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Technological and financial approaches to risk management in agriculture: an integrated approach*

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In the present paper, risk-management problems where farmers manage risk both through production decisions and through the use of market-based and informal risk-management mechanisms are considered. It is shown that many of these problems share a common structure, and that a unified and informative treatment of a broad spectrum of risk-management tools is possible within a cost-minimisation framework, under minimal conditions on their objective functions. Fundamental results are derived that apply regardless of the producer's preference towards risks, using only the no-arbitrage condition that agricultural producers never forego any opportunity to lower costs without lowering returns.

1. Introduction

One of the most heavily researched topics in agricultural economics is the behaviour of producers facing a stochastic technology and/or stochastic prices. Agricultural producers employ a variety of market-based and informal mechanisms to deal with price and production risk. A significant amount of literature has been published regarding many of these risk-management mechanisms. Examples include on-farm risk-management techniques such as crop diversification, the purchase or sale of financial instruments such as futures contracts, as well as exploitation of government policies such as price stabilisation and underwriting schemes, drought relief policies, and farm management deposits.

Three features of the published literature are particularly salient. First, distinct and largely separate literatures exist around each risk-management

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tool.[1] Thus, there is a large and growing literature on crop insurance and production decisions, and an even larger, but largely separate literature on incorporating futures and forward contracts into production decisions. Similarly, the government price stabilisation literature is largely distinct from the literature on the allocation of on-farm and off-farm labour. Even where government policies and market instruments might be seen as direct substitutes, as in the case of buffer-stock stabilisation and private storage arrangements, joint attention has been sporadic at best. As a result, it is often not clear whether the conclusions obtained in the literature reflect the general logic of optimisation, particularly cost minimisation, or depend on specific properties of individual risk-management tools.

Second, analysis in each of these areas usually employs restrictive assumptions on producer preferences and attitudes towards risk. It is normally (arguably, almost exclusively) assumed that preferences are characterised by either the mean-variance model or the expected-utility model. Particularly heavy reliance is placed on Sandmo's (1971) model of an individual maximising the expected utility of net returns and on the separable-effort expected utility specification popularised by Newbery and Stiglitz (1981). The reason behind the choice of models is not empirical. A good deal of evidence suggests both expected-utility and mean-variance models are unrealistic. Rather, the reason for employing restrictive assumptions is the difficulty of obtaining general results about the allocation of wealth between alternative investments without specific assumptions about the producer's risk preferences. When these restrictive assumptions are not enough, even more restrictive, and less empirically viable, assumptions are usually thought necessary to permit analysis.

Third, the treatment of multiple sources of risk has been limited. For example, many studies of production under uncertainty analyse producers using a single nonstochastic input to produce either a nonstochastic output in the presence of a stochastic output price or a single stochastic output in the presence of a nonstochastic output price. Such assumptions are convenient because they transform the producer decisionmaking problem into a slight generalisation of the simple portfolio-choice problem (Gollier 2001).

Even these very restrictive models sometimes seem too complex to be tractable. Thus, it is common practice to analyse production decisions separately and distinctly from decisions made on the risk-management tools. For example, in the published literature on area-yield insurance, it is standard

[1] Adequate or even representative citation for each of these areas would be extremely unwieldy and, therefore, we have elected to provide no detailed citations while acknowledging our indebtedness to the many authors who have contributed to these publications. Some specific contributions are noted in the examples in following text.

to model producer behaviour as solely consisting of the choice of an optimal level of protection for a risky asset (the producer's yield), over which the producer has no control. Few studies exist that simultaneously study the producer's self insurance decisions in planting and caring for his crop and his choice of enrollment in an area-yield insurance program. Similarly, there is a large amount of literature treating the hedging behaviour of producers in futures and forward markets in isolation from their stochastic production decisions. Again, structuring assumptions in this fashion allows these problems to be reduced to portfolio allocation problems familiar from finance theory.

In the present paper, we address each of these three salient characteristics positively. First, we attempt to show that many of these problems are formally equivalent. And, even if they are not formally equivalent, they share a common structure that can be exploited in analysing them. Therefore, important lessons learned in one area can be usefully applied in other areas. We do this by way of the introduction of a canonical model of producers facing both price and production uncertainty for a multiple input technology. We briefly survey a series of seemingly disparate examples, and show that all can be considered as special cases of a more generic model.

Second, rather than focussing on what different assumptions about risk preferences allow one to say, we take a more conservative approach and ask what can be said regardless of the producer's attitudes towards risk. It turns out that, once the model is properly framed, quite a lot can be said. We demonstrate this by studying producer behaviour under minimal assumptions on preferences. In particular, we only assume that producers prefer more income to less and that producers seek to minimise effort and other costs. The principle of arbitrage, when combined with these assumptions, offers insight even in the absence of any specific assumptions on the producer's attitudes towards risk. For our analysis there is no need to make any assumption about separability of producer preferences across states of Nature, as in expected utility analysis, or the presence or absence of any arbitrary degree of risk aversion.

In what follows, we first introduce a sketch of our canonical model. Then, we illustrate its broad applicability by showing how it applies to several apparently distinct examples. We then move on to a formal development and analysis of the resulting model.

2. A canonical model

We adopt a state-space approach to modelling uncertainty. Uncertainty is represented by a set

$$\Omega = \{1, 2, \dots, S\}, \tag{1}$$

where each element of Ω is referred to as a state of Nature, or more simply as just a state. Uncertainty is resolved by a neutral player, Nature, making a choice from Ω. Once that choice is made, all stochastic elements relevant to the individual decisionmaker we study are resolved.

We consider producers who in the sth state have income equalling

$$y_s = p_s z_s + ha_s, \tag{2}$$

where p_s is the stochastic price of the agricultural product, z_s is the stochastic production of the agricultural product, a_s is the payout in state s associated with the risk-management tool and h is the holding of the risk-management tool. To achieve this stochastic income producers must commit inputs measured by the vector $\mathbf{x}$ and purchase h units of the risk-management tool at a unit cost of v. The prices of the inputs in $\boldsymbol{x}$ are given by the vector $\boldsymbol{w}$. We restrict attention to single-output models.

There are two time-periods. At period 0, the producer faces a stochastic technology, represented for example by a stochastic production function and stochastic output prices. In addition, he can make purchases of a risk-management device, for example, a futures contract. He must, however, make his production choices and his risk-management choice before Nature makes a choice from Ω. Thus, his choices yield stochastic returns given by the vector $\boldsymbol{y} \in \Re^S$. In making these choices, he incurs costs of $\boldsymbol{wx} + vh$ in period 0. Once Nature makes its choice, uncertainty is resolved. Hence, in period 1, the producer receives the *ex post* amount y_s where the subscript s corresponds to Nature's choice from Ω.

Several comments are relevant at this point. First, as noted earlier, we only consider a single stochastic output. Our results generalise directly, however, to the case where agricultural production is multiple output in nature simply by replacing $p_s z_s$ with stochastic revenue from all sources of agricultural production. Second, here we typically restrict attention to a single risk-management tool, when in fact there is a broad array of risk-management tools available to agricultural producers. This is done solely to ensure that we have a single canonical model that can be reinterpreted in a number of different contexts, which the published literature has typically treated as distinct, by a simple renaming of variables. The more general case, where there exist multiple risk-management tools, has been treated extensively in Chambers and Quiggin (2002a) in the context of financial markets. Third, consistent with the tradition in the theoretical literature, the risk-management tools are modelled in a stylised fashion. Finally, in the above we have only talked about the case where the risk-management tool is linearly priced in a competitive market. In our discussion of on-farm and off-farm labour, we modify this assumption to allow non-linear pricing of the risk-management tool.

2.1 Example 1: forward and futures markets

Perhaps the most familiar risk-management tools are futures or forward markets for the commodity in question.[2] By purchasing a futures contract at a price of v in period 0, the producer obligates herself to deliver one unit of the commodity at the end of period 1 at the futures or forward price, denoted here by q. In practice, contracts are normally settled in cash rather than through physical delivery, and some contracts require settlement in this form. Thus, the farmer's state-contingent return from each unit of holding the forward contract is

$$a_s = q - p_s, \tag{3}$$

and the farmer's state-contingent income is given by

$$y_s = p_s z_s + h(q - p_s), \tag{4}$$

where h now denotes contracts purchased. Cost is measured by $\boldsymbol{wx} + vh$.

2.2 Example 2: yield-based futures contracts

Yield-based futures contracts operate similarly to price-based futures contracts except that by purchasing the contract the farmer makes a forward purchase or sale of a stochastic yield, determined according to the contract specification. Typically this yield is an estimate of the average yield over some area. Yield-based contracts are typically settled in cash terms with a nonstochastic payment, which we denote as $\tilde{p}$, made for each unit of the yield quote. If the contract yield in period 0 is b_0 and the stochastic contract yield in period 1 is denoted by $\boldsymbol{b} = (b_1, \dots, b_s)$, then, for a forward sale,

$$a_s = \tilde{p}(b_0 - b_s), \tag{5}$$

and

$$y_s = p_s z_s + h\tilde{p}(b_0 - b_s). \tag{6}$$

Thus, assuming $h > 0$, the farmer receives a positive payoff if area yield is less than the contract yield in period 0.

[2] Although it is possible to consider futures or forward contracts for any commodity with and without basis risk, we restrict attention here to the commodity that is produced by the farmer with no basis risk.

2.3 Example 3: put and call options for commodities

A put option conveys the right, but not the obligation, to sell one unit of the commodity at the strike price q after the state of Nature is revealed.[3] A call option conveys the right, but not the obligation, to buy one unit of the commodity at the strike price q.

Because the purchaser has no obligation to sell at the strike price, a put is exercised only if the strike price is greater than the prevailing market price in period 1. Then, exercising the right to sell at the strike price allows the producer to buy the commodity at p_s in the spot market and resell it for q for a profit. Therefore, the state-contingent return on the put option is

$$a_s = \max\{q - p_s, 0\}. \tag{7}$$

Similarly, a call is exercised only if the strike price is less than or equal to the prevailing market price. Thus

$$a_s = \max\{p_s - q, 0\}. \tag{8}$$

Notice the similarity and the dissimilarity between these option contracts and the futures and forward contracts modelled above. If the producer simultaneously buys a put option with a strike price of q and sells (writes) a call option at the same strike price then he or she creates an asset that yields a state-contingent return of

$$a_s = \max\{q - p_s, 0\} - \max\{p_s - q, 0\} = q - p_s. \tag{9}$$

Simultaneously buying a call and selling a put entitles the producer to the same stream of returns as buying one unit of the forward contract discussed in preceding text. However, because options are exercised only in a range bounded by the strike price, neither a put option nor a call option alone offers returns that are perfectly collinear with the forward or futures contract discussed. This lack of perfect collinearity, as is well recognised, implies that option contracts can be used to increase the range of risks covered by the risk-management tools. We return to the implications of this observation briefly in our penultimate section.

Some government-sponsored risk-management tools operate as option contracts. For example, price-underwriting schemes effectively operate by giving producers free put options on the price of the commodity. Similarly, a price-band stabilisation scheme might be seen as requiring producers to

[3] Here, for the sake of simplicity, we restrict attention to European options.

give the stabilisation authority a call option, with the strike price at the upper bound of the price band, in return for a put option, with the strike price at the lower bound of the price band.

2.4 Example 4: crop insurance

There are a variety of different crop-insurance products offered. We first consider the case of area-yield crop insurance first proposed by Halcrow (1949), later discussed in an Australian context by the Industries Assistance Commission (1978), and subsequently resurrected in the USA (Miranda 1991; Chambers and Quiggin 2002b).

Area-yield insurance works as an option expressed in units of the commodity yield. A purchaser of a contract is guaranteed a commodity payment if average yield over a specified area (the area yield) falls below a threshold level. If the area yield is above the threshold level, no payment is made. Therefore, we have

$$a_s = p_s \max\left\{b_0 - \frac{1}{N}\sum_{n=1}^{N} b_{sn}, 0\right\}, \tag{10}$$

where N is the number of farmers in the risk pool, b_{sn} is yield by individual n in state s, and b_0 is now the threshold level of yield that triggers actual payments.

It is, therefore, apparent that if the yield-based futures contract discussed takes the stochastic contract yield as the area yield, that is, if

$$b_s = \frac{1}{N}\sum_{n=1}^{N} b_{sn}, \tag{11}$$

then area-yield insurance operates as a yield-based put option on the area yield. Area-yield contracts are mathematically identical to other forms of risk-specific crop insurance, including rainfall insurance (Bardsley *et al.* 1984; Quiggin 1986), for which payouts are exogenous to the individual farmer.

Another common form of crop insurance is subsidised individual yield insurance. Suppose that acreage units are normalised to equal one. A popular form of yield insurance pays different returns depending upon the level of yield loss relative to a target yield. For example, one stylised form makes payments to farmers for losses below a program-determined threshold yield so that

$$a_s = \begin{cases} 0 & z_s > z_t \\ b(z_t - z_s) & \text{otherwise} \end{cases}, \tag{12}$$

where z_t is the threshold yield that triggers insurance payouts, whence

$$a_s = \max\{b(z_t - z_s), 0\}, \tag{13}$$

which corresponds to an options contract. Complete coverage below the threshold yield corresponds to $b = 1$. More complicated schemes involve different levels of coverage depending upon the magnitude of the loss and different options chosen by the participating farmer.

2.5 Example 5: on-farm and off-farm labour choice

Many Australian farm households do off-farm work. Off-farm work has been analysed empirically and theoretically as a method of matching household labour endowments to farm size and thereby mitigating problems of inadequate scale (Robinson *et al.* 1982; Quiggin and Vlastuin 1983). Off-farm work can also be viewed as a risk-management tool for agricultural producers. For simplicity, we assume that the farmer has already decided how many total hours to devote to labour, and call that amount H. Let the stochastic off-farm wage rate be denoted by $\boldsymbol{q} \in \Re_+^S$ with typical element q_s, which is beyond the farmer's control. Then, his state-contingent income from on-farm and off-farm income is given by

$$y_s = p_s z_s + hq_s, \tag{14}$$

where h now denotes the number of hours worked off farm.

Individualised yield insurance and on-farm, off-farm labour choice, when viewed as risk-management tools, differ slightly from the other risk-management tools that we have considered. In individualised yield insurance, the payout in any state of Nature is determined endogenously by the farmer's choice of z_s. In the on-farm, off-farm example, the risk-management tool is non-linearly priced because obtaining it requires selling a unit of labour off-farm. Because the opportunity cost of a unit of off-farm labour is its marginal cost in producing the agricultural commodity, this risk-management tool is non-linearly priced unless, of course, the agricultural technology is linear in on-farm labour.

2.6 Example 6: a riskless bond

Suppose that there exists a riskless bond, issued by the government, which yields a non-stochastic payout of $1 + r$ dollars in each state, at a cost of one dollar. Then,

$$a_s = 1 + r, \tag{15}$$

and state-contingent income is given by

$$y_s = p_s z_s + h(1 + r). \tag{16}$$

3. Stochastic technology and risk-management tool

We now consider a general model of a farmer facing a linear price for the risk-management tool. Producer preferences over income in the base period, y_0, which is certain, and income in period 1, which is stochastic, are given by $W(y_0, \boldsymbol{y})$, where $\boldsymbol{y} \in \Re^S$ is state-contingent income received in period 1. W is strictly increasing in y_0 and $\boldsymbol{y}$.

An example is given by time-separable expected utility preferences

$$W(y_0, \boldsymbol{y}) = u(y_0) + \beta \sum_s \pi_s u(y_s), \tag{17}$$

where u is a von Neumann–Morgenstern utility function, β is a discount factor and π_s is the subjective probability of state s, so that $\sum_s \pi_s = 1$. It is important to note, however, that the analysis in the present paper does not rely on the assumption that producers have expected-utility preferences or even that they have well-defined subjective probabilities regarding the states of nature. The analysis relies exclusively on the concept of cost-minimisation, so that the only behavioural assumption is that preferences are monotonic in net income, that is, that producers prefer more income to less.

The firm's stochastic production technology is represented by a single-product, state-contingent input correspondence.[4] Let $\boldsymbol{x} \in \Re_+^N$ be a vector of inputs committed prior to the resolution of uncertainty (period 0), and let $\boldsymbol{z} \in \Re_+^S$ be a vector of *ex ante* or state-contingent outputs also chosen in period 0. If state $s \in \Omega$ is realised (picked by 'Nature'), and the producer has chosen the *ex ante* input–output combination $(\boldsymbol{x}, \boldsymbol{z})$, then the realised or *ex post* output in period 1 is z_s.

The continuous input correspondence, $X : \Re_+^S \rightarrow \Re_+^N$, which maps state-contingent output vectors into input sets that are capable of producing that state-contingent output vector, is defined by

$$X(\boldsymbol{z}) = \{\boldsymbol{x} \in \Re_+^N : \boldsymbol{x} \text{ can produce } \boldsymbol{z}\} \tag{18}$$

We impose the following properties on $X(\boldsymbol{z})$:

[4] For a generalisation to the multiple-output case, see Chambers and Quiggin (2000, Chapter 4).

X.1 $X(\mathbf{0}) = \Re_+^N$ (no fixed costs), and $\mathbf{0} \notin X(z)$ for $z \geq \mathbf{0}$ and $z \neq \mathbf{0}$ (no free lunch).
X.2 $z' \leq z \Rightarrow X(z) \subseteq X(z')$.
X.3 $\lambda X(z) + (1-\lambda)X(z') \subseteq X(\lambda z + (1-\lambda)z') \quad 0 \leq \lambda \leq 1$.
X.4 X is continuous. (19)

Period 0 prices of inputs are denoted by $\boldsymbol{w} \in \Re_{++}^N$, and are non-stochastic. Output price is stochastic, and we denote by $\boldsymbol{p} \in \Re_{++}^S$ the vector of state-contingent output prices corresponding to the vector of state-contingent outputs. Producers take these state-contingent output prices and the prices of all inputs as given. The state-contingent revenue vector, denoted $\boldsymbol{p} \cdot \boldsymbol{z} \in \Re_+^S$, has typical elements of the form $p_s z_s$.

The farmer also has access to a risk-management tool, which offers per unit state-contingent payout equalling $\boldsymbol{a} \in \Re^S$ in time 1 in return for a non-stochastic per unit payment made in period 0 of v. Denote by h the firm's purchase of the risk-management tool. Because we want to be able to consider the case of producers going both long and short in various markets, we do not restrict the sign of h to be positive.[5]

Dual to $X(\mathbf{z})$ is the cost function, $c : \Re_{++}^S \times \Re_+^N \to \Re_+$ defined as

$$c(\boldsymbol{w}, \boldsymbol{z}) = \min_{\mathbf{x}} \{\boldsymbol{w}\boldsymbol{x} : \boldsymbol{x} \in X(\boldsymbol{z})\} \; \boldsymbol{w} \in \Re_{++}^N \tag{20}$$

if there exists an $\boldsymbol{x} \in X(\boldsymbol{z})$, and ∞ otherwise. If the input correspondence satisfies properties X, $c(\boldsymbol{w}, \boldsymbol{z})$ satisfies (Chambers and Quiggin 2000): $c(\boldsymbol{w}, \boldsymbol{z}) \geq 0$, $c(\boldsymbol{w}, 0_S) = 0$, and $c(\boldsymbol{w}, \boldsymbol{z}) > 0$ for $\boldsymbol{z} \geq \mathbf{0}$, $\boldsymbol{z} \neq \mathbf{0}$; $\boldsymbol{z}^0 \geq \boldsymbol{z} \Rightarrow c(\boldsymbol{w}, \boldsymbol{z}^0) \geq c(\boldsymbol{w}, \boldsymbol{z})$; and $c(\boldsymbol{w}, \boldsymbol{z})$ is convex on $\Re_+^S$ and continuous on the interior of the region where it is finite. For expositional clarity, we shall routinely strengthen continuity on $\Re_+^S$ to differentiability to permit the use of calculus-based arguments, and we shall assume that the cost structure is strictly increasing in state-contingent outputs. Denote the partial derivative of c with respect to z_s as $c_s(\boldsymbol{w}, \boldsymbol{z})$.[6]

4. Characterising optimal producer behaviour

Without making any specific assumptions on the farmer's risk preferences, consider any optimal choice of state-contingent income, call it $\boldsymbol{y}^*$. The monotonicity of preferences implies that the associated input allocation and risk-management tool must be chosen to satisfy

[5] Some risk-management tools incorporate short selling restrictions. For example, area-yield contracts as marketed in the USA only allow farmers to take a long position.

[6] Chambers and Quiggin (2002a) consider the case of non-differentiable and weakly monotonic cost structures in detail.

$$C(\boldsymbol{y}^*) = \min_{\mathbf{x},\mathbf{z},h} \boldsymbol{wx} + vh : \boldsymbol{x} \in X(\boldsymbol{z}),\ \boldsymbol{p} \cdot \boldsymbol{z} + h\boldsymbol{a} \geq \mathbf{y}^*, \tag{21}$$

where $\boldsymbol{wx} + vh$ is the current period (period 0) expenditure on inputs and asset purchases. Suppose instead that the farmer did not use the cheapest combination of inputs, state-contingent outputs, and holdings of the risk-management tool to assemble $\boldsymbol{y}^*$. Given the monotonicity of W, the farmer would then strictly gain by replacing his input choice with $C(\boldsymbol{y}^*)$. Any rational farmer would do this. Thus, in considering farmer behaviour, we focus exclusively on solutions of this generic cost minimisation problem.

By the principle of recursive optimisation, we can rewrite the cost problem above as

$$\begin{aligned} C(\boldsymbol{y}) &= \min_{\boldsymbol{x},\boldsymbol{z},h} \{\boldsymbol{wx} + vh : \boldsymbol{x} \in X(\boldsymbol{z}),\ \boldsymbol{p} \cdot \boldsymbol{z} + h\boldsymbol{a} \geq \mathbf{y}\} \\ &= \min_{\boldsymbol{z},h} \{\min_{\boldsymbol{x}} \{\boldsymbol{wx} : \boldsymbol{x} \in X(\boldsymbol{z})\} + vh : \boldsymbol{p} \cdot \boldsymbol{z} + h\boldsymbol{a} \geq \mathbf{y}\} \\ &= \min_{\boldsymbol{z},h} \{c(\boldsymbol{w}, \boldsymbol{z}) + vh : \boldsymbol{p} \cdot \boldsymbol{z} + h\boldsymbol{a} \geq \mathbf{y}\}. \end{aligned} \tag{22}$$

Before characterising the general solution to this problem, it is instructive to consider it in the simplest case where there are only two states of Nature, and where the output price is non-stochastic and equal to one.

Figure 1 illustrates the cost-minimisation problem. The vertical axis measures income in state 2, and the horizontal axis measures income in state 1. The random variable, $\boldsymbol{y}$, which represents the producer's desired amount of state-contingent income is represented by the vector labelled $\boldsymbol{y}$. The risk-management tool is depicted by the vector labelled $\boldsymbol{a}$ (as illustrated $a_2 > 0 > a_1$). This risk-management tool makes a positive payment to the producer in state 2, but requires a positive payment by the producer in state 1. Here, one might think of an insurance contract, whose net indemnity is positive in state 2, but negative in state 1. To simplify exposition, we set $\boldsymbol{p} = (1, 1)$, so that $\boldsymbol{y} = h\boldsymbol{a} + \boldsymbol{z}$, where $\boldsymbol{z}$ is state-contingent output.

To reach $\boldsymbol{y}$, the farmer must produce a state-contingent output consistent with $\mathbf{y}$, given the available risk-management tool. To visualise the possible production vectors that can be consistent with $\boldsymbol{y}$, start at $\boldsymbol{y}$ and consider what happens to state-contingent income if the producer buys one unit of the risk-management tool. Buying one unit of $\boldsymbol{a}$, that is, setting $h = 1$ changes his or her state-contingent income from (y_1, y_2) to $(y_1 + a_1, y_2 + a_2)$. This is represented in figure 1 by moving from point $\boldsymbol{y}$ to the north-west and exactly parallel to the vector $\boldsymbol{a}$. Similarly, if the producer sells one unit of the risk-management tool, then he moves from (y_1, y_2) to $(y_1 - a_1, y_2 - a_2)$. This is represented in figure 1 by moving from point $\boldsymbol{y}$ to the south-east and

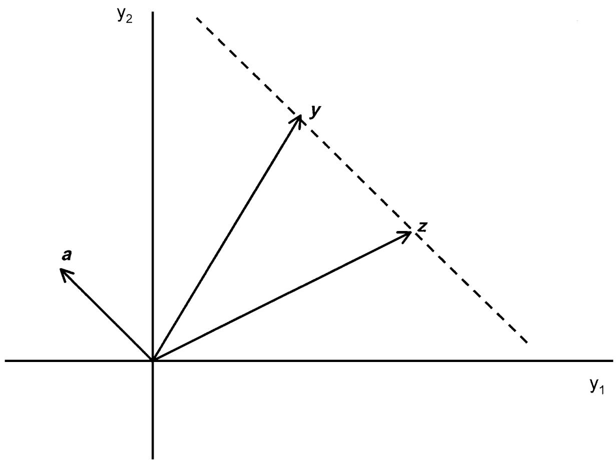

Figure 1 Optimal choice of $\boldsymbol{a}$ and $\boldsymbol{z}$.

exactly parallel to the vector $\boldsymbol{a}$. It follows, therefore, that any point on the dotted line that is parallel to $\boldsymbol{a}$ and that passes through $\boldsymbol{y}$ can be reached from $\boldsymbol{y}$ by buying and selling units of the risk-management tool. Conversely, the only state-contingent production vectors that allow one to reach $\boldsymbol{y}$ by buying and selling units of $\boldsymbol{a}$ are those lying on this same dotted line passing through $\boldsymbol{y}$. For production points to the south-east of $\boldsymbol{y}$ on this dotted line, the producer reaches $\boldsymbol{y}$ by buying units of $\boldsymbol{a}$ (going long), while points to the north-west entail the producer reaching $\boldsymbol{y}$ by selling $\boldsymbol{a}$ (shorting the market). In the absence of the risk-management tool, the farmer must set $\boldsymbol{z} = \boldsymbol{y}$. The farmer's choice problem is to pick $\boldsymbol{z}$ and h in a cost-minimising fashion.

Notice from figure 1 that as the individual buys or sells units of the risk-management tool, he or she is effectively trading off income in state 2 against income in state 1. Similarly, as individuals modify their state-contingent output bundle while holding cost constant, they are trading off, via the production technology, income in state 2 against income in state 1. Their cost-minimising mix of the risk-management tool and production to reach $\boldsymbol{y}$ will require that they equalise the rate at which the risk-management tool and the technology trade off these state-contingent incomes.

More formally, we have the following theorem, which is established in the Appendix. In the theorem and elsewhere in the remainder of the paper the partial derivative of C with respect to y_s is denoted $C_s(\boldsymbol{y})$. In particular, if we evaluate the partial derivatives at $\boldsymbol{y} + h\boldsymbol{a}$, we have

$$\frac{\partial}{\partial y_s}C(\boldsymbol{y} + h\boldsymbol{a}) = C_s(\boldsymbol{y} + h\boldsymbol{a})$$

$$\frac{\partial}{\partial h}C(\boldsymbol{y} + h\boldsymbol{a}) = \sum_{s=1}^{S} C_s(\boldsymbol{y} + h\boldsymbol{a})a_s. \tag{23}$$

$C(\boldsymbol{y})$ is increasing and convex in **y** with $C(\mathbf{0}) \leq 0$. If C is finite, then

$$\begin{aligned} &C(\boldsymbol{y} + h\boldsymbol{a}) = C(\boldsymbol{y}) + hv, \quad h \in \Re, \\ &C_s(\boldsymbol{y} + h\boldsymbol{a}) = C_s(\boldsymbol{y}), \qquad h \in \Re \\ &\sum_{s=1}^{S} C_s(\boldsymbol{y})a_s = v, \\ &\sum_{k=1}^{S}\sum_{s=1}^{S} C_{sk}(\boldsymbol{y})a_s a_k = 0. \end{aligned} \tag{24}$$

The monotonicity and convexity properties in the theorem are self explanatory and follow from the monotonicity and convexity properties of c. In the absence of any fixed costs, it is also clear that the cost of assembling a zero return vector is non-positive.[7]

The equalities in the theorem are probably less familiar. They are all consequences of the first equality, which we now discuss. The first equality says that adding h units of any risk-management tool to any $\boldsymbol{y}$ should increase its minimal cost by exactly hv. Suppose, for example, that

$$C(\boldsymbol{y} + h\boldsymbol{a}) > C(\boldsymbol{y}) + hv. \tag{25}$$

This cannot be consistent with rational behaviour. If it were true, a farmer currently at $\boldsymbol{y}$ could continue to 'assemble' $\boldsymbol{y}$ as before and purchase h units of the risk-management tool in the market for hv. This gives them title to a state-contingent income stream of $\boldsymbol{y} + h\boldsymbol{a}$, but at a cost less than $C(\boldsymbol{y} + h\boldsymbol{a})$. Because $C(\boldsymbol{y} + h\boldsymbol{a})$ has been defined as the minimal cost to the farmer of assembling $\boldsymbol{y} + h\boldsymbol{a}$ either by producing the stochastic output or using the risk-management tool, this can't happen. If the inequality were reversed, then

[7] Notice, however, that the theorem does not rule out $C(\mathbf{0}) < 0$ as equilibrium behaviour. Negative cost corresponds to a (non-stochastic) period 0 payment to the producer to assemble the non-stochastic asset yielding a zero return in all states of Nature. Because he or she can always do this by doing nothing, no rational farmer would ever turn down such a deal, and thus it cannot be ruled out simply on the basis of optimal farmer behaviour.

$$C(\boldsymbol{y} + h\boldsymbol{a}) - hv < C(\boldsymbol{y}). \tag{26}$$

Farmers now wishing to realise $\boldsymbol{y}$ in period 1 would gain by proceeding as follows. Construct a period 1 stochastic return of $\boldsymbol{y} + h\boldsymbol{a}$ through their production and risk-management activities at a period 0 cost $C(\boldsymbol{y} + h\boldsymbol{a})$. If they now sold h units of $\boldsymbol{a}$, they would leave themselves with a claim to $\boldsymbol{y}$ but at a cost less than $C(\boldsymbol{y})$.

Having obtained the first equality, it is easy to obtain the remaining three. The second equality follows by differentiating both sides of the first with respect to y_s. The third equality follows by differentiating the first equality once with respect to h and using the second equality to substitute $C_s(\boldsymbol{y})$ for $C_s(\boldsymbol{y} + h\boldsymbol{a})$. The fourth equality is obtained similarly with a further differentiation with respect to h.

The second equality has a visually intuitive interpretation. It says that as one proceeds in the direction of the risk-management tool in state-contingent income space (in figure 1, this is in the direction of the negatively-sloped dotted line segment connecting $\boldsymbol{z}$ and $\boldsymbol{y}$), one will cut successive isocost contours of $C(\boldsymbol{y})$ at points of equal slope.

Because of its importance, it is instructive to obtain the third equality in a slightly different fashion. The first-order conditions for an interior solution to the cost-minimisation problem in (21) require

$$\sum_{s=1}^{S} \frac{c_s(\boldsymbol{w}, \boldsymbol{z})}{p_s} a_s = v. \tag{27}$$

Note once again that this arbitrage result depends only on the logic of cost minimisation, and does not require any assumption about subjective probabilities or the functional form of preferences. It is particularly useful to consider the case when the risk-management tool is a bond, yielding a payoff of 1 in each state of nature. Then (27) becomes

$$\sum_{s=1}^{S} \frac{c_s(\boldsymbol{w}, \boldsymbol{z})}{p_s} = v. \tag{28}$$

The right-hand side is simply the cost of increasing revenue by one unit in every state of nature. This is the natural extension to the two-period case of the arbitrage condition derived by Chambers and Quiggin (2000, Chapter 5).

Applying the envelope theorem to the optimisation problem (21) gives

$$C_s(\boldsymbol{y}) = \frac{c_s(\boldsymbol{w}, \boldsymbol{z})}{p_s} \tag{29}$$

for all s. Together these conditions imply the third equality. This perspective also suggests a slightly different intuitive interpretation of the basic result.

The fourth equality is of interest in comparing a purely technological cost function with that derived in the presence of linearly-priced financial assets. Under standard convexity assumptions, the matrix of second derivatives for a purely technological cost function is negative semidefinite. By contrast, the matrix of second derivatives for a linear cost function is identically zero and therefore satisfies the fourth equality trivially.

In assembling $\boldsymbol{y}$, farmers always have at least three choices. They can assemble $\boldsymbol{y}$ simply by producing the state-contingent output vector

$$\boldsymbol{z} = \left(\frac{y_1}{p_1}, \dots, \frac{y_S}{p_S} \right). \tag{30}$$

They can assemble $\boldsymbol{y}$ by choosing the smallest holding of $\boldsymbol{a}$ that ensures

$$h\boldsymbol{a} \geq \boldsymbol{y}, \tag{31}$$

or they can assemble $\boldsymbol{y}$ by both producing and purchasing holdings of the risk-management tool. Varying holdings of $\boldsymbol{a}$ at the margin changes the state-contingent return by $\boldsymbol{a}$ at a marginal cost of v. In deciding whether to vary h or $\boldsymbol{z}$, rational producers would recognise that at an interior margin they can also achieve a marginal state-contingent return of $\boldsymbol{a}$ by varying $\boldsymbol{z}$ such that

$$\delta \boldsymbol{z} = \left(\frac{a_1}{p_1}, \dots, \frac{a_S}{p_S} \right). \tag{32}$$

By this variation in $\boldsymbol{z}$, farmers use the physical production technology to effectively replicate a single unit of the risk-management tool. The marginal cost of replicating $\boldsymbol{a}$ in this fashion is

$$\sum_{s=1}^{S} \frac{c_s(\boldsymbol{w}, \boldsymbol{z})}{p_s} a_s. \tag{33}$$

So, for example, if $\sum_{s=1}^{S} \{[c_s(\boldsymbol{w}, \boldsymbol{z})]/p_s\} a_s < v$, farmers can lower their cost by selling off one unit of the risk-management tool that they have 'replicated' in this manner. At the margin, this brings a cost savings of $v - \sum_{s=1}^{S} \{[c_s(\boldsymbol{w}, \boldsymbol{z})]/p_s\} a_s > 0$. Conversely, if the inequality were reversed the farmers could lower cost at the margin by lowering production by $\delta\boldsymbol{z}$ and acquiring the asset to replace the foregone production. Expression (27), therefore, reflects the basic requirement that farmers not miss any arbitrage opportunity that would permit them to lower cost while holding their state-contingent return constant.

It is of particular interest to note the connection between the physical arbitrage opportunities discussed here and the common notion of financial arbitrage. An arbitrage opportunity exists at (z, h) if there exists a (z', h') such that

$$c(\boldsymbol{w}, z') + vh' < c(\boldsymbol{w}, z) + vh \tag{34}$$

and

$$\boldsymbol{p} \cdot z' + h'\boldsymbol{a} \geq \boldsymbol{p} \cdot z + h\boldsymbol{a}. \tag{35}$$

Here (z', h') represents an arbitrage opportunity relative to (z, h) because it offers farmers a way to (weakly) increase their state-contingent return in each state of Nature while (strictly) lowering their cost. Seeing such an opportunity, a rational farmer would always adopt (z', h') in favour of (z, h) because it risklessly raises period 1 return while lowering cost. In any equilibrium, all such arbitrage opportunities must be eliminated (Chambers and Quiggin 2002a).

In marginal terms, an arbitrage opportunity exists at (z, h) if there exists a

$$\delta z = (h - h')\frac{\boldsymbol{a}}{\boldsymbol{p}}, \tag{36}$$

where $\frac{\boldsymbol{a}}{\boldsymbol{p}} = \left(\frac{a_1}{p_1}, \dots, \frac{a_S}{p_S}\right)$ such that

$$c(\boldsymbol{w}, z + \delta z) - c(\boldsymbol{w}, z) < v(h - h'). \tag{37}$$

The convexity of the farmer's cost function with respect to z implies that

$$\sum_{s=1}^{S} c_s(\boldsymbol{w}, z)\delta z_s \leq c(\boldsymbol{w}, z + \delta z) - c(\boldsymbol{w}, z). \tag{38}$$

Hence, if an arbitrage opportunity exists, then

$$(h - h')\sum_{s=1}^{S} \frac{c_s(\boldsymbol{w}, z)}{p_s} a_s < v(h - h'). \tag{39}$$

But so long as (27) holds, this last expression cannot be satisfied. Thus, (27) requires that no arbitrage opportunities exist.

Theorem 1 characterises rationality conditions that all meaningful equilibria for producers with monotonic preferences must satisfy, regardless of their risk preferences. This is a constructive view of these relationships that can be useful in a number of different contexts. Below, we show how these fundamental principles can be used to analyse equilibrium behaviour in different versions of the canonical model and how these basic principles extend to generalisations of the canonical model. In what remains of this section, however, we take a more prescriptive view of these conditions in the hope that it might highlight some potential practical applications of the theorem.

Any market equilibrium relationship can be recast as a virtual pricing rule. As an illustration of this point in relation to theorem 1, consider the policy problem of accurately pricing publicly-sponsored agricultural insurance. This topic was the subject of a lively debate within the Australian agricultural economics profession in the 1980s (Bardsley *et al.* 1984; Quiggin 1986).[8]

Instead of envisioning conditions in the theorem as equilibrium conditions constructed from the principle of optimisation, now view them as virtual pricing relationships and consider offering a rainfall insurance contract that pays $a_s \geq 0$ for each $s \in \Omega$. At the margin, the maximal price that farmers would be willing to pay for this contract is

$$v^* = \sum_s C_s(\boldsymbol{y})a_s, \tag{40}$$

where $\boldsymbol{y}$ is their state-contingent income vector in the absence of insurance. By (27), it follows that the appropriate price for the insurance contract can be derived as

$$v^* = \sum_s \frac{c_s(\boldsymbol{w}, \boldsymbol{z})}{p_s} a_s, \tag{41}$$

where $\boldsymbol{z}$ is now evaluated at its pre-insurance level.

In principle, the right-hand side of this expression can be constructed statistically from estimated versions of the stochastic production technology.

[8] Among the key issues was whether a public insurer should be less risk-averse with respect to rainfall than the farmers who might form a mutual insurance pool. Quiggin (1986) argued that because the correlation between rainfall and total government revenue is very small, a public insurer should be approximately risk-neutral. This point can be restated in terms of the cost–price ratios $c_s(\boldsymbol{w}, \boldsymbol{z})/p_s$ that prevail in equilibrium. In the absence of insurance, the equilibrium cost–price ratios for farmers will satisfy $c_s(\boldsymbol{w}, \boldsymbol{z})/p_s > \pi_s$ for low-rainfall states s, where π_s is the probability of such a state occurring. By contrast, in the presence of actuarially fair rainfall insurance $c_s(\boldsymbol{w}, \boldsymbol{z})/p_s = \pi_s$, and it is straightforward to show that farmers' welfare will be improved. This conclusion holds in the absence of transactions costs. As Bardsley *et al.* (1984) observe, if the price of rainfall insurance incorporates transactions costs, it might be unattractive to farmers.

Hence, once the issue of how best to estimate a representation of the technology is resolved, theorem 1 has a straightforward prescriptive component that can be applied in a potentially important policy context.

The asset price v^* derived in equation (41) can usefully be compared with the alternative of applying an option pricing rule such as the Black–Scholes formula. The main weakness of the latter approach is the requirement that the asset $\boldsymbol{a}$ be replicable in asset markets. This will not be the case for an agricultural insurance contract that contains what financial markets would view as idiosyncratic risk. The price v^* obeys the same fundamental no-arbitrage rules contained in the Black–Scholes formula, but it does not require the assumption that is needed to make the Black–Scholes approach relevant to an insurance contract, namely, that $\boldsymbol{a}$ be perfectly replicable within existing financial markets. Instead the formulation in (41) uses information on the production technology.

5. Non-linear pricing of risk management

This problem has a slightly different cost structure than the other examples, but the same basic principles apply with a slight modification of arguments. We need to modify the production model slightly to accommodate the existence of an input, which is simultaneously marketed at a stochastic price and consumed directly on the farm. Therefore, let h_f denote the number of hours worked on-farm and h the number of hours worked off-farm, and rewrite the input correspondence as:

$$X(\boldsymbol{z}, h_f) = \{\boldsymbol{x} \in \Re_+^N : \boldsymbol{x} \text{ can produce } \boldsymbol{z}, \text{ given } h_f \text{ hours worked}\}. \tag{42}$$

Associated with this respecification is the dual cost structure $c(\boldsymbol{w}, \boldsymbol{z}, h_f)$, where c is now decreasing in h_f.

The farmer's problem is now, in a slight abuse of notation, to

$$C(\boldsymbol{y}, H) = \min_{h_f, h, \boldsymbol{z}} \{c(\boldsymbol{w}, \boldsymbol{z}, h_f) : \boldsymbol{p} \cdot \boldsymbol{z} + h\boldsymbol{q} \geq \boldsymbol{y}, h + h_f = H\}, \tag{43}$$

where H is the total time available for productive effort, which is assumed here to be exogenously given. That is, $C(\boldsymbol{y}, H)$ is the minimum cost (given $\boldsymbol{w}$) for combinations $(\boldsymbol{z}, h)$ of on-farm output and off-farm labour that yield income $\boldsymbol{y}$ (given $\boldsymbol{p}$ and q).

It is no longer easy to assess the effect of shifting $\boldsymbol{y}$ in the direction of the risk-management tool. Thus, an exact analog to the first equality in theorem 1 does not apply. This change emerges from the fact that the risk-management tool, off-farm labour, is priced non-linearly by the farmer at its opportunity cost, which is its marginal return as on-farm labour.

Notice, however, that the general rule for the choice of the investment in the risk-management tool remains the same and, thus, the marginal analog to the third equality in theorem 1 does apply. First-order conditions for an interior solution require

$$-\sum_{s=1}^{S} \frac{c_s(\boldsymbol{w}, \boldsymbol{z}, H - h)}{p_s} q_s - c_h(\boldsymbol{w}, \boldsymbol{z}, H - h) = 0. \tag{44}$$

The envelope theorem implies that if $\boldsymbol{z}$ and h are cost-minimising choices for $\boldsymbol{y}$, given $\boldsymbol{w}$, $\boldsymbol{p}$ and q, then

$$C_H(\boldsymbol{y}, H) = c_h(\boldsymbol{w}, \boldsymbol{z}, H - h). \tag{45}$$

Hence, the first-order condition can be rewritten as

$$\sum_{s=1}^{S} C_s(\boldsymbol{y}, H) q_s = -C_H(\boldsymbol{y}, H). \tag{46}$$

The virtual valuation of the risk-management tool, off-farm labour, must equal the now non-linear, marginal price of the risk-management tool, which is the cost saving associated with a unit of on-farm labour.

If the off-farm labour market is riskless, with $q_s = q$ for all s, then this last condition becomes

$$q = -\frac{C_H(\boldsymbol{y}, H)}{\sum_{s=1}^{S} C_s(\boldsymbol{y}, H)}. \tag{47}$$

This result is perhaps more intuitive if viewed from a different perspective. When the off-farm labour market is riskless, agreeing to work off-farm is equivalent to investing in a riskless asset (for example, a savings account) that yields a period 1 non-stochastic return of q. The marginal cost to the farmer of one unit of time (how much he or she has to pay for each unit of the riskless asset) is his or her shadow price of a unit of time. This is captured by $C_H(\boldsymbol{y}, H)$. By receiving, in return, the right to a non-stochastic return of q dollars in period 1, the farmer can reduce output in each state of Nature by the amount q/p_s. The realised cost saving is $q\sum_{s=1}^{S} C_s(\boldsymbol{y}, H)$. The right-hand term in the last expression therefore measures the rate at which the technology transforms one hour of time worked in period 0 into riskless income in period 1. The equilibrium condition requires that this marginal technical rate of substitution between period 0 time and riskless period 1 income be equated to the riskless return from off-farm labour.

6. More than one risk-management tool

In reality, producers face an array of risk-management tools to control income risk. For example, commodities exist for which each of the above tools are simultaneously available. In fact, for many agricultural commodities, the simultaneous existence of these risk management tools is the norm and not the exception. Multiple risk-management tools increase the strategies available to farmers to manage income risk.

Two observations immediately come to mind. First, each of the optimality conditions developed must continue to apply when more than one tool is available. This realisation carries with it the second observation. The more risk-management tools that the producer has available, the more that can be said about their optimal behaviour and the world in which they operate.

Earlier we saw that the simultaneous presence of put and call options with a common strike price allows the farmer to 'create' his own forward contract by simultaneously buying a put option and writing a call option. Let us return to this result in the context of what we have learned and consider a farmer facing three risk-management tools: (i) a forward contract with no basis risk, (ii) a call option with a strike price equal to the forward contract price and (iii) a put option with the same strike price. Denote the period 0 prices for these contracts, respectively, as v_f, v_c and v_p.

By the obvious extension of theorem 1:

$$\sum_{s=1}^{S} C_s(\boldsymbol{y})(q - p_s) = v_f;$$

$$\sum_{s=1}^{S} C_s(\boldsymbol{y}) \max\{q - p_s, 0\} = v_p; \text{ and}$$

$$\sum_{s=1}^{S} C_s(\boldsymbol{y}) \max\{p_s - q, 0\} = v_c. \tag{48}$$

That is, the period 0 price of each contract must be equal to the sum of the state-contingent period 1 payoffs, weighted by the associated *ex ante* marginal costs C_s.

Subtracting the third equality from the second gives

$$\sum_{s=1}^{S} C_s(\boldsymbol{y})(q - p_s) = \sum_{s=1}^{S} C_s(\boldsymbol{y})[\max\{q - p_s, 0\} - \max\{p_s - q, 0\}] = v_p - v_c. \tag{49}$$

By comparing this expression with the first equality in (48), one sees that the three equilibrium conditions are consistent if and only if $v_f = v_p - v_c$. The implication is *not* that the farmer's production and risk-management equilibrate the forward price and the difference between the two option prices.[9] Recall that the price of each risk-management tool is exogenous to the farmer. Rather, this observation manifests the law of one price, and is a condition that must be satisfied by any market equilibrium.

Suppose, for example, that $v_f > v_p - v_c$ and consider any solution to the cost-minimisation problem. From that holding, the farmer could always keep his production level constant and then alter his holdings of the risk-management tools in the following way. Buy one put option and sell one call option at a total cost of $v_p - v_c$. As noted earlier, this 'creates' or replicates one unit of the forward contract. Having 'created' one unit of the forward contract, the farmer can now sell it and realise an arbitrage profit of $v_f - v_p + v_c > 0$ in period 0. This arbitrage profit lowers his cost. Moreover, because this process can be repeated an infinity of times, this is a money pump and thus $C(\boldsymbol{y})$ is driven to $-\infty$, implying that consumption in period 0 grows unboundedly large. Therefore, $v_f = v_p - v_c$ is a condition that must be satisfied if the three risk-management tools are not to present the farmer with financial arbitrages that will allow him or her to make an unboundedly large profit simply by taking the correct market position.

The conditions for an internal equilibrium manifest the same logic. Suppose, for example, that $v_f > v_p - v_c$, but that expressions (48) are satisfied. It was demonstrated in preceding text that purely financial transactions would allow the farmer to make an arbitrarily large arbitrage profit. However, because the farmer has access to a physical technology, he or she can also exploit the technology to take advantage of any potential arbitrage opportunities. In this instance, for example, instead of buying a put option the farmer could manufacture the put at a marginal cost of

$$\sum_{s=1}^{S} C_s(\boldsymbol{y}) \max\{q - p_s, 0\}. \tag{50}$$

The farmer could then sell one unit of the call option in return for its period 0 price of v_c. By this combination of a financial transaction (selling the call) and use of the physical technology (manufacturing the put), he or she has created a forward contract that can be used to replace one unit of the forward contract realising a period 0 profit of $v_f - v_p + v_c > 0$. If the farmer does not own any units of the forward contract, he or she can execute a trade in the forward market for a price of v_f again realising a period 0 profit. Either way, the result is lower period 0 cost. Thus, if $v_f \neq v_p - v_c$, the

[9] Obviously, the prices are determined by the aggregate production, consumption and risk-management decisions of all market participants, including farmers.

law of one price is violated. The farmer will always have an opportunity to lower period 0 cost either by a financial arbitrage or by a combination of a financial arbitrage and use of the physical technology. The deduced condition that equilibrium for the three risk-management tools is possible only if $v_f = v_p - v_c$ reflects the fact that violations of the law of one price across markets allow farmers to systematically drive their period 0 costs to $-\infty$, so that no economically meaningful solution to the cost-minimisation problem exists.

However, if $v_f = v_p - v_c$, the forward contract is redundant given the farmer's ability to 'create' a forward contract by taking the offsetting positions in the put and call options. In more familiar terms, the forward contract is in the 'span' of the put and call options.

To further illustrate, consider the case where there exist three states of Nature and two risk-management tools: a riskless asset with price equal to 1 and a forward contract priced at v_1. Recall again the *ex ante* interpretation of the state-contingent technology with inputs committed in period 0. The marginal cost of output in state s, denoted $c_s(\mathbf{w}, \mathbf{z})$ is therefore the resource commitment required in period 0 to raise output by one unit, contingent on the realisation of state s.

Equilibrium requires

$$\frac{c_1(\mathbf{w}, \mathbf{z})}{p_1} + \frac{c_2(\mathbf{w}, \mathbf{z})}{p_2} + \frac{c_3(\mathbf{w}, \mathbf{z})}{p_3} = 1 + r, \text{ and} \tag{51}$$

$$\frac{c_1(\mathbf{w}, \mathbf{z})}{p_1}(q - p_1) + \frac{c_2(\mathbf{w}, \mathbf{z})}{p_2}(q - p_2) + \frac{c_3(\mathbf{w}, \mathbf{z})}{p_3}(q - p_3) = v_1. \tag{52}$$

Suppose that this risk-management structure is enhanced by the creation of an option contract with a strike price of q and a period 0 price of v_2. Expression (51) and (52) are augmented by

$$\frac{c_1(\mathbf{w}, \mathbf{z})}{p_1}\max\{q - p_1, 0\} + \frac{c_2(\mathbf{w}, \mathbf{z})}{p_2}\max\{q - p_2, 0\} + \frac{c_3(\mathbf{w}, \mathbf{z})}{p_3}\max\{q - p_3, 0\} = v_2. \tag{53}$$

So long as

$$\min\{p_1, p_2, p_3\} < q < \max\{p_1, p_2, p_3\}, \tag{54}$$

expressions (51)–(53) can be solved for the optimal marginal costs $c_s(\mathbf{w}, \mathbf{z})$. Thus, any producer facing the same technology and the same risk-management options will choose their state-contingent production decisions to satisfy expressions (51) through (53). These three conditions constitute three equations in three unknowns (the three state-contingent outputs). None of these three conditions depends upon producers' attitudes towards risk either directly through the preference structure, *W*, or indirectly through their choice of $\boldsymbol{y}$. Because these conditions are independent of both *W* and $\boldsymbol{y}$, the equilibrium choice of the three unknowns (the three state-contingent outputs) can be characterised independently of producers' attitudes towards risk and independently of the level of $\boldsymbol{y}$. This manifests what is commonly known as a 'separation result', in the sense that farmers' production decisions are determined independently of their attitudes towards risk. This separation result has been established subject only to monotonicity conditions on farmers' preferences and not subject to any restrictions or assumptions on their risk attitudes. Furthermore, the separation result applies to a production technology where output is stochastically determined.

We close this section by noting that the phenomenon illustrated in expressions (51) through (53) is quite general. Consider, in particular, *S* option contracts written with the strike price of each equalling p_s. As long as there are no price redundancies, then we have *S* assets with the *kth* asset's return in state *s* equalling

$$a_{ks} = \max\{p_s - p_k, 0\}. \tag{55}$$

As is well known, these assets span the state space. If presented with such an asset structure, it follows by an immediate extension of the above arguments that all producers make the same production decisions regardless of their attitudes towards risk.

7. Concluding comments

In the present paper, we have examined the risk-management and production decisions of agricultural producers under minimal conditions on their objective functions. We have shown that a unified and informative treatment of a broad spectrum of risk-management tools is possible within a cost-minimisation framework. The fundamental results that we have obtained apply regardless of the producer's preferences towards risks and ensure that agricultural producers never forego any opportunity to lower costs without lowering returns. Our results encompass a wide range of partial results previously obtained for specific risk-management tools, under restrictive assumptions about sources of risk and the nature of preferences. Although many of these results rely, at least implicitly, on the no-arbitrage principle, the absence of a state-contingent framework obscures the essential logic.

The next stage in the analysis is to combine $C(\boldsymbol{y})$ with the producer's preference structure W in the determination of an optimal $\boldsymbol{y}$. That is, to consider the optimisation problem

$$\max_{\boldsymbol{y}} W(-C(\boldsymbol{y}), \boldsymbol{y}). \tag{56}$$

Problems of this type, with C satisfying a subset of the generic properties detailed in theorem 1, have been studied in detail in Chambers and Quiggin (2000), and the methods developed therein can be used to extend our results to consider the full impact that risk preferences play in determining equilibrium behaviour.

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Appendix

Proof of theorem 1: Continuity follows by the theorem of the maximum (Berge 1963). Let $(h', \boldsymbol{z}')$ be optimal for $\boldsymbol{y}' \geq \boldsymbol{y}$, then C non-decreasing follows because $(h', \boldsymbol{z}')$ is feasible for $\boldsymbol{y}$. To demonstrate convexity, let $(h', \boldsymbol{z}')$ and $(h'', \boldsymbol{z}'')$ be optimal for $\boldsymbol{y}'$ and $\boldsymbol{y}''$ respectively. By the linearity of the constraint sets, $(\lambda h' + (1 - \lambda)h'', \lambda \boldsymbol{z}' + (1 - \lambda)\boldsymbol{z}'')$ is feasible for $\lambda \boldsymbol{y}' + (1 - \lambda)\boldsymbol{y}''$. By the convexity of c,

$$c(\boldsymbol{w}, \lambda \boldsymbol{z}' + (1-\lambda)\boldsymbol{z}'') + v(\lambda h' + (1-\lambda)h'') \leq \lambda[c(\boldsymbol{w}, \boldsymbol{z}') + vh'] + (1-\lambda)[c(\boldsymbol{w}, \boldsymbol{z}'') + vh'']. \tag{A1}$$

Taking the minimum of the left-hand side yields convexity. That $C(\mathbf{0}) \leq 0$ follows from the absence of fixed costs after noting that $(h, \boldsymbol{z}) = (0, \mathbf{0})$ is feasible for $\boldsymbol{y} = \mathbf{0}$. To establish the first equality in the theorem:

$$\begin{aligned} C(\boldsymbol{y} + m\boldsymbol{a}) &= \min_{h,z}\{c(\boldsymbol{w}, \boldsymbol{z}) + vh: h\boldsymbol{a} + \boldsymbol{p} \cdot \boldsymbol{z} \geq \boldsymbol{y} + m\boldsymbol{a}\} \\ &= \min_{h,z}\{c(\boldsymbol{w}, \boldsymbol{z}) + vh: (h - m)\boldsymbol{a} + \boldsymbol{p} \cdot \boldsymbol{z} \geq \boldsymbol{y}\} \\ &= \min_{(h-m),z}\{c(\boldsymbol{w}, \boldsymbol{z}) + v(h - m): (h - m)\boldsymbol{a} + \boldsymbol{p} \cdot \boldsymbol{z} \geq \boldsymbol{y}\} + mv \\ &= C(\boldsymbol{y}) + mv. \end{aligned} \tag{A2}$$

The remaining equalities follow by differentiation as discussed in the text.