

ECONOMIC THEORY, APPLICATIONS AND ISSUES

Working Paper No. 29

**Linear Break-Even Analysis:
When is it Applicable to a Business?**

by

Clem Tisdell

April 2004



ISSN 1444-8890
WORKING PAPER ON

ECONOMIC THEORY, APPLICATIONS AND ISSUES

Working Paper No. 29

**Linear Break-Even Analysis:
When is it Applicable to a Business?***

by

Clem Tisdell†

April 2004

© All rights reserved

* First draft of paper for possible inclusion in *Contemporary Business Issues in the Indian Economy* edited by Professor Rashmi Agrawal in honour of Professor Lallan Prasad.

† School of Economics, The University of Queensland, Brisbane 4072 Australia
Email: c.tisdell@economic.uq.edu.au

WORKING PAPERS IN THE SERIES, *Economic Theory, Applications and Issues*, are published by the School of Economics, University of Queensland, 4072, Australia.

For more information write to Professor Clem Tisdell, School of Economics, University of Queensland, Brisbane 4072, Australia or email c.tisdell@economics.uq.edu.au

LINEAR BREAK-EVEN ANALYSIS: WHEN IS IT APPLICABLE TO A BUSINESS?

Abstract

Reviews linear break-even analysis as typically outlined in textbooks on managerial economics. It is claimed that a major shortcoming of these expositions is their failure to demonstrate in what circumstances linear break-even analysis is relevant for a business and in what circumstances it is inapplicable. This article helps rectify this situation. It points out that such analysis can not be applied to perfectly competitive firms. However, in special circumstances, it might apply to a purely competitive firm. It is highly relevant for businesses operating in oligopolistic conditions where the kinked demand curve applies. Furthermore, it is applicable if imperfectly competitive firms follow fixed price rules. On the other hand, if imperfectly competitive firms, such as monopolists, adopt flexible pricing (for example, prices to clear whatever quantity of product they have supplied to the market) the linearity assumption involved in this type of break-even analysis will, as a rule, be violated. This is so because the firm's total revenue will be a non-linear function of the quantity of the product supplied by the firm to the market. Nevertheless, because fixed price behaviour by businesses may be common, as well as constant average variable costs of production over a considerable range of output, linear break-even analyses has a considerable range of application to business.

LINEAR BREAK-EVEN ANALYSIS: WHEN IS IT APPLICABLE TO A BUSINESS?

1. Introduction

Most textbooks on managerial economics give some coverage of the factors that influence the ability of a business to break-even in its economic operations. Usually, however, this is explained using linear total revenue and linear total cost relationships for the business (see, for example, Salvatore, 2004, p.298; McGuigan et al., 2002, pp.379-383; Hershey, 2003, pp.308-310; Keat and Young, 2003; Appendix 9B). Yet, there is as a rule no discussion of the market or other conditions under which such analysis is applicable to a firm.

This is a serious omission because the relevant textbooks do not provide students with any guidance about the circumstances in which they can apply the analysis and they do not point out situations where application of the analysis is definitely inappropriate. It is the purpose of this short paper to rectify this deficiency. As will be seen, delineating the applicability of linear break-even analysis is not as straightforward as might be thought of first sight.

This paper proceeds by first briefly outlining linear break-even analysis as typically outlined in textbooks and illustrating the per-unit analysis that corresponds to this analysis based on the business' total revenue and total costs. This is followed by identification of business conditions in which linear break-even analysis can be applied and those in which it is inapplicable concluding observations follows.

2. Traditional Linear Break-Even Analysis

Traditional linear break-even analysis is based on the use of linear total revenue and total cost relationships. Such a situation is illustrated in Figure 1. There the total revenue of a business is shown by line OAC and its total cost is illustrated by the line DAF. The distance OD represents the firms fixed costs of production and the difference between this and line DAF represents the firm's total variable costs of production. The break-even point of the business corresponds to point A. It implies that if the firm produces and sells x_B of its product, it just breaks even in the circumstances illustrated. If it sells and produces more, it makes a profit, if less a loss.

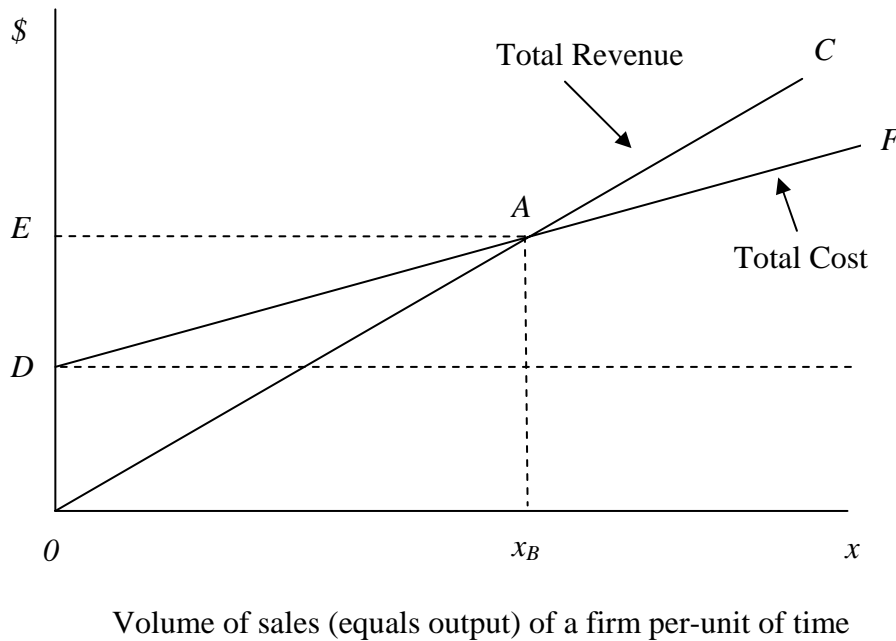


Figure 1: Diagram typically used to illustrate break-even analysis in textbooks on managerial economics

This can also be depicted algebraically. For example, where TR represents the total revenue of the firm, P the price per unit of its product and x indicates its volume of sales (equals volume of output)

$$TR = P x \tag{1}$$

Where a and b are coefficients, the business' total cost (TC) takes the form:

$$TC = a + b x \tag{2}$$

In this relationship a indicates the firm's total fixed cost of production and $b x$ specifies its total variable cost.

The business breaks even when $TR = TC$ that is when:

$$P x = a + b x \tag{3}$$

Or re-arranging, when:

$$x = \frac{a+b}{P} \quad (4)$$

Expression (4) indicates for example that other things equal, the business' break-even volume of sales will be greater the larger is its total fixed cost (a), the higher is its average variable cost (b), and the lower is the price of its product (P).

The relationship between the per-unit cost curves and revenue curves of a business and total break-even analysis is not usually specified in mainstream textbooks in managerial economics. However, it is useful to do this in order to determine the scope for applying the analysis.

The above total relationships imply that the firm's average revenue and marginal revenue can be represented by a horizontal line corresponding to the price of its product, P . Similarly, the firm's average variable cost can be shown by a horizontal line corresponding to the slope of its total cost curve, b . On the other hand, the firm's average fixed cost of production declines with its volume of sales and is equal to a/x .

This is illustrated in Figure 2. There the line designated $AR=P=MR$ represents the firm's average revenue and marginal revenue as a function of its volume of its sales. The line identifies by $AVC=MC$ represents its average variable costs (equals marginal cost of production in this case). The curve BCD specifies the average total cost of the business. The difference between this curve and the AVC line is equivalent to the average fixed cost of the firm in relation to its volume of output, and difference should correspond to a rectangular hyperbola.

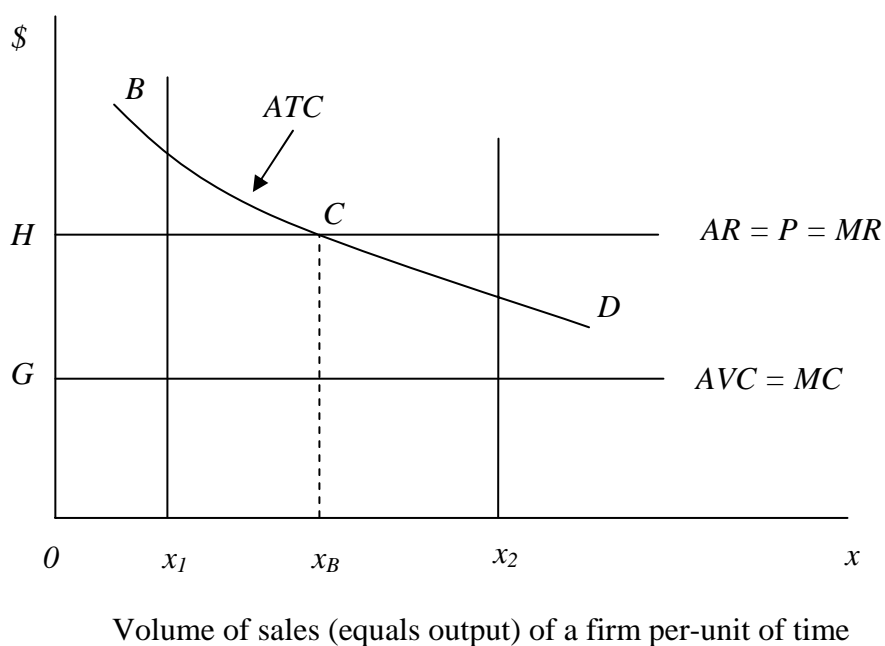


Figure 2: Break-even analysis using per-unit cost curves and assuming linear total revenue and total cost relationships. It is incompatible with the pure theory of perfect competition

In the case illustrated in Figure 2, the business just breaks even if it sells and produces x_B of its product. If it should be able to sell a greater quantity, it will make a profit. If it can only sell less than x_B , it makes a loss. In this case (as is always so in the linear break-even analysis, unless there is no volume of sales and production for which break-even is possible) just a single break-even point exists.

However, the business conditions for which this is the relevant model are not adequately outlined in present textbooks. Can the model be applied to firms operating under conditions of perfect competition? Is it applicable to businesses operating in oligopolistic or monopoly-like markets? Let us consider this aspect. In doing so, analysis using per-unit revenue and cost curves of firms will be found to be very useful.

3. Market Types, Market Behaviour and the Applicability of Linear Break-Even Analysis

The break-even model outlined above assumes that the firm has no control over its volume of sales, or that they are *exogenously* determined in the period for which the analysis is applied. In the latter situation, it can be viewed as a managerial planning device. But how applicable

is this linear technique for different types of market situations and different types of marketing behaviour of business managers?

First, we notice that the model is incompatible with the pure theoretical model of a perfectly competitive firm. This is because this theory assumes that the firm can sell an unlimited quantity of its product at the prevailing market price of it. The market does not constrain the volume of sales of the business. The case illustrated in Figure 2 would result in a profit-maximising perfectly competitive firm expanding its production without limit. It is eventually rising marginal cost of production that in all cases limits the size of a perfectly competitive firm, but this model assumes constant marginal costs.

However, there could be cases that are purely (not perfectly) comparative in nature where the firm is a price-taker and cannot sell an unlimited quantity of its product. The quantity of its product that a purely competitive business can sell may sometimes be largely exogenously determined in the short run. For example, the firm may not know whether it will be able to sell x_1 or x_2 of its product in the short-run case illustrated in Figure 2. In the former instance, it makes a loss and in the latter one, a profit.

Market competition that fits the linear break-even model well is that for the kinked demand curve model of oligopoly (Sweezy, 1939; Hall and Hitch, 1939). In oligopolistic industries, the average variable costs of production by a business are often constant or constant over a considerable range of production. Furthermore, an oligopolist is unlikely in a kinked demand curve situation to find it profitable to charge other than the customary or conventional price of the product in the industry. This **in effect** makes the oligopolist a price-taker.

In Figure 3, for example, the oligopolist may believe that the demand curve for his/her product will either be EFG or E'F'G' where OP_C represents the customary market price of this product. In the circumstances, it may not pay the oligopolist to deviate from that price. Consequently, the horizontal straight line P_CFFH becomes the oligopolist's **effective** average revenue or marginal revenue curve. This oligopolist's average variable cost curve may be a horizontal straight line as identified by AVC in Figure 3. Hence, this case fits well the type of linear break-even analysis depicted by Salvatore (2004, p.298) and in similar texts.

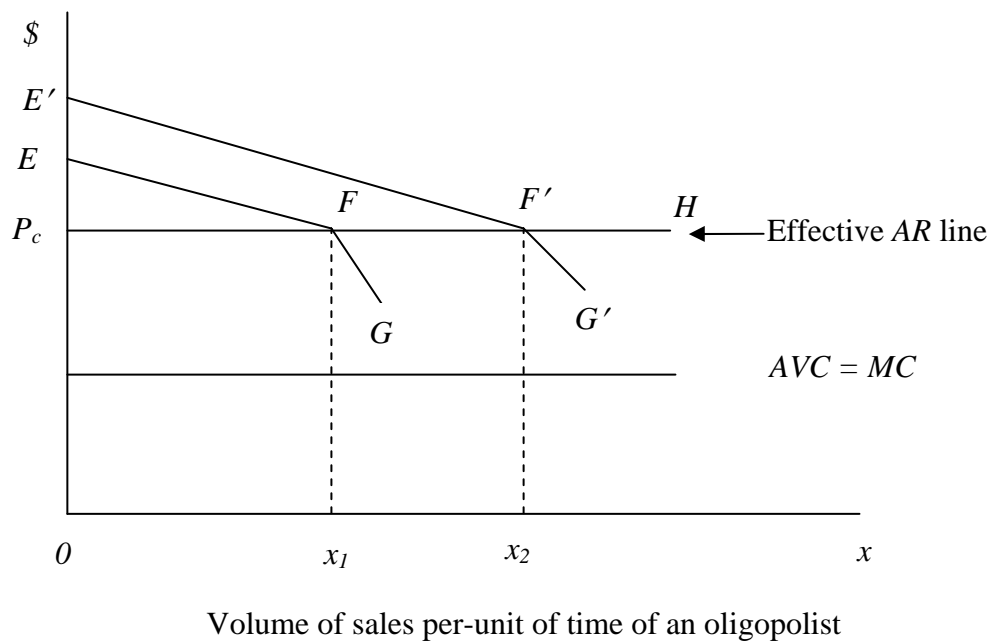


Figure 3: In an oligopolistic situation in which the kinked demand curve applies the firm's effective the average revenue curve may be a horizontal straight line and its total revenue relationships is linear

The type of situation in illustrated in Figure 4 accords more closely (than in the above in Figure 1) with the situation to be expected under conditions of pure or perfect competition. In Figure 4, the firm's total revenue curve is linear represented by line OE. Its total cost curve is non-linear and is represented by the curve ACDG. This total cost curve results in a U-shaped average cost curve.

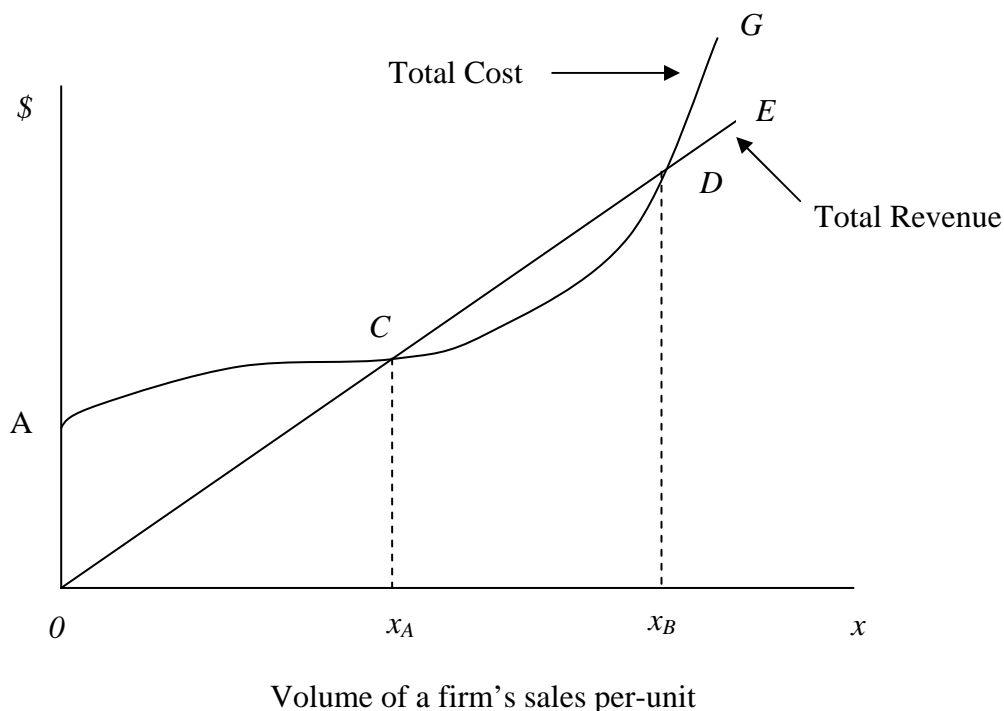


Figure 4: A case in which the total cost of production by a firm is non-linear. There are two break-even points. This case is compatible with perfect competition

In the case illustrated in Figure 4, the business has a lower break-even point, x_A and an upper one, x_B . It will make a profit if it sells and produces a quantity of its product between x_A and x_B . On the other hand, the business makes a loss if it sells less than x_A or more than x_B . Two break-even points are evident.

Nevertheless, even in this case, there might be only one point at which the business breaks even. This would happen if the firm's total revenue line should be just tangential to its total cost curve at a single point. This point can be found by rotating the line OE clockwise on the fixed point O until it just touches curve $ACDG$ at a single point.

A third possibility in this case is that there may be no point at which a firm can break even. The line OE could for example, be so far to the right of its total cost curve $ACDG$ in Figure 4 that it neither touches nor intersects $ACDG$.

Let us consider a different situation. It is possible for both the firm's total revenue curve and its total cost curve to be non-linear. The business' total revenue relationship will be non-linear, if it has a monopoly or has monopolistic market power. Such a case is illustrated in Figure 5. The firm's total revenue curve is shown by $OCDEF$. If the firm's total cost curve

is the one indicated by TC_1 , it has a lower break-even point x_A and an upper break-even point x_B . If on the other hand, the firm's total cost curve is as indicated by TC_2 , the firm has a single break-even point corresponding to point D_1 , namely x'_B . This would correspond to the typical long-run equilibrium position of a business operating in a large group monopolistically competitive market.

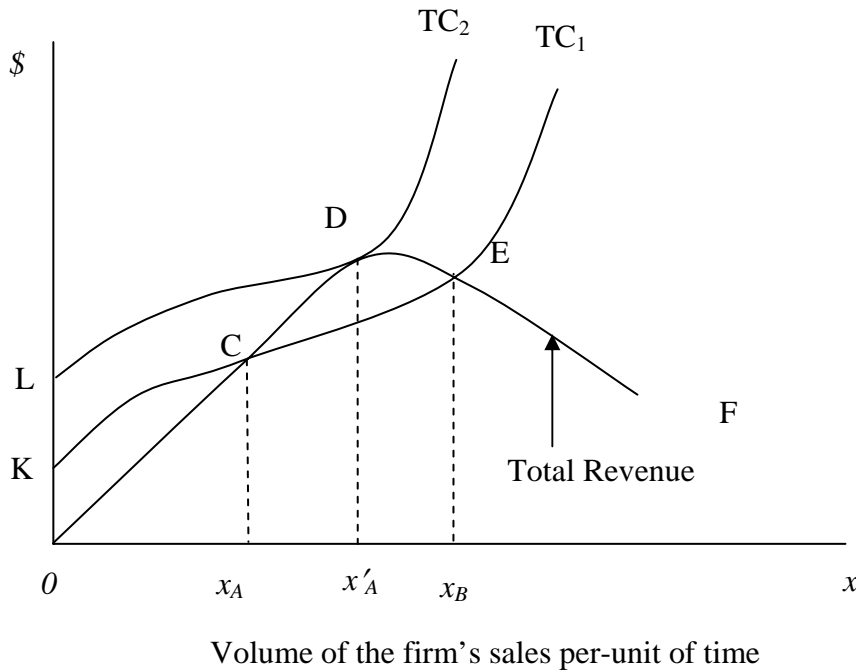


Figure 5: Illustration of break-even analysis for a case in which the firm's total revenue is non-linear. Such cases arise under conditions of monopoly as well as under monopolistic competition

Similar break-even relationships can occur if the firm's total cost curve is linear and its total revenue curve is non-linear.

Finally, we can now consider another set of conditions under which linear break-even analysis can be applied. It is relevant if the business follows a fixed price policy. The price of its product could for example be determined on a cost plus mark-up basis. The firm's price, at least in the short-run, could remain fixed even though it has some monopoly power. In its pricing policy, the firm may be following cost-plus pricing because it lacks knowledge of its demand curve or of the demand curve that will prevail for its product in the future (see Baumol and Quandt, 1964). In imperfectly competitive markets, the firm's average variable cost (average direct cost) production appears to be constant for a considerable variation in its

volume of output (see Hall and Hitch, 1939) and this results in a linear total cost curve for a wide range of levels of production. The fixed price behaviour of a firm effectively generates a linear total revenue relationship as a function of its volume of sales. Therefore, the linear break-even model can be applied.

This case can be illustrated in Figure 6. There the horizontal line identified by $AVC=MC$ represents the average variable costs of production of a firm with some market power. However, the firm is assumed to charge a fixed price of OC for its product, possibly arrived by adding a mark-up of HC/OH to its per-unit direct costs of production. The demand curve for the firm's product may vary and/or be poorly known. It may, for instance, be as identified by d_2d_2 or by d_1d_1 . If it is d_2d_2 , the firm will be able to sell x_2 of its product. If it is d_1d_1 , it will sell x_1 of its product. Similarly, the firm's volume of sales can be found for other possible demand curves. Hence, the **effective** average revenue curve for this business is as indicated by the line CG .

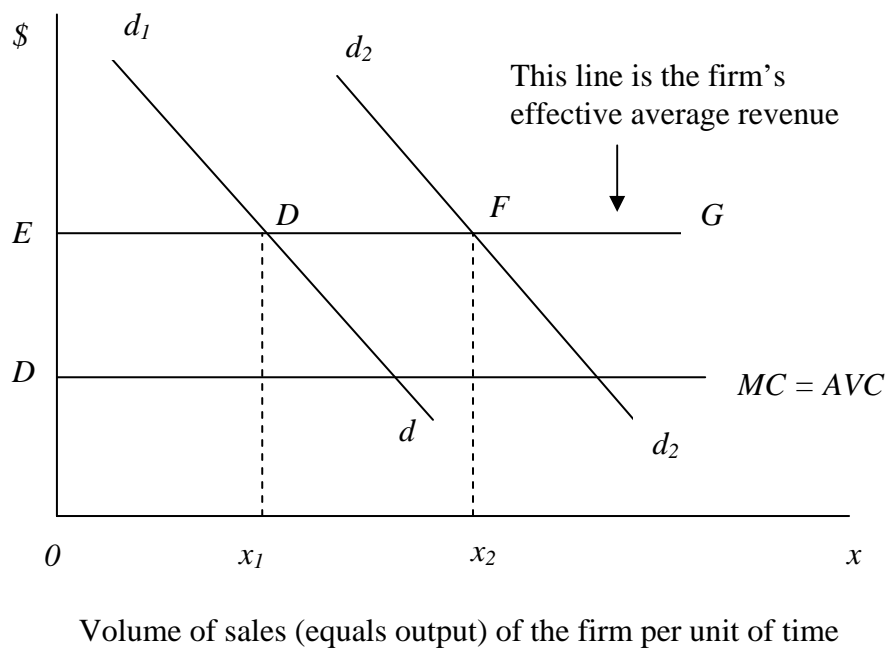


Figure 6: If a business adopts a fixed price policy this **effectively** makes its total revenue curve linear. If its average variable cost of production is constant, linear break-even analysis is applicable to its situation

In the above circumstances, linear break-even analysis can be applied. To find the business break-even point in Figure 6, average fixed costs of production is merely added to average variable costs in Figure 6 to provide the firm's average total cost curve. The point at which this average total cost curve crosses line *CG* yields the business' break-even point. Note that the linear analysis applies basically as a result of the behavioural assumption that the firm charges a fixed price.

4. Concluding Observations

Existing textbooks provide little explanation of conditions under which a business would find linear break-even analysis to be relevant for managerial decision-making. Such analysis cannot be applicable to a perfectly competitive firm but may have limited application to a purely competitive firm. Under conditions of imperfect competition, linear break-even analysis may be quite relevant in the kinked demand oligopolistic case and in circumstances where the business engages in fixed pricing, possibly for behavioural reasons. While linear break-even analysis can typify a considerable number of business cases, this paper also identifies cases where its linearity assumptions are inappropriate. Current textbooks do not adequately discuss the relevance of the linear approach to break-even analysis. This paper should be helpful, therefore, in addressing this shortcoming.

References

- Baumol, W. and Quandt, R. (1964) "Rules of Thumb and Optimally Imperfect Decisions", *American Economic Review*, Vol.54, pp.23-46.
- Hall, R. and Hitch, C. (1939) "Price Theory and Business Behaviour", *Oxford Economic Papers*, pp.12-45.
- Hirschey, M. (2002) *Managerial Economics*, 10th edn, South-Western, Mason, Ohio.
- Keat, P. G. and Young, P. K. Y. (2003) *Managerial Economics: Economic Tools for Today's Decision-Makers*, Prentice Hall, Upper Saddle River, New Jersey.
- McGuigan, J. R., Moyer, R. C. and Harris, F. (2002) *Managerial Economics: Applications, Strategy and Tactics*, 9th edn, South-Western, Mason, Ohio.
- Salvatore, D. (2004) *Managerial Economics in a Global Economy*, 5th edn, South-Western Mason, Ohio.
- Sweezy, P. (1939) "Demand under Conditions of Oligopoly", *Journal of Political Economy*, Vol. 47, pp.404-409.

ISSN 1444-8890
PREVIOUS WORKING PAPERS IN THE SERIES
ECONOMIC THEORY, APPLICATIONS AND ISSUES

1. Externalities, Thresholds and the Marketing of New Aquacultural Products: Theory and Examples by Clem Tisdell, January 2001.
2. Concepts of Competition in Theory and Practice by Serge Svizzero and Clem Tisdell, February 2001.
3. Diversity, Globalisation and Market Stability by Laurence Laselle, Serge Svizzero and Clem Tisdell, February 2001.
4. Globalisation, the Environment and Sustainability: EKC, Neo-Malthusian Concerns and the WTO by Clem Tisdell, March 2001.
5. Globalization, Social Welfare, Labor Markets and Fiscal Competition by Clem Tisdell and Serge Svizzero, May 2001.
6. Competition and Evolution in Economics and Ecology Compared by Clem Tisdell, May 2001.
7. The Political Economy of Globalisation: Processes involving the Role of Markets, Institutions and Governance by Clem Tisdell, May 2001.
8. Niches and Economic Competition: Implications for Economic Efficiency, Growth and Diversity by Clem Tisdell and Irmi Seidl, August 2001.
9. Socioeconomic Determinants of the Intra-Family Status of Wives in Rural India: An Extension of Earlier Analysis by Clem Tisdell, Kartik Roy and Gopal Regmi, August 2001.
10. Reconciling Globalisation and Technological Change: Growing Income Inequalities and Remedial Policies by Serge Svizzero and Clem Tisdell, October 2001.
11. Sustainability: Can it be Achieved? Is Economics the Bottom Line? by Clem Tisdell, October 2001.
12. Tourism as a Contributor to the Economic Diversification and Development of Small States: Its Strengths, Weaknesses and Potential for Brunei by Clem Tisdell, March 2002.
13. Unequal Gains of Nations from Globalisation by Clem Tisdell, Serge Svizzero and Laurence Laselle, May 2002.
14. The WTO and Labour Standards: Globalisation with Reference to India by Clem Tisdell, May 2002.
15. OLS and Tobit Analysis: When is Substitution Defensible Operationally? by Clevo Wilson and Clem Tisdell, May 2002.
16. Market-Oriented Reforms in Bangladesh and their Impact on Poverty by Clem Tisdell and Mohammad Alauddin, May 2002.
17. Economics and Tourism Development: Structural Features of Tourism and Economic Influences on its Vulnerability by Clem Tisdell, June 2002.
18. A Western Perspective of Kautilya's Arthashastra: Does it Provide a Basis for Economic Science? by Clem Tisdell, January 2003.
19. The Efficient Public Provision of Commodities: Transaction Cost, Bounded Rationality and Other Considerations.
20. Globalization, Social Welfare, and Labor Market Inequalities by Clem Tisdell and Serge Svizzero, June 2003.
21. A Western Perspective on Kautilya's 'Arthashastra' Does it Provide a Basis for Economic Science?, by Clem Tisdell, June 2003.
22. Economic Competition and Evolution: Are There Lessons from Ecology? by Clem Tisdell, June 2003.
23. Outbound Business Travel Depends on Business Returns: Australian Evidence by Darrian Collins and Clem Tisdell, August 2003.
24. China's Reformed Science and Technology System: An Overview and Assessment by Zhicun Gao and Clem Tisdell, August 2003.
25. Efficient Public Provision of Commodities: Transaction Costs, Bounded Rationality and Other Considerations by Clem Tisdell, August 2003.

26. Television Production: Its Changing Global Location, the Product Cycle and China by Zhicun Gao and Clem Tisdell, January 2004.
27. Transaction Costs and Bounded Rationality – Implications for Public Administration and Economic Policy by Clem Tisdell, January 2004.
28. Economics of Business Learning: The Need for Broader Perspectives in Managerial Economics by Clem Tisdell, April 2004.