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Government Intervention and Financial Fragility

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Abstract

This paper studies a model in which banks decide on the projects in which they invest, and the banks to which or from which they obtain loans. Thus, the links (network) created between banks is endogenous. Each bank is characterized by parameters which define the return on its projects, the withdrawal rate of its depositors and its equity available for investments. Maturity mismatch of balance sheets forces a fraction of assets to be prematurely liquidated, at a fire sale cost. The paper focuses on the impact of government intervention, which alleviates this cost by increasing the recovery rate of assets. The fragility of a network is measured by the number of bank failures following shocks of two kinds: first a shock to a single bank, second a simultaneous shock to all banks. The first leads to a ranking of the banks similar to that used by Google to rank websites: the higher its ranking the greater the degree of vulnerability induced by the bank. The vulnerability of the network to simultaneous shocks depends on the probability distribution of the banks characteristics: the more dispersed the distribution the greater its vulnerability. Government intervention increases the vulnerability of the network, the increase being greater the more dispersed the characteristics of the banks. Banking systems with similar leverage can have different degrees of vulnerability, highlighting the importance of networks.

Keywords: Government intervention; financial fragility; financial networks; contagion.

JEL Classification: G1; G2; G3.

1 Introduction

The financial crisis of 2007-2009 brought to the fore the question of what should be the role of government in times of distress. On the one side there were some advocating in favor of an intervention - a bailout of financial institutions - so that a collapse of the financial system could be avoided, whereas on the other there were those claiming that banks should be let at their own fate, since their failure was nothing else than a result of a reckless, greedy behavior, in way that a bailout would do nothing but exacerbate the moral hazard problem.

What makes the government sensitive to the bailout of a particular institution is the risk of financial contagion, or the probability that its failure will trigger a chain reaction, leading to the demise of several other banks. A financial system associated with a higher probability of bank failures can be said to be more fragile, and in this case an intervention by the government is conceivably more necessary than in others.

Contagion might follow after the distress of one bank, due to either solvency or liquidity issues, if there exists a mechanism allowing for the transfer of losses. That is usually the case among banks, since its part of their business to make transactions with each other, e.g., interbank loans. A debtor bank facing problems could simply renege on the payment of the loan taken, causing a loss to the creditor institution. If the creditor in its turn cannot bear the debtor's default, the loss will propagate further, characterizing a situation of contagion.

Large institutions tend to be the ones with more connections in the market, potentially having a stronger knock-on effect in case they fail. In a network of banks, those tend to be the first to receive assistance, under the so called too-big-to-fail policy, TBTF, by which the government tries to prevent a system's collapse after a bank is hit by a shock.

The question addressed here is whether a government intervention policy might lead to a more fragile network, where fragility is viewed as being related to failures that are not caused by direct exposure to shocks but rather due to contagion. This is done by first constructing a network through an interaction process, where banks decide whether or not to be connected. Upon that, shocks are imposed, and the number of banks in distress is calculated, with the total cases of indirect failures giving a measure of fragility. The structure of the network turns to depend on government's policy and, therefore, so does fragility, allowing one to study the effects of intervention.

The model developed is thought of as an economy divided into several regions, with their own idiosyncrasies in terms of investment opportunities and consumers' preferences. There is a representative bank in each region, responsible for taking deposits and making investments. Investments can be made in either large or small projects, with larger projects commanding a higher payoff but also demanding an extra level of initial capital, which can be secured only if a loan from another bank is taken.

Embedded in the framework is the banks' maturity mismatch problem, due to the financing of long-term assets with short-term liabilities. Bank are assumed to be short of capital to service depositors, which forces them to liquidate a fraction of their assets - project or loans - before they are ripe. This premature liquidation comes at a fire-sale cost: there is a penalty applied to any fraction of an asset sold before maturity. The fire-sale cost is taken to be proportional to the size of the asset, so that large projects are sold at a higher discount - or, in other words, have a lower recovery rate - than small projects and loans.

Intervention then takes place by having the government to alleviate the fire-sale cost incurred by banks, and such a policy is assumed to be implemented in a more pronounced way for larger assets. This is the way that one views a too-big-to-fail policy as being present in the framework developed.

The differences across regions in terms of projects' payoffs and preferences of depositors, combined with the fire-sale cost and government's intervention policy, make some assets to be more profitable than others, according to the region where the bank is located. That will determine the network structure resulting from a process of pairwise meetings of banks, with a link between any two nodes representing a loan. The point is that intervention might lead to the creation of links that otherwise would be inexistent, in a way that the structure of the network will depend on government's intervention policy.

After the formation of the network, the degree of financial fragility is assessed by the number of indirect bank failures caused by having banks' assets hit by shocks. Upon the shocks, the payoff of projects turn to be only a fraction of what was specified when those projects were undertaken. These are perturbations of the network, in the sense of being probability zero events that banks are not prepared for and, therefore, they immediately cause losses as soon as they take place.

Intervention makes the number of links in a network to be at least as high as that of

a network formed under no government. Ceteris paribus, that means the fragility of the former will be at least as high as that of the later, since more banks get exposed to the possibility of contagion. However, even though it is not necessarily the case, a network with a higher number of links might also have a greater overall networth, which means a thicker cushion to absorb shocks and, hence, a smaller number of failures. Therefore, at least theoretically, the effect of government intervention on financial fragility is not straightforward.

In order to understand how the impacts of intervention vary with the parameters of the model, simulations are performed for 3 different economies, designed to represent varying stages of financial development. For all of them, the results show that government intervention leads to a mucher higher percentage of indirect bank failures relative to the total cases of distress, and that effect is much more pronounced the less advanced the economy is. Not only that, leverage might be similar across networks obtained under different government policies, even though they present starkingly different degrees of fragility, highlighting the importance of the structure of the system where banks operate and, indirectly, of government intervention for, fragility.

The paper is structured as follows: following it is presented a simple example that illustrates the main idea of the model; the review of the literature and the contributions/limitations are discussed in subsections 1.2 and 1.3, respectively; section 2 details the model and its primitives; section 3 discusses the link formation process and the implications of government intervention for the network structure; section 5 gives the balance-sheet characterization of the financial system represented by the network, introduce shocks and how they can be studied in an input-output analysis flavor; section 6 proposes differents measures of financial fragility, and a way of ranking banks that happens to be analogous to the PageRank algorithm used by Google to rank websites, detailing also the 3 types of economies designed for the exercise in comparative statics, with qualitative results from the simulations following.

1.1   Example

Consider two banks, A and B, representing distinct regions of an economy, in a 3-period world, $t = 0, 1, 2$. These banks have, at $t = 0$, the opportunity to invest in local projects paying r_A and r_B, respectively, at $t = 2$. Without loss of generality, assume that $r_A >$

$r_B > 2$. Both projects demand an initial investment of \$2, which can be partially supplied by households, who deposit an amount of \$1 at $t = 0$ - the other \$1 required to start a project can only be obtained by a loan from the other bank, to be repaid at $t = 1$. Assume banks have zero opportunity cost of lending so that the same \$1 obtained at $t = 0$ is paid back at $t = 1$.

Households withdraw their money from banks at $t = 1$ and, after an investment is made, banks can obtain the \$1 demanded by depositors only by liquidating a fraction of their projects. This premature liquidation, at $t = 1$, occurs at a fire-sale cost, so that a project which is worth r_i at $t = 2$ can only be transacted by ρr_i at $t = 1$, for $i = A, B$. Thus, upon investing in projects, banks need to sacrifice, at $t = 1$, a fraction α_i of their investments, so to obtain the amount to service depositors and to pay back the loan,

$$
\alpha_i \rho r_i = \overbrace{1}^{\text{Depositors}} + \overbrace{1}^{\text{Loan}},
$$
$$
\Leftrightarrow \quad \alpha_i = \frac{2}{\rho r_i}, \quad i = A, B. \tag{1}
$$

Thus, the profit banks can realize out of projects, at $t = 2$, is

$$
\Pi_i = (1 - \alpha_i) r_i
$$
$$
\Leftrightarrow \quad \Pi_i = r_i - \frac{2}{\rho}, \quad i = A, B. \tag{2}
$$

In this way, banks would consider to invest only if the recovery rate ρ is such that $\Pi_i > 0$, for $i = A, B$, or, equivalently,

$$
\rho > \underline{\rho_i} := \frac{2}{r_i}, \quad i = A, B. \tag{3}
$$

The possible scenarios are possible:

- Both A and B want to invest if $\rho > \underline{\rho_B} > \underline{\rho_A}$;

- Only A considers investing if $\underline{\rho_B} > \rho > \underline{\rho_A}$; and

- No bank wants to invest if $\underline{\rho_B} > \underline{\rho_A} > \rho$.

Consider now the possibility of government intervention, meaning a subsidy of a fraction γ of the loss due to the fire-sale cost incurred by banks, $1 - \rho$. Upon intervention,

thus, the recovery rate of projects increases to $\rho + \gamma(1 - \rho)$, and a project that at $t = 1$ was worth ρr_i is now valued at $[\rho + \gamma(1 - \rho)]\, r_i$, for $i = A, B$. The fraction of projects needed to be prematurely liquidated to servive depositors and pay loans is now given by

$$
\alpha_i^G \left[\rho + \gamma(1 - \rho)\right] r_i = \overbrace{1}^{\text{Depositors}} + \overbrace{1}^{\text{Loan}},
$$
$$
\Leftrightarrow \quad \alpha_i^G = \frac{2}{\rho + \gamma(1 - \rho)}, \quad i = A, B. \tag{4}
$$

Analogously to the previous scenario, the profit banks can realize out of projects, at $t = 2$, is now

$$
\Pi_i^G = \left(1 - \alpha_i^G\right) r_i
$$
$$
\Leftrightarrow \quad \Pi_i^G = r_i - \frac{2}{\rho + \gamma(1 - \rho)}, \quad i = A, B. \tag{5}
$$

The condition for banks to be willing to invest in projects, $\Pi_i^G > 0$, for $i = A, B$, is

$$
\rho > \underline{\rho_i^G} := \frac{1}{1 - \gamma}\left(\frac{2}{r_i} - \gamma\right), \quad i = A, B. \tag{6}
$$

As before, the following are the possible scenarios adventing from the investment conditions for banks A and B:

- Both A and B want to invest if $\rho > \rho_B^G > \rho_A^G$;

- Only A considers investing if $\rho_B^G > \rho > \rho_A^G$; and

- No bank wants to invest if $\rho_B^G > \rho_A^G > \rho$.

Given the subsidy, the condition for banks to be willing to invest in projects is always easier to be satisfied with intervention than without, i.e., for any $\gamma > 0$. Not only that, intervention might lead to bank lending in circumstances where otherwise there would not be any. For instance, if the parameters are such that

1. $r_A > r_B > 2$

2. $\frac{2}{r_A} > \gamma > \left(\frac{r_A - r_B}{r_B}\right)\left(\frac{2}{r_A} - 2\right)$

3. $\frac{2}{r_A} > \rho > \left(\frac{1}{1 - \gamma}\right)\left(\frac{2}{r_B} - \gamma\right)$

then it follows that

$$\underline{\rho_B} > \underline{\rho_A} > \rho > \rho_B^G > \rho_A^G, \tag{7}$$

i.e., under no intervention banks would not be willing to invest in projects whereas both would like to take a loan in case the government is present. One can check that taking $r_A = 4$, $r_B = 3$, $\gamma = .415$ and $\rho = .465$ satisfy all the above.

Thus, if one assigns all the bargaining power to bank A, Figure 1 depicts the two types of network that would emerge under different intervention policies of the government:
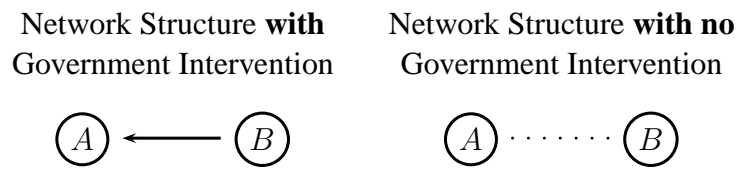

Figure 1: Network Structure and Government Intervention.

These two structures have different implications for financial fragility, as it will be studied later. For instance, with no intervention, banks A and B are not exposed to an eventual ex-post risk posed by projects, when they do not deliver their expected payoffs. With banks connected, however, a failure at $t = 1$ of bank A in fulfilling its promise to pay back the loan taken from B, due to the anticipation of a problem in its project, could potentially lead to contagion.

The full-fledged version of the model has many banks, with two investment opportunities available - large and small projects - on top of the possibility of lending. Also, banks are equipped with different levels of capital and varying amounts of households withdrawing their deposits before projects are ripe, causing a maturity mismatch problem. As in the example, government intervention takes the form of a subsidy that allows banks to alleviate their fire-sale costs, and might turn profitable investment opportunities that otherwise would not be. This will lead to different network structures, with distinct levels of financial fragility associated to them.

1.2   Related Literature

Since the main theme of the paper is how government intervention can lead to financial fragility, the related literature is traced to that on financial crisis, contagion, government

intervention and networks. For each of these, the papers closer in spirit to the present work are highlighted.

In the financial crisis literature, the main reference is the classic Diamond and Dybvig (1983) work on bank runs. It is from their paper the idea of having depositors with consumption needs arising stochastically. Differently from Diamond and Dybvig, though, the framework developed has multiple banks and studies how distress in one institution spreads to others, where the mechanism of contagion are the links across banks due to loan agreements. Another important reference is Shleifer and Vishny (1992), providing the rationale for having in the model a fire-sale cost adventing from the early liquidation of projects, which gives room and motivates government intervention, in a way that a fraction of the loss incurred by banks can be recouped.

Network theory has been increasingly used to study different issues in economics and finance. Allen and Babus (2009) provide an account with specific applications to finance and Schweitzer, Fagiolo, Sornette, Vega-Redondo, Vespignani, and White (2009) point to new directions for research. One approach is to study how systemic risk is associated with different types os networks, or the susceptibiity to contagion those structures have - the failure of one institution leading to that of others. Some papers along these lines are: Rochet and Tirole (1996), where banks have a role in monitoring each other and whose closure decisions after being hit by a shock are interlinked; Kiyotaki and Moore (1997), with a chain of firms borrowing and lending giving rise to systemic risk, in case some of them become temporarily illiquid and cause others to get into financial difficulties as well; Allen and Gale (2000b), which shows that similarly efficient networks have varying degrees of robustness to shocks; Freixas, Parigi, and Rochet (2000), with credit lines across banks in different regions being the channel for contagion; Eisenberg and Noe (2001), developing a model of a clearing system and providing a measure of systemic risk based on the number of "waves" of default necessary to cause the bankruptcy of a bank; Lagunoff and Schreft (2001), where diversification leads agents to have their portfolios linked, with these connections being subject to be broken as a result of asset reallocation following shocks, eventually leading to additional disruptions and a crisis; Cifuentes, Ferrucci, and Shin (2005), which studies how contagion might be processed not only through direct balance sheet exposure among banks but also via asset prices, addressing also the consequences that might advent from prudential regulation; Nier, Yang, Yorulmazer, and Alentorn

(2007), first modelling banking systems as random graphs and then simulating changes in the underlying parameters, in order to assess the resilience of the strucutre to different shocks; Brusco and Castiglionesi (2007), with liquidity coinsurance potentially bringing down a bank in case it is paired with a not well capitalized institution that engages in excessive risk taking; more recently, Caballero and Simsek (2011), with a model showing how uncertainty about the network structure, denominated *complexity*, exacerbates shocks and makes banks to behave more precautiously, leading to fire-sales and crises; Zawadowski (2011), who uses a network framework to model bilateral over-the-counter contracts, showing that banks underinsure against counterparty risk by not incorporating the network externalities they impose on third parties once they fail; Haldane and May (2011), drawing on an analogy of the banking system with ecologic food webs and networks where infectious diseases spread; and Duffie (2011), discussing a network-based approach to monitor systemic risk.

What these papers do not provide is an explicit mechanism that leads to the formation of financial networks. Some that do are: Leitner (2005), with banks forming links that allow transfers of endowments to be made, which in turn prevents the failure of less wealthy members that could cause the demise of the entire network; Babus (2009), using a game-theoric approach to endogenously derive a network that can provide insurance to the possibility of contagion; Castiglionesi and Navarro (2011), where banks decide whether to join a network that makes it possible for them to coinsure each other against liquidity shocks, through the granting of credit lines that in turn result in them being connected; and Cohen-Cole, Patacchini, and Zenou (2012), which model the network formation process as a Cournot competition in the lending market, showing that banks prefer to be linked with others that are more rather than less connected.

Related to government intervention, some papers that study the effects on the behavior of private institutions from government's policy in periods of distress are: Huang and Xu (1999), which model the 1997 East Asia financial crisis as a result of soft budget constraints, SBCs, a term originally coined by Janos Kornai to refer to the support by the government to private enterprises in transition economies, but increasingly observed in capitalist societies as well, with a handful of unintended consequences (for a review of the theory, Kornai, Maskin, and Roland (2003); for the classic model that captures the SBC concept, Dewatripont and Maskin (1995); for an application to banking crises,

Mitchell (2000)); Schneider and Tornell (2004), which, in a two-sector economy, show that a contract enforceability problem induces bailout guarantees to nontradables, propelling that sector to initiate crises; Gorton and Huang (2004), where government bailouts can be an efficient mechanism for the recapitalizion of banks in times their assets are hit by a negative shock, by means of providing liquidity in the secondary market for projects and hence guaranteeing they can fetch a price better than what a fire-sale would entail; Corsetti, Guimarães, and Roubini (2006) and Morris and Shin (2006), studying models of catalytic finance in which, rather than moral hazard, support from institutions of the like of the IMF ends up being pivotal for countries to engage in costly but necessary reforms; Acharya and Yorulmazer (2007), analysing the implications of intervention policies that are designed to be implemented only in systemic crises - those that affect a significant portion of the banking industry - resulting in a too-many-too-fail type of guarantee by the government, which in turn increases the likelihood of distress in the banking system; Ennis and Keister (2009), where ex post efficient interventions to bank runs might generate self-fulfilling crises episodes that destabilize the banking system; Farhi and Tirole (2010) and Diamond and Rajan (2011), both with models analysing intervention through changes in the interest rate, Farhi and Tirole showing that a policy whereby the government responds to crises decreasing the interest rate leads to more maturity mismatch in the economy and, hence, exposure to liquidity shocks, and Diamond and Rajan arguing that such a policy turns out to be better than the alternatives, in particular a bailout of a specific institution.

The sources of distress analysed are payoff shocks, i.e., just out of a sudden banks realize they will get from projects only a fraction of what was originally specified. However, the paper abstracts from providing a reason for why such shocks take place. Some papers in that regard, to cite a few, are: related to bubbles, Allen and Gale (2000a) and Abreu and Brunnermeier (2003); the leverage cycle theory, Geanakoplos (1997), Geanakoplos (2003) and Geanakoplos (2010); liquidity spirals, Brunnermeier and Pedersen (2009); imperfect information, Morris and Shin (1998) and He and Xiong (2011); flight to quality, global imbalances and sudden stops, Caballero and Krishnamurthy (2008), Mendoza and Quadrini (2010) and Mendoza (2010), respectively. Greenlaw, Hatzius, Kashyap, and Shin (2008) and Brunnermeier (2009) describe the chronology of events leading to the financial crisis of 2007-2009.

1.3 Contributions of the Paper

The main contribution of the paper is to provide a framework based on networks to address the question of whether government intervention can lead to financial fragility. The framework developed is essentially an attempt to combine the Diamond and Dybvig (1983) model of bank runs with the Allen and Gale (2000b) model of financial contagion, with two additional components: government intervention and network formation.

In so doing, it provides a mechanism where government's policy is crucial in the banks' decision to become or not connected, i.e., create links. To the best of one's knowledge, this is the first paper that models the formation of a network of banks with an explicit role for the government in shaping the final structure. Obviously, as pointed out in the previous section, it is neither the first paper to model the formation of financial networks nor government intervention, it only puts both themes together.

With the government being pivotal in the process of network formation, the framework provided allows one to study the effects of particular policies, in particular a too-big-to-fail type of intervention, whereby large enterprises (projects in the model) enjoy a higher level of assistance than small ones. The financial crisis of 2007-2009 produced a lot of debate about this type of intervention and the consequences ensuing from that. Several papers discuss how government bailouts can lead to moral hazard, but the effects on the incentives for banks being connected, using a network framework as it is proposed, seems to be novel.

The model proposed is very tractable and allows one to perform different kinds of simulations, in order to assess the implications of alternative government policies. Not only that, one can also use it as a stress test tool, to measure how the network is vulnerable to the distress of particular financial institutions. All in all, and as the simulations show, the point is that knowledge of the network structure of the financial system - as far as financial fragility is concerned - is as valuable of an information as are more standard measures, such as the leverage ratio. Being the government crucial for network formation process, so crucial it is its policy in determining the degree of financial fragility.

However tractable the model is, it is also a very stylized way of seeing reality. For instance, the network formation process implicitly assumes that banks are myopic, in the sense that they make decisions without considering other banks they meet in the future. Also, one abstracts from the budget constraint of the government, which is viewed

simply as an institution with deep pockets that is there offering a subsidy to banks, to offset some of their costs. The plausibility of the government behaving in such a way is obviously questionable, and in full fledged model it would need to be reconsidered. Another important aspect is that, by having no uncertainty, the model abstracts from the default risk that banks calculate when they decide whether or not to lend to other institutions. The huge volume of literature on default risk highlights how important such an aspect is but, for the sake of tractability, it is not considered in the framework proposed.

2 Model

Consider a 1-good (\$), 3-period economy, $t = 0, 1, 2$, divided in an even number N of regions, $N = \{1, \ldots, N\}$. Every region $i \in N$ has a representative bank $B_i \in \mathbb{B}$, with $\mathbb{B} = \{B_1, \ldots, B_N\}$ representing the set of banks, each with a secured endowment (equity) of $e_i > 0$ per transaction they engage in, as it will be clarified later.

Every region i has $N-1$ continuums of depositors, $\mathbb{D}^i = \left\{D_1^i, \ldots, D_{N-1}^i\right\}$, each of them of unit mass. Depositors are endowed with \$1 and have Diamond-Dybvig preferences, i.e., they face uncertainty regarding when their consumption will take place, formalized by having the following utility function:

$$U_i\left(c_1, c_2\right) = \begin{cases} c_1, & \text{with probability} \quad \omega_i, \\ c_2, & \text{with probability} \quad 1 - \omega_i. \end{cases} \tag{8}$$

For any continuum D_j^i in region i, with probability ω_i a depositor will consume at $t = 1$, denoted by c_1 and denominated henceforth early depositor, whereas with probability $1 - \omega_i$ she will consume at $t = 2$, denoted by c_2 and denominated late depositor. Uncertainty is resolved at $t = 1$ and the probability of consuming at $t = 1$ or $t = 2$ varies across regions. Figure 2 illustrates the continuums of depositors for a region $i \in N$.

Any representative bank has available an infinite supply of two types of long-term investment opportunities, namely a large and a small project. At $t = 2$, the large project pays r_i^* whereas the small yields r_i, the former for an investment of \$2 and the later of \$1, at $t = 0$. By assumption, $r_i^* > r_i$, for any $i \in N$. The cash flows of projects available to bank $B_i \in \mathbb{B}$ are represented in Figure 3.

Projects are available only to the representative bank of the respective region, i.e., a bank $i \in N$ cannot invest in projects other than the ones in its own region - cross-
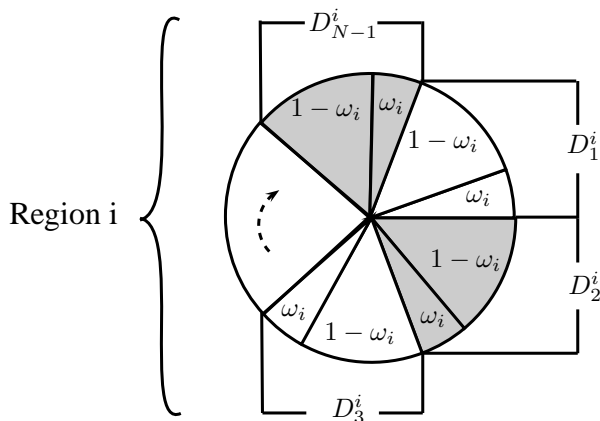
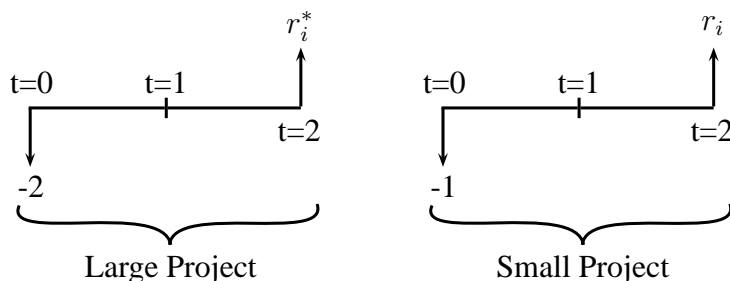Figure 2: Region i and its continuums of depositors.

Figure 3: Cash flows of region i's projects.

region investment is ruled out[1]. At a discount, projects can be partially liquidated before maturity, i.e., at $t = 1$. Banks are also allowed to borrow (long-term) from other banks and they also have available at $t = 0$ a short-term asset that pays zero interest rate. Depositors do not have access to either long-term projects or short-term assets, and are forced to deposit their endowments in the local bank, by means of a deposit contract that allows withdrawals at will. This imples that banks can borrow from depositors at a zero interest rate.

2.1 Banks Interaction Process and Arrival of Depositors

Banks interact with each other in order to take advantage of differences across regions in the payoff offered by small and large projects, since cross-region investment is ruled out. The interaction protocol of banks is assumed to be such that, at $t = 0$, a specific number of rounds of interaction takes place, with banks meeting pairwise. The number of rounds is such that, at the end of the interaction process, banks will have met each other once

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[1]One way of thinking about this is that projects require an expertise that only local banks have.

and only once. Given an even number N of banks, at $t = 0$ there will be $N - 1$ rounds of interaction. For example, with four banks, $N = 4$, the rounds of interaction will be as in Figure 4:

Round 1: $B_1 \longleftrightarrow B_2$; $B_3 \longleftrightarrow B_4$

Round 2: $B_1 \longleftrightarrow B_3$; $B_2 \longleftrightarrow B_4$

Round 3: $B_1 \longleftrightarrow B_4$; $B_2 \longleftrightarrow B_3$
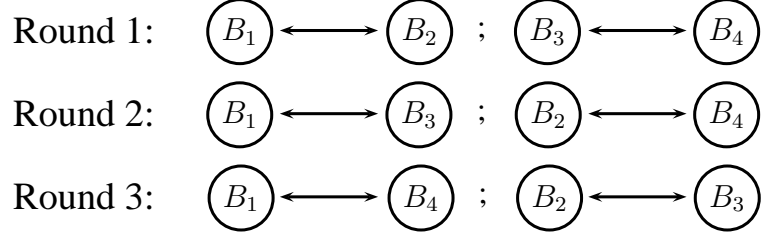
Figure 4: Interaction process of banks for $N = 4$.

Synchronized with the interaction process of banks is the arrival process of depositors. Depositors show up sequentially at their local banks, in a time fashion matching the way that banks meet each other. In every round of interaction, one of the $N - 1$ continuums of depositors in each region arrives at the local bank and, given that the interaction process of banks is composed of $N - 1$ rounds, by the end of that all depositors will have shown up at their respective institutions.

2.2   Network Structure

At every round of interaction, a bank will both (i) receive depositors and (ii) meet another bank, making an investment in either a loan or a project. One important assumption is that banks decide during the round of interaction how to allocate the $1 received from depositors. This implies banks behaving in a myopic way, since they cannot keep that dolar for more profitable transactions to come in the future. At each round of interaction, therefore, banks have to decide whether to:

 (i) Invest the $1 received in a small project;

 (ii) Borrow $1 more and invest the total in a large project;

(iii) Lend $1 to the other bank.

Figure 5 illustrates, for a particular round, the possibilities arising from a meeting between banks i and j. The blue dashed line represents an investment in a small project, the green in a large project in region j - by means of a loan agreement between bank
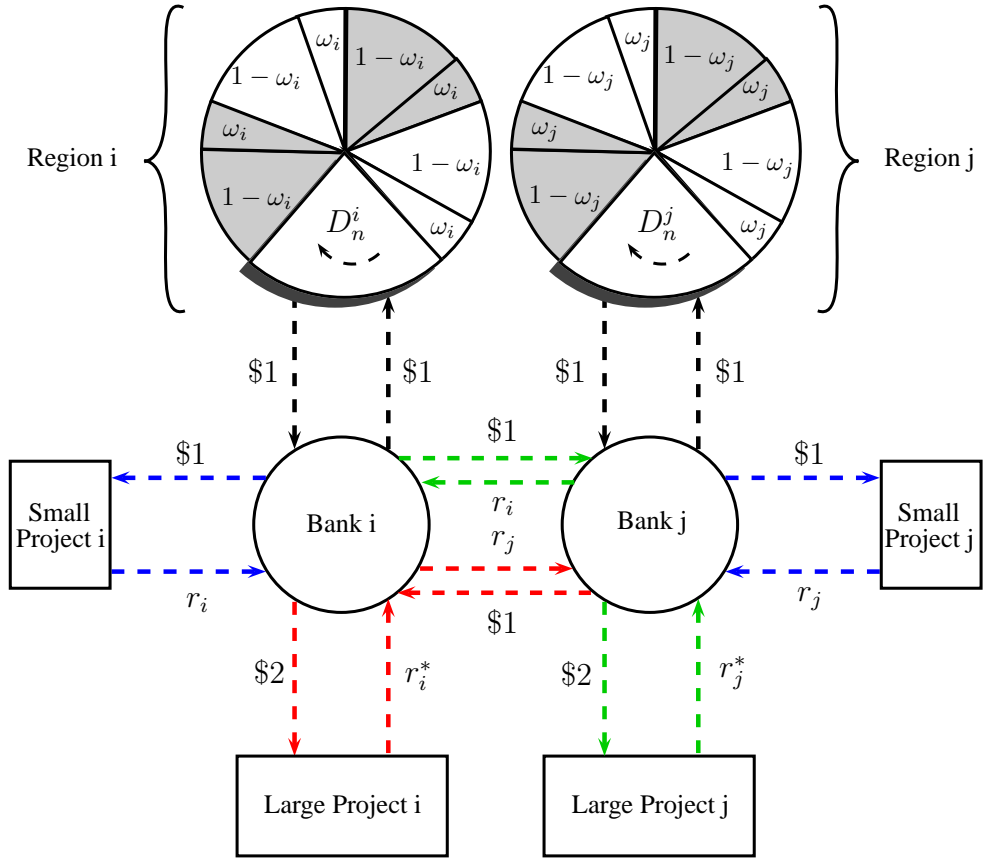
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Figure 5: Portfolio decision of banks at a particular round of interaction.

i (lender) and bank j (borrower) - whereas the red an investment in a large project in region i - by means of a loan agreement between bank i (borrower) and bank j (lender).

The condition for a loan agreement to take place is the payment by the borrower (interest plus principal) to be at least as large as the opportunity cost of the lender. The opportunity cost of the lender is due to the fact that, by disposing of \$1, the possibility of investing in a small project is foregone. Therefore, in any loan agreement, the borrower must pay to the lender at least the payoff the later would get by investing in a small project.

Upon the meeting of banks i and j, the double-headed arrows in Figure 4 become one of the following:

(i) $(B_i) \longrightarrow (B_j)$: B_i lends to B_j;

(ii) $(B_i) \longleftarrow (B_j)$: B_i borrows from B_j;

(iii) $(B_i) \cdots (B_j)$: No loan agreement between B_i and B_j.

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Therefore, with N banks, after $N-1$ rounds of interaction like the one depicted in Figure 5, many types of network structures might emerge. As an example, Figure 6 illustrates possible strucutures for $N = 4$.
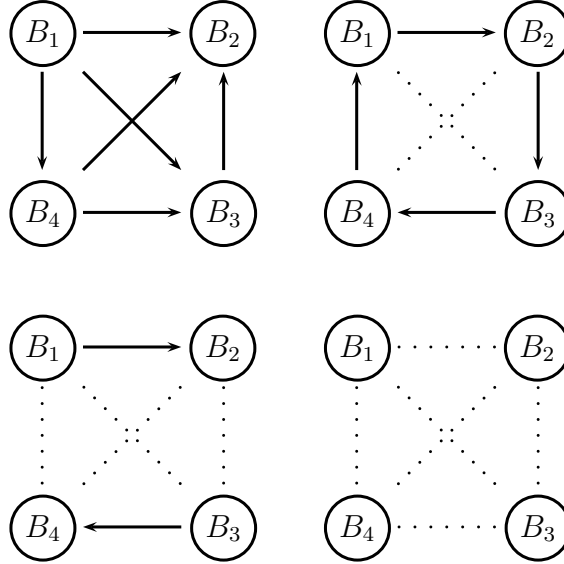

Figure 6: Examples of networks at the end of the interaction process ($N = 4$).

2.3 Maturity Mismatch

To finance an investment in either a small project or a loan, banks need deposits and, in case of large projects, to borrow money from other banks. Since assets payoff only in the long-term whereas a fraction of deposits is withdrawn in the short, banks are exposed to the problem of maturity mismatch, i.e., the use of short-term funds to finance long-term assets.

Any bank $i \in N$ has available and endowment of e_i, for every transaction they engage in, either an investment in a project or a loan. However, banks are assumed to be wealth constrained, meaning that e_i is not sufficient to cover withdrawals by early depositors, ω_i. Therefore, for any bank $i \in I$, it holds that $e_i < \omega_i$.

Being wealth constrained, early withdrawals can be met only by having banks prematurely liquidating a fraction of their investments. The early liquidation of investments comes at a fire-sale cost, though. How costly it is to liquidate assets is assumed to depend on the size of the investment made, in the following way:

(i) Large projects have a recovery rate of ρ^*: one unit of investment in a large project paying r_i^* at $t = 2$ is worth $\rho^* r_i^*$ at $t = 1$, with $0 < \rho^* < 1$;

(ii) Small projects have a recovery rate of ρ: one unit of investment in a small project paying r_i at $t = 2$ is worth ρr_i at $t = 1$, with $0 < \rho < 1$.

Since loans are always the size of an investment in a small project, $1, the cost of early liquidating them is taken to be the same as the one incurred with small projects, ρ. Another important assumption is that large projects are more costly to be early liquidated than small ones and loans, i.e.,

$$0 < \rho^* < \rho < 1. \tag{9}$$

Government intervention, as discussed next, is a way of alleviating the costs imposed on banks due to the early liquidation of assets.

2.4   Government Intervention

The early liquidation of investments is costly and, depending on how severe that is, banks might prefer to make small rather than large investments.

For reasons abstracted from, it might be in the interest of the government to reduce the investment cost of banks. Intervention is thought of as if there was a secondary market allowing for the purchase of distressed assets and, by actively participating on that, the government would be able to increase the recovery rate of banks' investments. It is assumed that intervention changes the recovery rates as the following:

(i) For large projects, $\rho^* + \gamma^* (1 - \rho^*)$: one unit of investment in a project paying r_i^* at $t = 2$ is worth $[\rho^* + \gamma^* (1 - \rho^*)] r_i^*$ at $t = 1$;

(ii) For small projects, $\rho + \gamma (1 - \rho)$: one unit of investment in a project paying r_i at $t = 2$ is worth $[\rho + \gamma (1 - \rho)] r_i$ at $t = 1$.

Therefore, with γ and γ^* defined as the government intervention parameters for small and large investments, respectively, under no intervention, i.e., $\gamma^* = \gamma = 0$, the original recovery rates apply, ρ for small projects and ρ^* for large ones. On the other hand, with full government participation, $\gamma^* = \gamma = 1$, there is no fire-sale cost to be incurred when projects are liquidated.

In order to capture the effects of what is thought as too-big-to-fail type of policy, it is assumed that large investments command more support from the government than small

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ones, i.e., $\gamma^* > \gamma$. Despite such a policy, however, large investments are still assumed to be more costly to liquidate than small ones, i.e., even with government intervention they still command a lower recovery rate:

$$\rho + \gamma (1 - \rho) > \rho^* + \gamma^* (1 - \rho^*). \tag{10}$$

2.5 Timeline of Events

With the ingredients of the model in hand, the timeline of events is the following:

- $t = 0$:

  1. Banks meet pairwise, giving rise to a network structure after the interaction process. At each round of meetings banks decide:
     (i) Whether or not to form a link (make a loan or borrow);
     (ii) How much to invest in the short-term asset;
     (iii) How much of the long-term asset (project or loan) to liquidate in order to meet early withdrawals.

- $t = 1$:

  1. Banks execute the liquidation strategy;
  2. Together with the investment in the short-term asset, proceeds are used to pay early depositors.

- $t = 2$:

  1. Payoffs from long-term assets (projects and loans) are realized, with the fraction not early liquidated accruing to banks;
  2. Banks pay late depositors and clear positions with other banks, consuming the remainings as profits.

The next section details the link formation process of banks.

3 Link Formation

After all rounds of interaction, the network structure that arises is the result of banks' borrowing and lending decisions made at each of the pairwise meetings they participate. Every time a loan is made a link in the network is formed. For that purpose, consider any two arbitrary banks, i and j. Without loss of generality, one focus on each of the three possible choices of bank i when meeting bank j.

3.1 Investment in a Small Project

If bank i is to invest in a small project, the budget constraints to be satisfied in each period are:

$$1 + b_i \leq 1 + e_i \quad \text{(BC at } t = 0)$$

$$b_i + \alpha_r^i r_i \left[\rho + \gamma(1 - \rho)\right] \geq \omega_i \quad \text{(BC at } t = 1) \tag{11}$$

$$(1 - \alpha_r^i) r_i = (1 - \omega_i) + e_i + \pi_i \quad \text{(BC at } t = 2)$$

where:

- π_i: profit of bank i with an investment in a small project;

- b_i: investment in the short-term asset (bond that pays no interest);

- α_r^i: fraction of the small project to be liquidated at $t = 1$.

The budget constraint at $t = 0$ expresses that total expenses, i.e., investment in the small project, \$1, plus investment in the short-term asset, b_i, cannot exhaust the amount of total resources available, namely deposits, \$1, and equity disbursement, e_i. At $t = 1$, the revenue from the short-term asset, b_i, plus the proceeds from the liquidation of a fraction of the small project, $\alpha_r^i r_i \left[\rho + \gamma(1 - \rho)\right]$, should suffice to service early depositors, ω_i. Finally, at $t = 2$, the fraction not liquidated of the small project, $(1 - \alpha_r^i) r_i$, must allow the bank to meet the demands from late depositors, $1 - \omega_i$, plus the amount owed to equity holders, e_i. What is left from the payoff of the small project after paying late depositors and equity holders constitutes the profit of the bank, π_i.

Since bank i does not want to (i) leave resources unused and (ii) liquidate more than what is necessary of the small project, the budget constraints at $t = 0$ and $t = 1$ will be binding, from which one can solve for b_i and α_r^i, yielding:

$$b_i = e_i, \tag{12}$$

$$\alpha_r^i = \frac{\omega_i - e_i}{r_i \left[\rho + \gamma\left(1 - \rho\right)\right]}. \tag{13}$$

Substituting into the expression for bank i's profit, π_i becomes:

$$\pi_i = r_i - \left\{\left(1 - \omega_i\right) + e_i + \frac{\omega_i - e_i}{\left[\rho + \gamma\left(1 - \rho\right)\right]}\right\}$$

$$\Leftrightarrow \quad \pi_i = r_i - \underline{r_i}, \tag{14}$$

where

$$\underline{r_i} := \left(1 - \omega_i\right) + e_i + \frac{\omega_i - e_i}{\left[\rho + \gamma\left(1 - \rho\right)\right]}. \tag{15}$$

Obviously, bank i would be willing to accept deposits that it could channel to a small project as long as $\pi_i \geq 0$, i.e., $r_i \geq \underline{r_i}$, which is an assumption maintained for any $i \in N$.

3.2 Investment in a Large Project

If bank i is to invest in a large project, a loan agreement has to be established so that \$1 is borrowed from bank j. The budget constraints are then modified in the following way:

$$2 + b_i \leq 2 + e_i \quad \text{(BC at } t = 0\text{)}$$

$$b_i + \alpha_{r*}^i r_i^* \left[\rho^* + \gamma^*\left(1 - \rho^*\right)\right] \geq \omega_i \quad \text{(BC at } t = 1\text{)} \tag{16}$$

$$\left(1 - \alpha_{r*}^i\right) r_i^* = \left(1 - \omega_i\right) + e_i + y_{ij} + \pi_{ij}^* \quad \text{(BC at } t = 2\text{)}$$

$$y_{ij} \geq r_j \quad \text{(IR of the Lender)}$$

Differently from an investment in a small project, in the budget constraint at $t = 0$ there is now one extra \$1 coming from the loan taken and, as a result, at $t = 2$ there is

an extra expense representing the loan[2] that bank i has to repay bank j, an yield denoted by y_{ij}. The profit of the bank in a large project in turn depends from whom the lender is and, accordingly, is denoted by π_{ij}^*.

An additional constraint in bank i's problem is the **individual rationality constraint** of bank j, the lender, labeled IR. By extending a loan, the lender incurs an opportunity cost equal to the payoff it could get by investing in a small project. Therefore, bank j is willing to participate in a loan agreement as long as $y_{ij} \geq r_j$. Without loss of generality, the borrower is assumed to have all the bargaining power and, as such, offers the minimum interest on the loan, at which the lender is indifferent between lending or not[3], resulting in $y_{ij} = r_j$.

Analogously to an investment in a small project, the budget constraints at $t = 0$ and $t = 1$ bind, for there is no reason that would lead bank i to leave resources unused and neither to over liquidate its long-term project - that is costly after all. Adding to that the fact that bank i pays the minimum interest to the lender, the following results:

$$y_{ij} = r_j, \tag{17}$$

$$b_i = e_i, \tag{18}$$

$$\alpha_{r^*}^i = \frac{\omega_i - e_i}{r_i^* \left[\rho^* + \gamma^* \left(1 - \rho^*\right)\right]}. \tag{19}$$

Bank i's profit with a large project when borrowing from bank j, π_{ij}^*, is then:

$$\pi_{ij}^* = r_i^* - \left\{ (1 - \omega_i) + e_i + r_j + \frac{\omega_i - e_i}{\left[\rho^* + \gamma^* \left(1 - \rho^*\right)\right]} \right\}.$$

3.3 Investment in Loans

From the assumptions that (i) the discount parameter for loans is the same applied to small projects and (ii) the payoff on a loan is the same as the one in a small project, *loans and small projects are perfect substitutes*. The same analysis used for small projects, therefore, applies to the case of loans: for bank i, the investment in the short-term asset

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[2]Principal plus interest.

[3]One way of breaking the indifference point towards bank j extending a loan would be to impose a cost due to asymetric information with investments in projects. Since, presumably, market forces lead banks to be more scrutinized than projects, a loan to another bank would be preferable to an equivalent investment in a small project, *ceteris paribus*.

and the fraction of the long-term investment to be liquidated before maturity - *this time loans instead of a small project* - will be the same, and so will be its profits. For practical purposes, therefore, investments in loans and small projects are indistinguishable.

4 Banks' Portfolio Allocation

Comparing the profits that could be obtained in each of their investment opportunities, banks i and j will decide whether to (i) stay in **autarky**, i.e., invest separately in their respective small projects, or (ii) engage in a **loan agreement**, allowing the borrower to invest in a large project and the creditor in a loan. The possible outcomes after the pairwise meeting of any two banks i and j are summarized in the following:

1. **Bank i wants to borrow from bank j, but not vice versa:** the former is better-off investing in a large project, $\pi_{ij}^* > \pi_i$, whereas the opposite is true for the later, $\pi_j > \pi_{ji}^*$. From expressions (14) and (20), that is the case if the following holds:

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}, \qquad (20)$$

   where the first inequality represents the fact that $\pi_{ij}^* > \pi_i$ and the second that $\pi_j > \pi_{ji}^*$.

2. **Bank j wants to borrow from bank i, but not vice versa:** Analogous to the previous condition, but now with bank j being the one interested in borrowing, $\pi_{ji}^* > \pi_j$, and bank i in lending, $\pi_i > \pi_{ij}^*$:

$$\frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} > \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \qquad (21)$$

   where the first inequality represents the fact that $\pi_{ji}^* > \pi_j$ and the second that $\pi_i > \pi_{ij}^*$.

3. **Both banks i and j want to borrow:** In this case, it holds that $\pi_{ij}^* > \pi_i$ and $\pi_{ji}^* > \pi_j$, translated into:

$$\min\left\{\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}\right\} > \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]}. \qquad (22)$$

In this scenario, the tie is broken by favouring the bank which is to make the larger profit from borrowing: if $\pi_{ij}^* > \pi_{ji}^*$, which from (20) is equivalent to

$$\left[r_i^* - (1 - \omega_i) - e_i - r_j\right] - \left[r_j^* - (1 - \omega_j) - e_j - r_i\right] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^*(1 - \rho^*)}, \quad (23)$$

then bank i ends up borrowing from bank j; otherwise, i.e., if $\pi_{ji}^* > \pi_{ij}^*$,

$$\left[r_j^* - (1 - \omega_j) - e_j - r_i\right] - \left[r_i^* - (1 - \omega_i) - e_i - r_j\right] > \frac{(\omega_j - e_j) - (\omega_i - e_i)}{\rho^* + \gamma^*(1 - \rho^*)}, \quad (24)$$

then it is bank j who borrows from bank j.

4. **Neither bank i nor bank j wants to borrow:** in this scenario, it follows that both banks are better-off investing in their respective small projects, with $\pi_i > \pi_{ij}^*$ for bank i and $\pi_j > \pi_{ji}^*$ for bank j, equivalent to:

$$\frac{[\rho + \gamma(1 - \rho)] - [\rho^* + \gamma^*(1 - \rho^*)]}{[\rho + \gamma(1 - \rho)][\rho^* + \gamma^*(1 - \rho^*)]} > \max\left\{\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}\right\}. \quad (25)$$

In this scenario, therefore, banks remain in autarky.

4.1 Government Intervention and Network Structure

The conditions for the formation of links across banks given previously hold for any arbitrary set of parameters. In particular, the conditions are true for $\gamma^* = \gamma = 0$, the case where there is no government intervention. One is interested in seeing how adding government intervention, $\gamma^* \geq \gamma > 0$, affects the structure of a network, in particular regarding the number of links, i.e., loan agreements, that banks engage in.

Proposition 1. *In any network, government intervention has a non-decreasing effect on the number of links across banks.*

Proof: See Appendix.

Proposition 1 establishes that government intervention does not brake links across banks that would already exist otherwise, and eventually it actually lead banks to engage

in new loan agreements. One might wonder if it is possible after intervention that an otherwise borrower would become a lender, considering an arbitrary pair of banks. According to Proposition 2 that follows, the answer to this question is yes.

Proposition 2. *Consider a network of banks formed without government intervention. If two arbitrary banks engage in a loan agreement, the identity of the borrower and the lender might change if one considers the network that would prevail with the participation of the government.*

Proof: See Appendix.

Government intervention, therefore, is crucial for the shape of the network structure emerging from the interaction between banks. As a corollary of Lemma 7, it also follows that under the so called too-big-to-fail policy, TBTF, $\gamma^* > \gamma > 0$, the incentives for the creation of links increases vis-a-vis what one would obtain in case government intervention was to be the same for both large and small projects, $\gamma^* = \gamma > 0$.

Corollary 3. *Under a TBTF policy, $\gamma^* > \gamma > 0$, the incentives created by the government for the banks to get connected are even stronger.*

Proof: See Appendix.

5 Characterization of the Financial System and Shocks

Following the pairwise meetings of banks in each round of interaction, the network structure - the financial system - can be describe by the following matrix:

$$
X = \begin{bmatrix}
0 & \chi_{12} & \cdots & \chi_{1N} \\
\chi_{21} & 0 & \cdots & \chi_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\chi_{N1} & \chi_{N2} & \cdots & 0
\end{bmatrix},
\tag{26}
$$

where χ_{ij} is an indicator function such that $\chi_{ij} = 1$ if i lends to j and 0 otherwise. From now on, one assumes that the quantities being discussed are for an arbitrary bank i, with $i \in N$. The row-sum gives the number of debtors, i.e., the number of loans made, $n_L^i = \sum_{j \in N} \chi_{ij}$, whereas the column-sum gives the number of creditors which, equivalently, is the number of large projects undertaken, $n_{r*}^i = \sum_{j \in N} \chi_{ji}$. Since in every

round of interaction banks invest in an asset, and there are $N - 1$ of those rounds, the number of small projects can be obtained by $n_r^i = (N - 1) - n_L^i - n_{r*}^i$.

The primitives of the model are:

$$
r^* = \begin{bmatrix} r_1^* \\ r_2^* \\ \vdots \\ r_N^* \end{bmatrix} \quad r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} \quad \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_N \end{bmatrix} \quad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix}. \tag{27}
$$

Recall that banks cannot serve early depositors only with equity disbursements, i.e, $\omega_i > e_i$, for any $i \in N$. This leads to a premature liquidation of a fraction α_r^i of an investment in either a loan or a small project, as in (13), and a fraction α_{r*}^i in case the investment is in a large project, as in (19). At $t = 2$, therefore, there is $a_L^i = r_i \left(1 - \alpha_r^i\right) n_L^i$ in loans, $a_{r*}^i = r_i^* \left(1 - \alpha_{r*}^i\right) n_{r*}^i$ in large projects and $a_r^i = r_i \left(1 - \alpha_r^i\right) n_r^i$ in small projects. Assets are then written as $a^i = a_L^i + a_{r*}^i + a_r^i$.

Liabilities are composed by the amount owed to late depositors, loans taken from other banks, i.e., debt, and networth, which is equity plus profits. There is \$1 collected from depositors in all the $N - 1$ rounds of interaction, and the fraction of these deposits that remains to be claimed at $t = 2$ is $1 - \omega_i$. Therefore, the amount owed to depositors is $l_\omega^i = (N - 1)\left(1 - \omega_i\right)$. Debt is owed to other banks due to loans taken in order to invest in large projects and, therefore, are written as $l_d^i = \sum_{j \in N} \chi_{ji} r_j$.

For the networth, equity is given by $l_e^i = (N - 1) e_i$, since there is an equity disbursement in all the rounds of interaction. Profits are obtained from investments, composed of loans and projects. Loans and small projects yield the same profit, π_i, and since there are n_L^i loans and n_r^i small projects, profits from these two assets is $\left(n_L^i + n_r^i\right) r_i$. From large projects, the profit is π_{ij}^* when the loan making the investment feasible is taken from bank j and, therefore, profits generated from going large are $\sum_{j \in N} \chi_{ji} \pi_{ij}^*$. Profits are then $l_\pi^i = \left(n_L^i + n_r^i\right) r_i + \sum_{j \in N} \chi_{ji} \pi_{ij}^*$. Networth is writen as $W^i = l_e^i + l_\pi^i$ and, therefore, liabilities in the balance-sheet are $l^i = l_\omega^i + l_d^i + W^i$.

In this way one can obtain, for any network formed with or without the participation of the government, the level of assets, investments in small and large projects, debt of banks, networth, leverage and other measures related to the balance-sheet of banks and that representing the entire financial network. An interesting feature of the participation of the government in the network formation process is that the total networth of the

financial system does not necessarily increase. This is a corollary from Propositions 1 and 2:

Corollary 4. *Let $\underline{\pi}^G := \min\left\{\pi_1^G, \ldots, \pi_N^G\right\}$ and $\overline{\pi} := \max\left\{\pi_1, \ldots, \pi_N\right\}$ denote the minimum payoff of a small project out of all the banks, with and without intervention, respectively. If $\underline{\pi}^G > \overline{\pi}$, then government intervention leads to an increase in the networth of the network of banks, $\sum_{i \in N} W^i$. Otherwise, that is not necessarily the case.*

Proof: See Appendix.

Corollary 4 is important for the study of the effects of government intervention on the capacity of networks to absorb shocks - to be defined next - given that the only cushion banks have to deal with losses is their networth.

The shocks to be introduced affect the payoffs that banks are entitled to receive from their investments in projects, either large or small. For instance, if bank i made an investment in a large project with a payoff of r_i^*, a shock of δ_i^* implies that it receives $r_i^*(1 - \delta_i^*)$ instead. If the investment was in a small project, the payoff is $r_i(1 - \delta_i)$ rather than r_i, considering a shock of δ_i. The vectors of shocks in the payoffs of small and large projects are given by, respectively:

$$\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_N \end{bmatrix} \quad \delta^* = \begin{bmatrix} \delta_1^* \\ \delta_2^* \\ \vdots \\ \delta_N^* \end{bmatrix}. \tag{28}$$

Upon being hit by shocks, banks face a loss in the asset side of their balance-sheets, whereas their liabilities remain the same. The loss is written as $\Delta^i = \Delta_L^i + \Delta_{r^*}^i + \Delta_r^i$, explained in the sequence. The loss with small projects is $\Delta_r^i = a_r^i \delta_i$, whereas with large projects $\Delta_{r^*}^i = a_{r^*}^i \delta_i^*$.

The loss with loans, Δ_L^i, will depend on the bankruptcy status of bank i's debtors and, therefore, it is endogenously determined. A bank is bankrupt if it cannot fulfill entirely its obligations with debtors and households, i.e., if the losses incurred are greater than the networth, $\Delta^i > W^i$.

For instance, assume that bank i has lent to bank j, i.e., $\chi_{ij} = 1$, and bank j is bankrupt. The losses spread by bank j are given by $\Delta^j - W^j > 0$, first evenly distributed among debtors, which are owed the amount l_d^j, and, if greater than that, spread also to

households. Therefore, for every unit borrowed, bank j pays only[4] $\left(\Delta^j - W^j\right)/l_d^j$. Upon that, the indirect loss to bank i, which expected to receive a payment of r_i for the fraction $(1 - \alpha_r^i)$ of the loan still in its balance-sheet, is $r_i\left(1 - \alpha_r^i\right)\left(\Delta^j - W^j\right)/l_d^j$. Therefore,

$$\Delta_L^i = r_i\left(1 - \alpha_r^i\right)\sum_{j \in N}\chi_{ij}\frac{\left(\Delta^j - W^j\right)^+}{l_d^j} \tag{29}$$

represents the indirect losses suffered by bank i upon the eventual default of its debtors, where $()^+$ denotes the positive part. The system of equations that endogenously determines $\widetilde{\Delta}^i := \left(\Delta^i - W^i\right)^+$ - bank's loss in excess of networth - is:

$$\widetilde{\Delta}^1 = \left[r_1\left(1 - \alpha_r^1\right)\sum_{j \in N}\chi_{1j}\frac{\widetilde{\Delta}^j}{l_d^j} + \Delta_{r*}^1 + \Delta_r^1 - W^1\right]^+, \tag{30}$$

$$\widetilde{\Delta}^2 = \left[r_2\left(1 - \alpha_r^2\right)\sum_{j \in N}\chi_{2j}\frac{\widetilde{\Delta}^j}{l_d^j} + \Delta_{r*}^2 + \Delta_r^2 - W^2\right]^+, \tag{31}$$

$$\vdots$$

$$\widetilde{\Delta}^N = \left[r_N\left(1 - \alpha_r^N\right)\sum_{j \in N}\chi_{Nj}\frac{\widetilde{\Delta}^j}{l_d^j} + \Delta_{r*}^N + \Delta_r^N - W^N\right]^+. \tag{32}$$

In matrix form, this system is written as:

$$\widetilde{\Delta} = \left(\widetilde{X}\widetilde{\Delta} + \Delta_{r*} + \Delta_r - W\right)^+, \tag{33}$$

where $\widetilde{X}\widetilde{\Delta}$ represents the vector of indirect losses, Δ_{r*} of losses in large projects, Δ_r in small and w the networth, the dimensions of these vectors being $N \times 1$. The matrix $\widetilde{X}$ is given by:

$$\widetilde{X} = \begin{bmatrix} 0 & \frac{r_1\left(1-\alpha_r^1\right)\chi_{12}}{l_d^2} & \cdots & \frac{r_1\left(1-\alpha_r^1\right)\chi_{1N}}{l_d^N} \\ \frac{r_2\left(1-\alpha_r^2\right)\chi_{21}}{l_d^1} & 0 & \cdots & \frac{r_2\left(1-\alpha_r^2\right)\chi_{2N}}{l_d^N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{r_N\left(1-\alpha_r^N\right)\chi_{N1}}{l_d^1} & \frac{r_N\left(1-\alpha_r^N\right)\chi_{N2}}{l_d^2} & \cdots & 0 \end{bmatrix}, \tag{34}$$

which equivalently can be written as

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[4]One assumes that depositors are protected by deposit insurance so the focus is on the case where $\Delta^j - W^j \leq l_d^j$.

$$\widetilde{X} = \mathrm{diag}\left(\begin{bmatrix} r_1\left(1-\alpha_r^1\right) \\ r_2\left(1-\alpha_r^2\right) \\ \vdots \\ r_N\left(1-\alpha_r^N\right) \end{bmatrix}\right)\begin{bmatrix} 0 & \chi_{12} & \cdots & \chi_{1N} \\ \chi_{21} & 0 & \cdots & \chi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_{N1} & \chi_{N2} & \cdots & 0 \end{bmatrix}\mathrm{diag}\left(\begin{bmatrix} \frac{1}{l_d^1} \\ \frac{1}{l_d^2} \\ \vdots \\ \frac{1}{l_d^N} \end{bmatrix}\right). \qquad (35)$$

The matrix $\widetilde{X}$ is simply a weighted version of X, the matrix representing the network structure, row-weights being the expected payoff on loans made still on banks' balance-sheet, $r_i\left(1-\alpha_r^i\right)$, whereas column-weights are each bank's per unit amount owed to total debt, $1/\sum_{k\in N}\chi_{ki}r_k$. From (29), an element i,j of $\widetilde{X}$ represents the per unit loss that bank j imposes on bank i, conditional on i having lent to j, i.e., $\chi_{ij} = 1$.

One proceeds with a thought experiment and assume that banks have zero networth, i.e., $W^i = 0$, for $i \in N$. Expression (33) is then written as:

$$\widetilde{\Delta} = \widetilde{X}\widetilde{\Delta} + \Delta_{r^*} + \Delta_r, \qquad (36)$$

or, if some conditions to be discussed are satisfied, as

$$\widetilde{\Delta} = \left(I - \widetilde{X}\right)^{-1}\left(\Delta_{r^*} + \Delta_r\right). \qquad (37)$$

Following Takayama (1985), by the conditions being met one means a positive answer to:

1. For any given $c \geq 0$, is there an $\widehat{\Delta}$ such that $\widehat{\Delta} = \left(I - \widetilde{X}\right)^{-1}c$? Is such a $\widehat{\Delta}$ unique?

2. Is the matrix $\left(I - \widetilde{X}\right)$ nonsingular? If so, is it the case that $\left(I - \widetilde{X}\right) \geq 0$?

The following lemma is used in giving an answer to the above questions. One recalls that a $n \times n$ matrix B has a dominant diagonal if there are positive numbers $d_1, d_2 \ldots, d_n$ such that $d_j |b_{jj}| > \sum_{i\neq j} d_i |b_{ij}|$, for $j = 1,\ldots,n$.

Lemma 5. *The $N \times N$ matrix $B := \left(I - \widetilde{X}\right)$ has a dominant diagonal.*

Proof: See Appendix.

From theorems 4.C.3, 4.C.4 and 4.C.6 in Takayama (1985), $\left(I - \widetilde{X}\right)$ having a dominant diagonal implies that the answers to the existence and nonsigularity questions raised above are all positive and, not only that, $\left(I - \widetilde{X}\right)^{-1}$ can be written as:

$$\left(I - \widetilde{X}\right)^{-1} = \sum_{k=0}^{\infty} \left(\widetilde{X}\right)^k = I + \widetilde{X} + \widetilde{X}^2 + \widetilde{X}^3 + \dots . \tag{38}$$

Combining (37) and (38), it follows that:

$$\widetilde{\Delta} = (\Delta_{r^*} + \Delta_r) + \widetilde{X}(\Delta_{r^*} + \Delta_r) + \widetilde{X}^2(\Delta_{r^*} + \Delta_r) + \widetilde{X}^3(\Delta_{r^*} + \Delta_r) + \dots . \tag{39}$$

The terms in the series (39) can be interpreted in the following way: $(\Delta_{r^*} + \Delta_r)$ corresponds to the first wave of shocks banks receive from investments in small and large projects. Given the assumption that the networth of banks is negligible, every bank is bankrupt upon being hit by shocks of any dimension and, therefore, default by debtors on loans taken will ensue. Banks defaulting on loans will cause a second wave of shocks, $\widetilde{X}(\Delta_{r^*} + \Delta_r)$, that adds to the first one due to direct losses. The matrix $\widetilde{X}$ provides the factors according to which a unit loss of debtors - column header banks - is spread among creditors - row header banks. Multiplying $\widetilde{X}$ by $(\Delta_{r^*} + \Delta_r)$, therefore, transforms the per unit losses in total losses.

The second wave of shocks might lead to a third one in which, for example, a debtor bank 1 spreads losses to a creditor bank 2, which in turn is a debtor of bank 3, this last on its own being a debtor of bank 1 - in other words, if there is a collection of banks connected in a circular chain. The interpretation of the additional terms in (39) is analogous and, as the simulations performed in the next section show, they tend to die fast since, in the great majority of cases, only a handful of banks concentrate most of the loans made in the economy. All the rest just make loans or invest in small projects, in this way not constituting channels for contagion that would give rise to terms of higher order in (39).

The analogy with input-output analysis comes from the elements of $\widetilde{X}$ being viewed as the inputs necessary to produce the loss generated by banks, $\widetilde{\Delta}$. For, from the analysis preceding (29) one knows that $r_i\left(1 - \alpha_r^i\right)\left(\widetilde{\Delta}^j\right)/l_d^j$ represents the unitary loss produced by bank j and imposed on bank i - conditional on $\chi_{ij} = 1$, i.e., bank i having lent to bank j. Therefore,

$$\widetilde{\Delta}^j \left[\frac{r_1\left(1 - \alpha_r^1\right)\chi_{1j}}{l_d^j} + \dots + \frac{r_N\left(1 - \alpha_r^N\right)\chi_{Nj}}{l_d^j} \right] \tag{40}$$

is the total loss produced by bank j. To make up such a loss one sums up its individual pieces, i.e.,

$$\underbrace{\widetilde{\Delta}^j \left[\frac{r_1 \left(1 - \alpha_r^1\right) \chi_{1j}}{l_d^j} \right]}_{\text{Bank } j\text{'s loss channeled to bank } 1} + \ldots + \underbrace{\widetilde{\Delta}^j \left[\frac{r_N \left(1 - \alpha_r^N\right) \chi_{Nj}}{l_d^j} \right]}_{\text{Bank } j\text{'s loss channeled to bank } N}, \qquad (41)$$

so that, for any i, $r_i \left(1 - \alpha_r^i\right) \chi_{ij}/l_d^j$ corresponds to the unit contribution of the loss imposed on bank i in the making of the loss produced by bank j. These input factors of production are elements of the j's column in $\widetilde{X}$, as it would be in a standard input-output matrix.

6 Measures of Financial Fragility

The network structure automatically allows one to obtain the balance-sheet of banks, in which assets and liabilities can be characterized. The assets represent how banks are exposed to shocks, since they show the composition of portfolios in the different classes of investments, i.e., large projects, small projects, and/or loans. Liabilities might indicate how harmful a bank can be, since they detail to whom and in which amount banks owe money. The networth gives a measure of the health of a bank, since it provides what a bank has - assets - minus what it owes - debt.

The networth is, thus, a first proxy to evaluate how fragile a bank is to shocks, i.e., its vulnerability to a sudden decrease in the payoff of its investments. The network's networth is obtained by summing up that of its individual members. Thus, one possible way of characterizing the degree of fragility of a network would be by calculating the maximum size of a shock it could absorb without causing bank failures. That this maximum shock turns to be the network's networth is guaranteed by a celebrated result of Ford & Fulkerson - the max-flow min-cut theorem. Before introducing that, one superficially touches upon some terminology.

In network flow theory, the shocks hitting projects would be deemed *sources*, whereas banks' networth, *sinks*. With that one can define a *flow*, i.e., a function over the links and nodes of the structure so that every shock hitting a bank is channeled to its sink and, if that is not large enough, to other banks - conditional on the existence of a link that allows the shock to be passed on. The total size of the shock crossing the network - the sum of the individual banks' shocks - is always preserved.

Partitioning a network in a way that sources are split from sinks defines a *cut* - or, to be precise, an *s-t cut*. The capacity of an s-t cut is the size of the maximum flow that

can go from the source to the sink side of the partition, i.e., the maximum amount the souces can generated so that all the sinks get filled. Thus, the capacity of an s-t cut is determined by the sum of the capacity of its individual sinks. The Ford & Fulkerson's theorem establishes what the maximum flow is:

Theorem 6. *(Ford & Fulkerson, 1956) The maximum value of an s-t flow is equal to the minimum capacity over all s-t cuts.*

In the setup used for the study of contagion, there is only one partition separating sources - projects - from sinks - networth - and therefore one concentrates on a single s-t cut, whose capacity is defined by the sum of banks' networths. Thus, the maximum loss a network can absorb without causing bank failures, $\overline{\Delta}$, is given by the sum of the networths of its individual members, i.e., $\sum_{i \in N} W^i$, otherwise at least one bank fails.

Corollary 4 establishes that total networth does not necessarily increase with government participation and, thus, it cannot be said either that the very same intervention leads to a higher capacity of the network to absorb shocks. Also, a network with a higher number of links is not a foregone conclusion for less fragility since, as stated in Proposition 1, intervention potentially leads to that but, as just said, does not necessarily increase the capacity of the network to absorb shocks.

Networth alone does not take into account the possibility of contagion, which is important to be considered in any measure of financial fragility. Instead of focusing only on direct failures that are due to shocks in projects, one has to think also of indirect cases of bankruptcy where the source of distress comes from banks being creditors of non-performing institutions.

One approach is to consider, for a given network, the number of bank failures following banks' shocks, one at a time, in the payoff of their projects. This constitutes a stress test of the network, in order to see how robust the overall structure is to problems its individual members might face. By considering the total cases of distress, direct and indirect, one takes into account contagion, as not only fails due to payoff shocks are considered but also those due to debt default.

In this way, consider a network with N banks, and take an arbitrary bank i facing shocks in its projects. The set

$$D^i := \left\{ j \neq i \,\middle|\, \Delta^j > W^j \right\} \tag{42}$$

contains all those banks that become distressed as a result of contagion, since the only bank facing direct shocks is bank i. The cardinality of this set, $|D^i|$, thus, gives the total number of indirect failures following bank i's shocks. For contagion to ensue, a necessary condition is that bank i is bankrupt itself and, therefore, if $|D^i|$ is positive, the total number of bank failures is $1 + |D^i|$. The index

$$f^i := \begin{cases} 1 + |D^i| & \text{if } D^i \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \tag{43}$$

gives, therefore, a measure of the relative fragility of the network to bank i, for a particular realization of shocks that it faces. By doing the same for every bank $j \neq i$ in the network and combining all the results, for instance taking

$$f := \sum_{i \in N} f^i, \tag{44}$$

one has a measure of the overall fragility of the network relative to the individual failure of its members. To get more robust results, one perform multiple simulations drawing different shocks for every bank $i \in N$.

To fix the idea, consider the following example of a network with 6 banks, generated under $\rho^* = 0.05\rho$, $\gamma^* = 0.8$ and $\gamma = 0.3$, and its non-government counterpart, i.e., the one obtained in the same way but with $\gamma^* = \gamma = 0$. The other parameters used are given by Table 1:

	$\mathbf{r}^*$	$\mathbf{r}$	ω	e
Bank 1	3.23	1.19	0.05	0.04
Bank 2	3.00	1.01	0.17	0.12
Bank 3	2.22	1.19	0.15	0.11
Bank 4	2.55	1.08	0.09	0.07
Bank 5	2.97	1.00	0.15	0.12
Bank 6	2.71	1.21	0.14	0.10

Table 1: Parameters used for the generation of the networks in Figure 7.

Figure 7 gives the networks obtained under such a set of parameters. After performing $1,000$ simulations for each bank, where both shocks hitting large and small projects, δ and δ^*, respectively, are drawn from independent $U[0,1]$ uniform distributions, Table 2
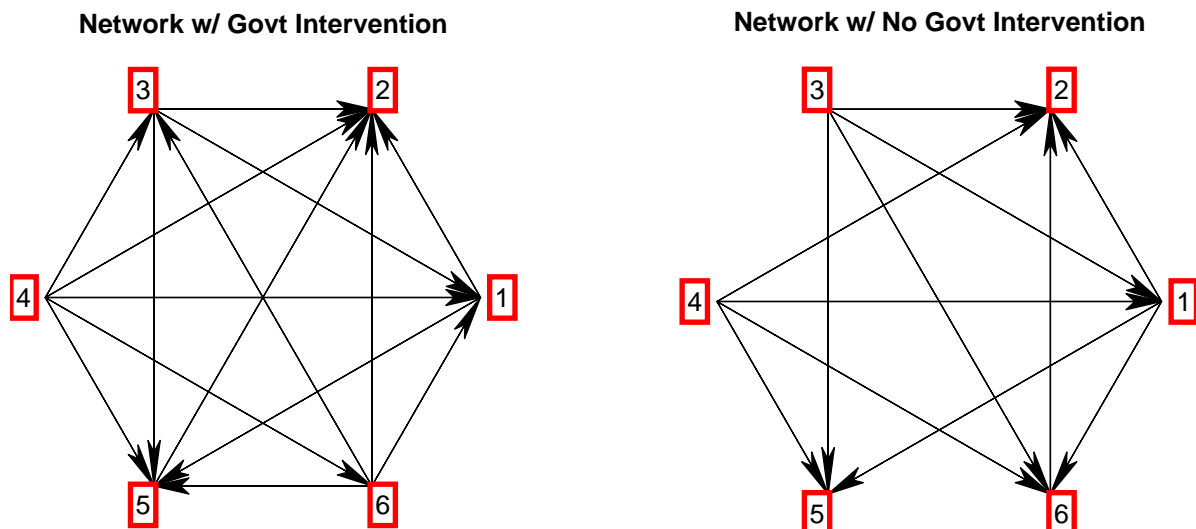
Figure 7: Networks generated under the parameters of Table 1.

gives the number of failures induced by shocks in each of the banks, split in indirect and direct cases of distress. Some other measures, like the leverage ratio, LR, and the number of (inward) links, are included.

Government						No Government					
Bank	LR	Links	Ind Fails	Dir Fails	Networth	Bank	LR	Links	Ind Fails	Dir Fails	Networth
5.00	0.69	4.00	492.00	660.00	4.08	6.00	0.82	3.00	703.00	895.00	1.82
3.00	0.79	2.00	474.00	664.00	1.86	5.00	0.68	3.00	450.00	624.00	3.67
2.00	0.63	5.00	475.00	633.00	6.04	2.00	0.62	4.00	273.00	472.00	5.32
1.00	0.67	3.00	454.00	609.00	3.79	4.00	0.91	0.00	0.00	572.00	0.50
6.00	0.81	1.00	249.00	449.00	1.42	1.00	0.70	2.00	0.00	203.00	2.81
4.00	0.91	0.00	0.00	0.00	0.50	3.00	0.83	0.00	0.00	0.00	0.89

Table 2: Ranking of banks according to the number of failures.

For the network obtained under intervention, calculating (44) one has $f = 5,159$ and, for its non-government counterpart, $f = 4,192$, across a total of 100,000 simulations. If one is to consider these as measures of fragility, in this particular example the network under intervention would be deemed as more fragile.

The number of both indirect and direct failures happens to be large in the government intervention network so that, even considering one of them instead of the total as a measure of fragility, it would still lead to te same result. However, this is not necessarily the case given that intervention, at the same time potentially leading to a larger number of links and, thus, causing banks to become more exposed to indirect cases of distress, can also increase network's networth, making it more robust to direct shocks. The first

increases the number of failures whereas the second, decreases it. Which of these effects dominates is not straightforward, and neither is the direction of changes in the measure of financial fragility following intervention.

For any given network, a question that can be adressed using this methodology refers to which node is deemed to be the most important in terms of financial fragility, i.e., which is the bank that, once hit by shocks, will bring about the largest number of failures, direct and indirect. An analogous question is tackled by search engines in the internet, where the goal is not to rank banks but, rather, to show the results of a particular query, in a way that links considered to be the most relevant - according to specific algorithms - appear first. This is one reason why some engines are more popular than others, i.e., they have a better way of ranking relevant information.

PageRank, developed by Larry Page, is the algorithm used by the Google Internet search engine. The algorithm seeks to rank a *hyperlinked set* of documents by means of assigning to each of them a measure, or weight, that summarizes their relative importance, as described in Chiang (2012). A network is a hyperlinked set, with edges, or links, going from nodes to nodes. The PageRank algorithm assigns a weight to a node as a function not of the number of nodes leading to it but, rather, the number of nodes leading to the nodes that are leading to it, and so on and so forth, in a recursive way.

Such an idea finds its use in the way banks should be ranked in terms of their relative importance to financial fragility: for the same reason that a webpage with many links pointing to it might not be the most relevant in an internet query, a bank with the highest number of connections does not necessarily need to be the one with the highest potential for causing failures, thus not the most relevant for financial fragility. What should make a bank to be pivotal in a particular network is its own financial health - networth - taken together with that of banks it is connected to. Contagion is easier to ensue in cases where weak banks operate together, since the failure of one will most probably bring about the failure of the other. This is a what makes a variant of the PageRank algorithm to be useful in a ranking of banks where ones focus on their relative importance to financial fragility.

By simulating shocks, one bank at a time, and calculating the number of failures resulting, one analogously does for the network of banks what the PageRank algorithm does for the internet web. It is in this sense that f implicitly embeds on it the idea

used by Google to rank webpages. It should not come as a surprise, therefore, that in a ranking of banks according to such a measure of fragility, the most important one will not necessarily be that with the largest number of connections, or inward arrows.

Going back to Table 2, the bank most highly ranked does not happen to be the one with the highest number of links, whatever the policy under consideration. In the particular case of intervention, bank 5 lags behing bank 2 and, with no government, bank 6 is tied with number 5 but has fewer links than bank 2. What makes bank 5 and bank 6 to lead the rankings for the two networks is that both are creditors of banks that have a relatively small networth so that, whenever 5 and 6 fail, another bank will probably become distressed too. Bank 5 has a relatively high networth but not bank 6, which has a cushion to absorb shocks higher only when compared to that of the banks it is debtor - a typical example of weak banks operating together.

Also, from Table 2 one notices that neither bank 5 or 6 has either the highest leverage ratio, LR_i, given by

$$LR_i := 1 - \frac{W^i}{a^i} = \frac{\overbrace{l_\omega^i + l_d^i}^{\text{Total debt}}}{\underbrace{a^i}_{\text{Total assets}}}, \tag{45}$$

or the lowest networth across the banks in the network they belong to. For instance, bank 5 is only the 4th most leveraged bank, and has the 2nd highest networth, whereas bank 6 occupies the 3rd position in terms of leverage and the 4th in terms of networth. This shows how important the network structure turns to be if one aims at spotting those banks that can cause a great deal of a problem once they become distressed. By focusing only on more traditional financial indicators, like leverage and networth, might potentially be misleading, and for sure it would be in the example at hand.

However insightful it might be for the study of financial fragility and in determining the relative importance of banks, it would be more plausible to expect shocks to hit the entire network, with banks being affected at different levels. Therefore, instead of a single bank facing losses in its projects being the source of distress, every bank becomes part of the problem as soon as they all are hit by shocks. Not all will fail and spread contagion, but some might do only because of third parties defaulting on their debt. In this regard, consider the following sets:

$$D := \left\{ i \in N \,\middle|\, \Delta^i > W^i \right\} \tag{46}$$

and

$$D' := \left\{ i \in N \,\middle|\, \Delta^i_{r^*} + \Delta^i_r > W^i \right\}. \tag{47}$$

Set D contains all those banks that fail as a result of either direct shocks or contagion, whereas D' has in it those that would be deemed bankrupt even in the absence of contagion. Obviously one has that $D' \subseteq D$, allowing a third set to be defined,

$$D'' := D \backslash D', \tag{48}$$

which in turn contains only banks that fail due to contagion. Analogously to (44),

$$f := |D| = |D'| + |D''| \tag{49}$$

is a measure of financial fragility capturing how banks are exposed to direct shocks and indirect failures, due to them belonging to a particular network. Contrasting f obtained in networks with and without intervention, one gathers how important government's policy is for financial fragility. Also, it is necessary to split the total number of bank failures in cases of direct and indirect distress, since intervention might lead to opposite changes in those numbers that otherwise would go unnoticed. For instance, intervention might increase network's networth, leading to a smaller number of direct bank failures but, since it might also increase the number of links, it can make banks more prone to become distressed due to contagion.

The framework developed has many parameters and, therefore, it is important to perform a comparative statics exercise in order to understand why and when intervention matters. Differently from the case where one aimed at stressing a specific network to obtain a measure of its robustness to shocks in individual banks, one is now interested in understanding how different set of parameters lead to different effects coming from an intervention policy.

One way to specify changes in the set of parameters is by making them vary in a way as if resembling economies at different stages of financial development. Thus, by studying the effects of intervention under each of these set of parameters, one might have something to say about how the impact of government's policy is related to the context

where an economy belongs, and which variables in its environment matter for policy and are conducive to different outcomes. The following section digs into that.

6.1 Simulations and Financial Development

In the framework developed, government intervention is a way of allowing banks to recoup a fraction of their costs, due to the early liquidation of projects, which in turn is caused by the maturity mismatch between the assets and the liabilities side of balance-sheets. The parameters involved in this calculation are projects' payoffs, fraction of early depositors and capitalization of banks, not forgetting those that represent the fire-sale costs and government's policy. One would expect, therefore, the effect of intervention to be related to the level at which these parameters are specified.

Instead of just running simulations and relating changes in the outcomes from government's policy to changes in the parameters, one can impose a finer partition on the support of the parameters' distributions and associate to the economy generated from those a certain degree of financial development. That would give a way of looking at the effects of government's policy in economies at different levels of advancement. Recall that the parameters of the model are:

- r^*: return on large projects;

- r: retutn on small projects;

- ω: fraction of early depositors;

- e: equity (capitalization);

- ρ and ρ^*: fire-sale parameters for small and large projects, respectively; and

- γ and γ^*: government intervention for small and large projects, respectively.

The economies that are to represent different cases of financial development are labeled *high, middle and low income* economies. They are distinguished according to the following assumptions:

1. The distribution of payoffs across the economies is ranked according to the following:

$$U_l^{r^*} \quad fosd \quad U_m^{r^*} \quad fosd \quad U_h^{r^*} \tag{50}$$

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for large and

$$U_l^r \quad fosd \quad U_m^r \quad fosd \quad U_h^r \tag{51}$$

for small projects, i.e., the distribution for the low income economy *first-order stochastically dominates* that for a middle one, which in turn dominates that for a high income economy - no matter the size of the project. It is such because the higher the income the more developed the economy is assumed to be and, as such, the harder it should be to find projects yielding high payoffs, as those would had been exploited in earlier stages of development;

2. For the distribution of early depositors, it is imposed that:

$$U_l^\omega \quad fosd \quad U_m^\omega \quad fosd \quad U_h^\omega, \tag{52}$$

i.e., depositors in low income economies are more predisposed to withdraw early as they have less of a cushion in terms of money and, therefore, are potentially more exposed to liquidity needs, as opposed to depositors in other economies where the income level is assumed to be higher;

3. Banks are on average less capitalized the lower the income of the economies where they are inserted, which translates to:

$$U_h^e \quad fosd \quad U_m^e \quad fosd \quad U_l^e. \tag{53}$$

Regarding the other parameters, one assumption previously made was that the profitability of small projects, expression (14), is nonnegative, otherwise banks would not even be willing to take deposits from retailers. In this way, for any specification of r^*, r, w and e, and taking the level of government intervention in small projects, γ, as given, the fire-sale parameter ρ is endogenously determined so that no bank is at a loss by going small. Having ρ endogenously determined, the fire-sale parameter for large projects, ρ^*, is set so that it represents a fraction of the one for small projects. In the simulations performed, that captures circumstances when having to liquidate a large project before maturity gets more and more costly relative to liquidating a small one.

By setting the parameters as above one can make an inter comparison of economies and see how they are differently affected by government's policy. However, one is also interested in seeing how different policies affect the same economy, i.e., an intra comparison. For that purpose, four different levels of government intervention for large projects are considered, namely *high*, γ_h^*, *medium*, γ_m^*, *low*, γ_l^*, *and indistinguishable*, γ_i^*, the last one being named as such since on it the level of intervention in large and small projects is the same. Obviously it must be that:

$$\gamma_h^* > \gamma_m^* > \gamma_l^* > \gamma_i^* = \gamma. \tag{54}$$

The parameter corresponding to government intervention for small projects, γ, is in turn simulated at three different levels, *high*, γ_h, *medium*, γ_m, *and low*, γ_l, so that one can control for the robustness of the results, with:

$$\gamma_h > \gamma_m > \gamma_l. \tag{55}$$

Table 3 gives the specifications used to generate the set of parameters for the simulations. For the columns one has:

- ρ^*/ρ: gives the relation between ρ^* and ρ, the fire-sale parameters for large and small projects, respectively, showing how severe it is to liquidate a fraction of the large project before maturity relative to liquidating a fraction of the small one. In particular, for $\rho^*/\rho = .05$, a fraction of the small project at $t = 1$ is worth only ρ whereas for a large project it is only 5% of that, i.e., $\rho^* = .05\rho$ (recall that ρ is endogenously determined);

- $\overline{\gamma^*}$: sets the upper bound on the government intervention parameter for large projects, γ^*, and from it the different policy levels in (54) are derived, i.e., for a high level of intervention, $\gamma_h^* = \overline{\gamma^*}$, for a medium level of intervention, $\gamma_m^* = \gamma + 2\left(\overline{\gamma^*} - \gamma\right)/3$, and for a low level of intervention, $\gamma_i^* = \gamma$;

- γ: the government intervention parameter for small projects;

- N: number of banks;

- $\overline{r^*}$, $\overline{r}$, $\overline{\omega}$, $\overline{\delta}$ and $\overline{\delta^*}$: upper bound on the support of the distribution of payoffs for large projects, small projects, early depositors, shocks for small projects and shocks

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for large projects, respectively.

Simulation	ρ^*/ρ	$\overline{\gamma^*}$	γ	N	$\overline{r^*}$	$\overline{r}$	$\overline{\omega}$	$\overline{\delta}$	$\overline{\delta^*}$
I	.05	.80	.30	10	6	2	.50	1	1
II	.10	.80	.30	10	6	2	.50	1	1
III	.15	.80	.30	10	6	2	.50	1	1
IV	.20	.80	.30	10	6	2	.50	1	1
V	.05	.80	.50	10	6	2	.50	1	1
VI	.10	.80	.50	10	6	2	.50	1	1
VII	.15	.80	.50	10	6	2	.50	1	1
VIII	.20	.80	.50	10	6	2	.50	1	1
IX	.05	.80	.70	10	6	2	.50	1	1
X	.10	.80	.70	10	6	2	.50	1	1
XI	.15	.80	.70	10	6	2	.50	1	1
XII	.20	.80	.70	10	6	2	.50	1	1

Table 3: Parameters used in the simulations performed.

In every simulation one has a 1,000 draws for each possible level of intervention (large projects), and that is for each type of economy. Since there are 4 of those levels, namely γ_h^*, γ_m^*, γ_l^* and γ_i^*, and three types of economy, every simulation involves 12,000 draws from uniform distributions as specified below. Let $k = 1, 2, 3$ denote the high, middle and low income economies, respectively. The random parameters for the simulations are then generated in the following way:

- Payoffs of **large projects** are drawn from

$$U_k^{r^*} := U\left[\overline{r}, r_k^*\right],\qquad(56)$$

with $r_k^* = \overline{r} + k(\overline{r^*} - \overline{r})/3$;

- Payoffs of **small projects** from

$$U_k^r := U\left[1, r_k\right],\qquad(57)$$

with $r_k = 1 + k(\overline{r} - 1)/3$;

- Fractions of **early depositors** from

$$U_k^\omega := U\left[0, \omega_k\right], \tag{58}$$

with $\omega_k = k\overline{\omega}/3$;

- **Capitalization** ok banks is taken to be

$$e_k = k\frac{\omega_k}{4}; \tag{59}$$

where ω_k is drawn as previously described for economy k;

- Shocks to **small** and **large projects** from

$$U^\delta := U\left[0, \overline{\delta}\right] \tag{60}$$

and

$$U^{\delta*} := U\left[0, \overline{\delta^*}\right], \tag{61}$$

respectively.

With all the parameters in place, one obtain network structures for the high, middle and low income economies, according to varying levels of intervention (large projects). Coupled with those are the networks obtained under the same parameters but where intervention is nonexistent, after all the objective is to determine the effects of intervention vis a vis no government. For that, the metric to be used is the total number of defaults, f, as defined in (49). That is obtained by imposing the shocks on small and large projects for each bank, in every network, and calculating how that spreads and leads to contagion. The number of failures is obtained and averaged out, after all the simulations in a particular type of economy and for a specific level of intervention are done. The results obtained are analysed in the sequel.

6.2 Qualitative Results from Simulations

Some qualitative statements can be made upon the results obtained from the simulations. The first one is:

Government intervention leads to more financial fragility.

The level of financial fragility in economies under no intervention is persistently higher compared to those in networks obtained without the participation of the government. For low income economies, the financial fragility indicator is at least 40% higher when the government is present. The difference gets to be small only when intervention does not change much the recovery rate of projects, which is the case when one has in the same scenario a high income economy and a relatively small fire-sale cost.

The number of direct bank failures is not as much affected by government intervention as is the number of indirect cases of distress. Intervention increases both, for all types of economies, but the number of indirect bank failures associated with structures under participation of the government is significantly higher.

The following is a result related to the higher number of indirect failures under government intervention and that can be viewed as a numerical validation of Proposition 1:

Government intervention leads to a persistently higher number of links.

Indirect bank failures are due to contagion, when otherwise healthy institutions become distressed as a result of third parties not fulfilling their debt obligations. Therefore, a necessary condition for indirect failures is the existence of links across institutions, since those represent loans taken as a way of financing larger projects. Proposition 1, by saying that intervention does not brake links across banks - although it might change the identity of the borrower and the lender - thus helps in the understanding of why network structures generated under the government's presence have a higher number of indirect failures - basically, because they have more links.

The next result is on its turn a consequence of Corollary 4, and it also has an impact on the number of bank failures:

Government intervention leads to a persistently higher level of total net-worth.

This, in theory, should partially offset the increased exposure of banks to third part failures discussed above, since networth is the cushion that banks have to absorb shocks. On average, the simulations show that total networth under government intervention is persistently higher than without, for all types of economies, and in particular for the less developed types. Thus, one in principle would expect networks under intervention to be associated with a smaller number of failures.

What happens is that, even though increasing the total networth of banks, intervention makes them to diversify less their risks, since banks that otherwise would be investing in small projects - in their respective regions and with their respective idiosyncratic shocks - now concentrate their risks in the very same institution to which loans are made. When an institution that becomes a debtor to many others is hit by a large shock, its higher networth - coming from investments in large projects - is not enough to prevent the shocks it receives from rippling across the network.

For the creditors, thus, it becomes as if being hit by those very same large shocks. Recall that banks' projects are hit by shocks coming independent uniform distributions, so that the probability of a single bank being hit by a large shock is higher than that of two or more being hit at the same time by a corresponding large shock. This is what makes a higher number of banks to succumb to large shocks when intervention is in place.

All the more surprising is the fact that, for the very same type of economies where it is verified the highest increase in networth upon intervention, it is also observed the highest increase in the level of financial fragility - and that is for the less developed type. The way these economies are designed - having a higher fraction of early depositors and being less capitalized - makes their structure to be much more dependent on government's policy, and that is why for them the results become much more significant in a government versus non government comparison.

That the effects of government intervention are more significant in the less developed economies does not mean that it has no impact on the more developed type. For instance, the total number of bank failures in consistently higher in the later than in the former, it is only that, as highlighted in the previous paragraph, less developed economies are built upon parameters that make their structure much more dependent on government's participation.

One traditional financial indicator that is often looked upon as a way of measuring the degree of exposure to shocks is the leverage ratio. The higher the leverage ratio, the higher is the fraction that debt represents of total assets and, therefore, the more trouble would have a bank to fulfill its promises as soon as it finds itself in a difficult financial situation. The next result states the impact of government intervention on the leverage ratio:

> *Government intervention leads to no significant changes in the average leverage ratio of a network.*

The simulations show that intervention does not have significant effects in the leverage ratio, regardless of the economy's type. The reason is that, in the framework proposed, government leads to an almost proportional increase in both assets and networth and, thus, the leverage ratio barely changes, even though assets and networth are.

This result, combined with the fact that intervention brings much more fragility, shows how misleading it can be if one regards only leverage as a measure of the healthiness of the financial system. In another words, it shows how important considerations about the network structure might be for financial fragility.

Another result from the simulations concerns the effects of different levels at which the government implements its intervention policy. Using the too-big-to-fail terminology to designate those instances where the support offered by the government for large projects is higher than the equivalent one for small investments, one has that:

> *A too-big-to-fail intervention policy does not have much impact on neither financial fragility nor total networth.*

Despite financial fragility being quite the same under different intervention levels, it is still persistently higher for less developed economies. The same does not hold for total networth, as the difference across economies gets flat as the fire-sale cost to liquidate large projects becomes less severe. That means that, unless the recovery rate of large projects is relatively low, if the aim of the government is to increase banks' total networth, then offering more subsidies to large projects is a waste of money and, in case one has at hand a less developed economy, it will come at the expense of more financial fragility.

One parameter that was varied across the simulations is the fire-sale cost of large projects relative to that of small ones. Regarding that, the results show that:

Government intervention has a more significant impact on financial fragility and banks' total networth when the fire-sale cost of large projects is relatively large.

This result is due to the fact that the fire-sale cost is one of the main drivers - together with government's policy - of the recovery rate of projects. When the fire-sale cost of large projects is relatively small, government intervention might not be enough to tip the balance in favor of larger investments. This in turn makes the number of links in government versus non-government networks to be similar, and thus no much impact of intervention one gets to notice on financial fragility and total networth, since these are mainly dependent on the number of links.

As already mentioned, one should keep in mind that it is not the case that intervention becomes irrelevant when the fire-sale cost of large projects is relatively small but, rather, that it becomes less significant. For instance, in the simulations using less developed economies one still get a level of financial fragility more than 40% higher with intervention than without.

Appendix

A.1 Proof of Proposition 1

The following lemma is used in the proof of the proposition:

Lemma 7. *For any $1 > \rho > \rho^* > 0$ and $1 > \gamma^* \geq \gamma \geq 0$, the following is satisfied:*

$$\frac{\rho - \rho^*}{\rho \rho^*} \geq \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]}. \tag{62}$$

Proof: Consider the function $h(\gamma, \gamma^*)$ defined by

$$h(\gamma, \gamma^*) := \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]}. \tag{63}$$

For $\gamma^* = \gamma$ it follows that

$$h(\gamma, \gamma) = \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma(1-\rho^*)]} := H(\gamma). \tag{64}$$

Taking the derivative of H yields

$$H'(\gamma) = -\frac{(\rho - \rho^*)\{[\rho^* + \gamma(1-\rho^*)] + (1-\gamma)(1-\rho^*)[\rho + \gamma(1-\rho)]\}}{\{[\rho + \gamma(1-\rho)][\rho^* + \gamma(1-\rho^*)]\}^2} < 0, \tag{65}$$

the inequality following from $\rho > \rho^*$. For $\gamma \geq 0$, it follows therefore that $H(0) \geq H(\gamma)$, i.e,

$$\frac{\rho - \rho^*}{\rho \rho^*} \geq \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma(1-\rho^*)]}. \tag{66}$$

For $\gamma^* \geq \gamma$ one has that

$$\frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma(1-\rho^*)]}{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]} \geq 1 \tag{67}$$

and

$$\frac{[\rho + \gamma(1-\rho)][\rho^* + \gamma(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]} \leq 1, \tag{68}$$

therefore

$$\frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma(1-\rho^*)]} \geq \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]}. \tag{69}$$

Combining (66) with (69) implies that

$$\frac{\rho - \rho^*}{\rho\rho^*} \geq \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]}, \tag{70}$$

which is the desired result. □

Now one proceeds proving Proposition 1: for an even number of banks, N, consider the original network formed without government intervention, $\gamma = \gamma^* = 0$, and take arbitrarily two of them, say banks i and j. Without loss of generality, assume that

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}. \tag{71}$$

One is interested in what would be the network if, instead, government intervention, $\gamma^* \geq \gamma > 0$, was in place. From Lemma 7 it follows that:

$$\frac{\rho - \rho^*}{\rho\rho^*} > \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]}. \tag{72}$$

If in the original network banks i and j do not have a link, (25) and (71) imply that

$$\frac{\rho - \rho^*}{\rho\rho^*} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}. \tag{73}$$

Following intervention, however, either

$$\frac{\rho - \rho^*}{\rho\rho^*} > \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i}, \tag{74}$$

which from (25) implies that banks still do not want to transact, or

$$\frac{\rho - \rho^*}{\rho\rho^*} > \frac{r_i^* - (r_i + r_j)}{(\omega_i - e_i)} \geq \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}, \tag{75}$$

which from (20) implies that bank i borrows from bank j and a link is established, or

$$\frac{\rho - \rho^*}{\rho\rho^*} > \frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{[\rho + \gamma(1-\rho)] - [\rho^* + \gamma^*(1-\rho^*)]}{[\rho + \gamma(1-\rho)][\rho^* + \gamma^*(1-\rho^*)]}, \tag{76}$$

which from (22) implies that both banks want to borrow and that a new link will be created, independently of the identities of the borrower and the lender. Therefore, if in the original network banks i and j do not share a link, with government intervention they will either continue not transacting or instead will create a link.

If in the original network banks i and j do have a loan agreement, then (25) and (71) imply that

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho\rho^*}. \tag{77}$$

Following intervention, either

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho\rho^*} > \frac{[\rho + \gamma(1 - \rho)] - [\rho^* + \gamma^*(1 - \rho^*)]}{[\rho + \gamma(1 - \rho)][\rho^* + \gamma^*(1 - \rho^*)]} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}, \tag{78}$$

which from (20) results in bank i borrowing from bank j and a link is kept in place, or

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho\rho^*} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{[\rho + \gamma(1 - \rho)] - [\rho^* + \gamma^*(1 - \rho^*)]}{[\rho + \gamma(1 - \rho)][\rho^* + \gamma^*(1 - \rho^*)]}, \tag{79}$$

which from (22) means that both banks want to borrow and, no matter what the identities of borrower and lender, they engage in a transaction and a link remains.

Therefore, if in the original network banks i and j do not transact, with intervention they might create a link and, if they do transact, they keep transacting and a link keep existing across the banks. By the arbitrariness of banks i and j the result follows. $\qquad\square$

A.2 Proof of Proposition 2

Consider the original network with an even number N of banks, being formed without government intervention. Take arbitrarily two banks, say i and j, and assume without loss of generality that

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j}. \tag{80}$$

For the sake of the argument, assume also that i borrows from j in the original network, with $\gamma^* = \gamma = 0$. That means, from (20) and (22), that either

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho\rho^*} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \tag{81}$$

holds or simultaneously that

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{\rho - \rho^*}{\rho\rho^*} \tag{82}$$

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and

$$[r_i^* - (1 - \omega_i) - e_i - r_j] - \left[r_j^* - (1 - \omega_j) - e_j - r_i\right] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^*}. \qquad (83)$$

Consider first (81). From Lemma 7, one knows that, with government, $\gamma^* \geq \gamma > 0$,

$$\frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]}. \qquad (84)$$

Therefore, upon intervention it follows that either

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \qquad (85)$$

or

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho \rho^*} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{[\rho + \gamma (1 - \rho)] - [\rho^* + \gamma^* (1 - \rho^*)]}{[\rho + \gamma (1 - \rho)] [\rho^* + \gamma^* (1 - \rho^*)]} \qquad (86)$$

holds. If (85) is the case, from (20) it follows that bank i continues borrowing from j. If (86), bank i will keep borrowing from bank j and not the converse as long as (23) is satisfied, i.e.,

$$[r_i^* - (1 - \omega_i) - e_i - r_j] - \left[r_j^* - (1 - \omega_j) - e_j - r_i\right] > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)}. \qquad (87)$$

However, one can show that, if

$$\left[(r_i^* - r_j) - \left(r_j^* - r_i\right)\right] \left\{ \frac{\rho^* + \gamma^* (1 - \rho^*)}{1 - [\rho^* + \gamma^* (1 - \rho^*)]} \right\} < (\omega_i - e_i) - (\omega_j - e_j) < \left(r_i^* - r_j^*\right) \left(\frac{\rho \rho^*}{\rho - \rho^*} \right) \qquad (88)$$

then (80) is satisfied whereas (87) is not, i.e., without intervention bank i borrows from bank j, and with government the opposite is true - bank i becomes the lender and bank j the borrower.

In the second case, i.e., if both (82) and (83) hold simultaneously, the fact that $\rho^* < \rho^* + \gamma^* (1 - \rho^*)$ implies that

$$\frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^*} > \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^* (1 - \rho^*)} \qquad (89)$$

and, therefore, (83) being satisfied automatically implies that (23) also is, and hence that with intervention bank i is kept as the borrower and bank j the lender. $\qquad \square$

A.3   Proof of Corollary 3

As in the proof of Lemma 7, from (69) one has that

$$\frac{[\rho + \gamma\,(1-\rho)] - [\rho^* + \gamma\,(1-\rho^*)]}{[\rho + \gamma\,(1-\rho)]\,[\rho^* + \gamma\,(1-\rho^*)]} > \frac{[\rho + \gamma\,(1-\rho)] - [\rho^* + \gamma^*\,(1-\rho^*)]}{[\rho + \gamma\,(1-\rho)]\,[\rho^* + \gamma^*\,(1-\rho^*)]}. \tag{90}$$

For $\gamma^* > \gamma > 0$, the right-hand side of the above inequality is the criteria for banks to choose between investing in large or small projects. Running through all possible scenarios after banks' pairwise meetings - expressions (20), (21), (22) and (25) - one immediately sees that the incentives for both banks to invest in a large project - and hence borrow money - are large with the TBTF policy as opposed to the case where $\gamma^* = \gamma > 0$. $\qquad \square$

A.4   Proof of Corollary 4

Denote by π^*_{ij} the profits of bank i with a large project when it borrows from bank j and by π_i with a small one, and analogously for bank j. By adding a superscript G one has the same variables but for the case with government intervention. From Proposition 1, the number of links across banks do not decrease when a network is formed with intervention, compared to the one obtained without the government. From Proposition 2, however, if (86) holds,

$$\frac{r_i^* - (r_i + r_j)}{\omega_i - e_i} \geq \frac{\rho - \rho^*}{\rho\rho^*} > \frac{r_j^* - (r_j + r_i)}{\omega_j - e_j} \geq \frac{[\rho + \gamma\,(1-\rho)] - [\rho^* + \gamma^*\,(1-\rho^*)]}{[\rho + \gamma\,(1-\rho)]\,[\rho^* + \gamma^*\,(1-\rho^*)]}, \tag{91}$$

i.e., *without* the government bank i borrows from bank j and the last is actually better-off either lending or investing in a small project - $\pi^*_{ij} > \pi_i$ and $\pi_j > \pi^*_{ji}$ - but *with* intervention both banks prefer an investment in a large project - $\pi^*_{ij}{}^G > \pi_i^G$ and $\pi^*_{ji}{}^G > \pi_j^G$ - it might well be that $\pi^*_{ji}{}^G > \pi^*_{ij}{}^G$,

$$\left[r_i^* - (1 - \omega_i) - e_i - r_j\right] - \left[r_j^* - (1 - \omega_j) - e_j - r_i\right] < \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^* + \gamma^*\,(1 - \rho^*)}, \tag{92}$$

which from (24) implies that, with government, the role of banks is switched - bank i becomes the lender and bank j the borrower. Given that $\rho^* + \gamma^*\,(1 - \rho^*) > \rho^*$, (92) implies that

$$\left[r_i^* - (1 - \omega_i) - e_i - r_j\right] - \left[r_j^* - (1 - \omega_j) - e_j - r_i\right] < \frac{(\omega_i - e_i) - (\omega_j - e_j)}{\rho^*}, \tag{93}$$

which in turn means that, without the government, the profit of bank j with a large project was also bigger than that realized by bank i, or that $\pi_{ji}^* > \pi_{ij}^*$. Therefore, *without* government one has that

$$\pi_j > \pi_{ji}^* > \pi_{ij}^* > \pi_i, \tag{94}$$

whereas *with* intervention

$$\pi_{ji}^{*\,G} > \max\left\{\pi_j^G, \pi_{ij}^{*\,G}\right\} \quad \text{and} \quad \pi_{ij}^{*\,G} > \pi_i^G. \tag{95}$$

Therefore, in the particular cases of banks i and j, networth increases with government intervention only if

$$\pi_{ji}^{*\,G} + \pi_i^G > \pi_{ij}^* + \pi_j. \tag{96}$$

From $\pi_{ji}^{*\,G} > \pi_{ij}^{*\,G}$ and $\pi_{ij}^{*\,G} > \pi_{ij}^*$ one has that $\pi_{ji}^{*\,G} > \pi_{ij}^*$. If $\underline{\pi}^G > \overline{\pi}$, then $\pi_i^G > \underline{\pi}^G > \overline{\pi} > \pi_j$ leads to $\pi_i^G > \pi_j$ and, therefore, (96) is satisfied, i.e., intervention leads to a networth improvement - but not necessarily otherwise. From the arbitrariness of banks i and j, thus, the result follows. $\qquad \square$

A.5 Proof of Lemma 5

The matrix B is given by:

$$B := \left(I - \widehat{X}\right) = \begin{bmatrix} 1 & -\dfrac{r_1\left(1-\alpha_r^1\right)\chi_{12}}{\sum_{k\in N}\chi_{k2}r_k} & \cdots & -\dfrac{r_1\left(1-\alpha_r^1\right)\chi_{1N}}{\sum_{k\in N}\chi_{kN}r_k} \\ -\dfrac{r_2\left(1-\alpha_r^2\right)\chi_{21}}{\sum_{k\in N}\chi_{k1}r_k} & 1 & \cdots & -\dfrac{r_2\left(1-\alpha_r^2\right)\chi_{2N}}{\sum_{k\in N}\chi_{kN}r_k} \\ \vdots & \vdots & \ddots & \vdots \\ -\dfrac{r_N\left(1-\alpha_r^N\right)\chi_{N1}}{\sum_{k\in N}\chi_{k1}r_k} & -\dfrac{r_N\left(1-\alpha_r^N\right)\chi_{N2}}{\sum_{k\in N}\chi_{k2}r_k} & \cdots & 1 \end{bmatrix}.$$

One needs to show that there are positive numbers $d_1, d_2 \ldots, d_n$ such that $d_j\,|b_{jj}| > \sum_{i\neq j} d_i\,|b_{ij}|$, for $j = 1, \ldots, n$, i.e.:

$$d_1 > \sum_{i \neq 1} d_i |b_{i1}| = \sum_{i \neq 1} d_i \frac{r_i \left(1 - \alpha_r^i\right) \chi_{i1}}{\sum_{k \in N} \chi_{k1} r_k},$$

$$d_2 > \sum_{i \neq 2} d_i |b_{i2}| = \sum_{i \neq 2} d_i \frac{r_i \left(1 - \alpha_r^i\right) \chi_{i2}}{\sum_{k \in N} \chi_{k2} r_k},$$

$$\vdots$$

$$d_N > \sum_{i \neq N} d_i |b_{iN}| = \sum_{i \neq N} d_i \frac{r_i \left(1 - \alpha_r^i\right) \chi_{iN}}{\sum_{k \in N} \chi_{kN} r_k}.$$

Suppose that $d_1 = d_2 = \ldots = d_N = d$. The above then becomes:

$$1 > \sum_{i \neq 1} \left(1 - \alpha_r^i\right) \frac{r_i \chi_{i1}}{\sum_{k \in N} \chi_{k1} r_k},$$

$$1 > \sum_{i \neq 2} \left(1 - \alpha_r^i\right) \frac{r_i \chi_{i2}}{\sum_{k \in N} \chi_{k2} r_k},$$

$$\vdots$$

$$1 > \sum_{i \neq N} \left(1 - \alpha_r^i\right) \frac{r_i \chi_{iN}}{\sum_{k \in N} \chi_{kN} r_k}.$$

For any i, one knows that $0 < \alpha_r^i < 1$ or, equivalently, $0 < \left(1 - \alpha_r^i\right) < 1$, which implies that:

$$1 = \frac{\sum_{i \neq 1} r_i \chi_{i1}}{\sum_{k \in N} \chi_{k1} r_k} > \sum_{i \neq 1} \left(1 - \alpha_r^i\right) \frac{r_i \chi_{i1}}{\sum_{k \in N} \chi_{k1} r_k},$$

$$1 = \frac{\sum_{i \neq 2} r_i \chi_{i2}}{\sum_{k \in N} \chi_{k2} r_k} > \sum_{i \neq 2} \left(1 - \alpha_r^i\right) \frac{r_i \chi_{i2}}{\sum_{k \in N} \chi_{k2} r_k},$$

$$\vdots$$

$$1 = \frac{\sum_{i \neq N} r_i \chi_{iN}}{\sum_{k \in N} \chi_{kN} r_k} > \sum_{i \neq N} \left(1 - \alpha_r^i\right) \frac{r_i \chi_{iN}}{\sum_{k \in N} \chi_{kN} r_k},$$

as one wanted to show. $\qquad\qquad\qquad\square$

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