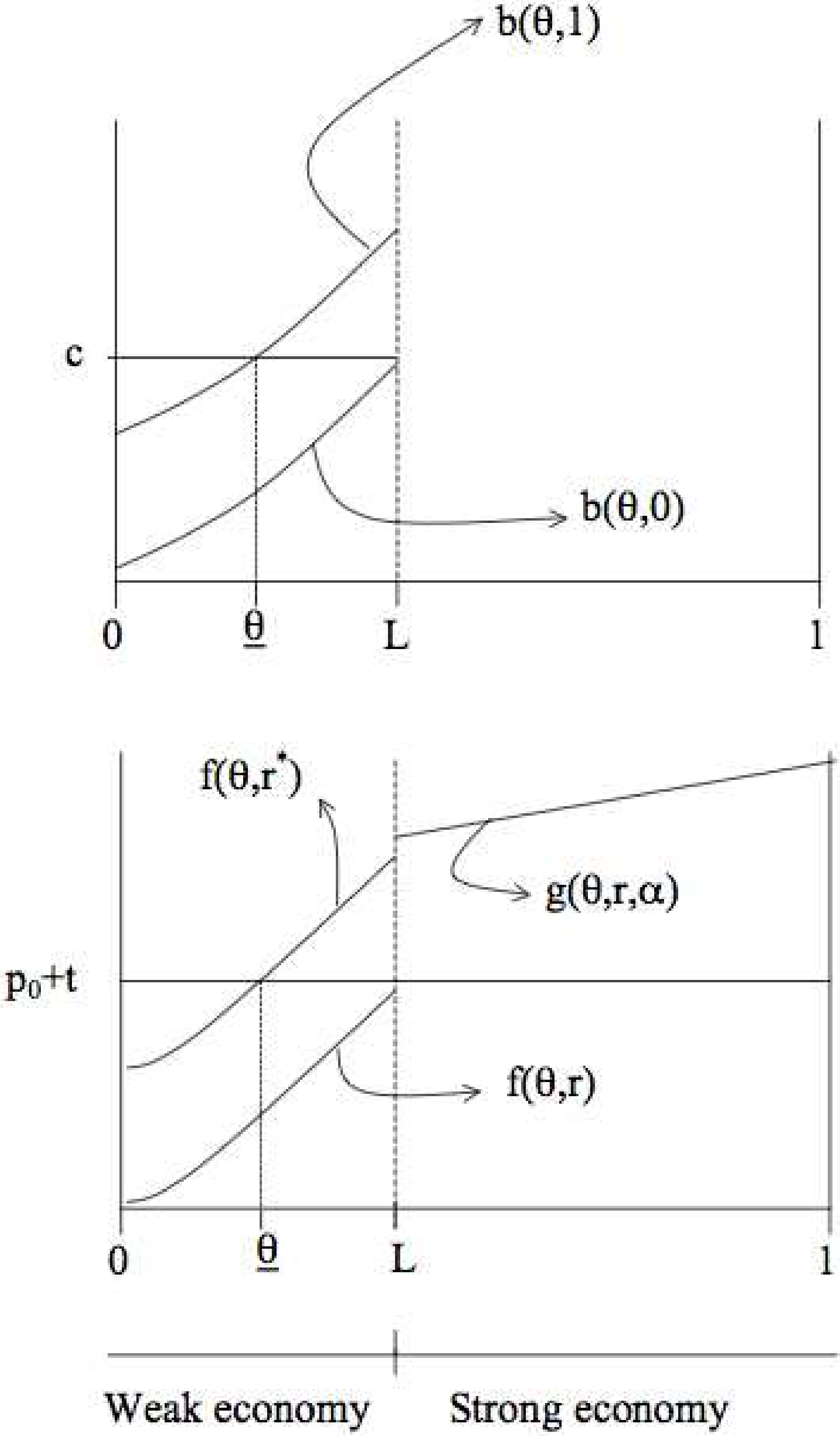

Figure 1: Bubble function for $\alpha = 0$ and $\alpha = 1$.

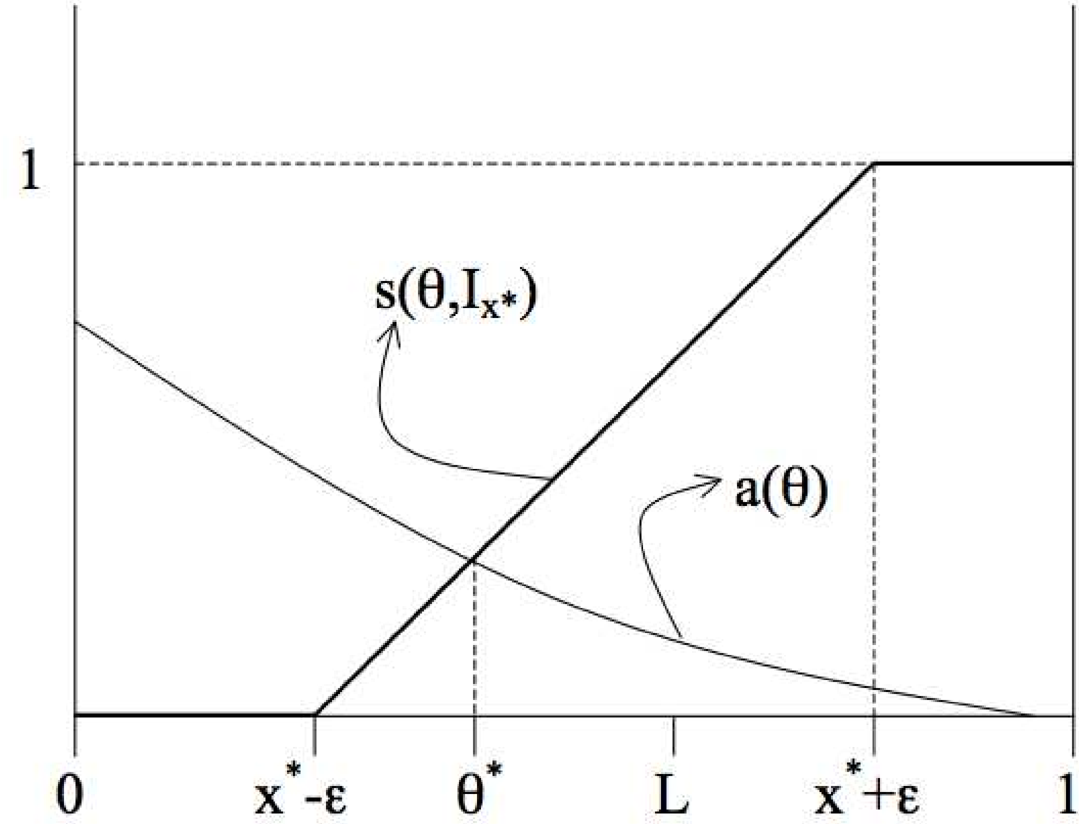

Figure 2: Mass of speculators buying the asset and critical mass requiring government intervention.

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