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Ambiguous contracts: a syntactic approach*

Abstract

We focus on syntactic aspects of differential awareness that give rise to contractual disputes. Boundedly rational parties use a common language, but do not share a common understanding of the world, leading to ambiguity in both syntactic and semantic forms. In contractual relationships, ambiguity leads to disagreement and disputes. We show that the agents may prefer simpler less ambiguous contracts when facing potential disputes.

JEL Classification: D80, D82

Key words: ambiguity, bounded rationality, expected uncertain utility, incomplete contracts

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1 Introduction

The fact that ambiguous contractual terms can lead to incomplete but nevertheless *ex ante* efficient contracts Mukerji (1998), Mukerji & Tallon (2001), Board & Chung (2007, 2009), Filiz-Ozbay (2010) and Grant, Kline & Quiggin (2012, 2013). In these papers, contracts are modelled as state-contingent acts, with incompleteness arising from the fact that some states may be non-contractible or from state-contingent preferences that are ambiguous, in the technical sense that there exists no well-defined probability distribution over the state space. The language in which contracts are written is either not specified or is derived from the state space.

In this paper, we begin with a syntactic approach, in which the set of contingencies and the set of actions expressible in a common language available to the two parties are taken as primitive. In this approach, a contract is a set of conditional actions, built up using contingencies that can be expressed using the contractual terms available in the common language. We consider contracts between two parties using the same contractual language, but with possibly different interpretations of the contingencies specified in the contract. We define possibility of dispute relations that specify the pairs of contingencies over which the two players might be in dispute.

It is natural for a party to consider the range of outcomes that might arise given the ambiguity he or she perceives to be associated with the range of possible interpretations by the other party. We show how this can give rise to preferences that may be represented by the ε-contamination model commonly used to represent preferences averse to state-contingent ambiguity. Thus, our approach establishes a connection between aversion to syntactic or linguistic ambiguity (the sense in which the term 'ambiguity' is normally found in ordinary usage) and semantic or state-contingent ambiguity (the sense in which the term is commonly used in decision theory).

The paper is organized as follows. In section 2, we set up a formal test-based language in which contracts are specified and derive a representation for preferences in the absence of ambiguity. Next, in Section 3, we develop the concept of contractual ambiguity, and derive preferences over ambiguous contracts using an ε-contamination model. In Section **??** we apply our model to give some results on liquidated damages contracts. In Section 4 we discuss the implications of our analysis and its relationship to the existing literature on incomplete contracts and bounded rationality.

2 Tests, actions and contracts

We consider two parties $i = 1, 2$. Following the approach of Blume et al. (2006), we assume that both players have access to a non-empty set of primitive test propositions $T_0 = \{t_1, ..., t_K\}$ and a set of actions A_0. Let T denote the closure of T_0 under conjunction ($\wedge$) and negation ($\neg$). We use $t \vee t'$ as an abbreviation for $\neg(\neg t \wedge \neg t')$.

For the semantics, we follow Blume et al. (2006) and use a state space that is equivalent to their set of atoms over primitive tests. We set $S^i = S = \{0, 1\}^K$ for $i = 1, 2$, with $|S| = 2^K$.

Hence a state s is a vector of zeroes and ones (a binary number) where the k^{th} component of s denotes the result of test t_k in state s, with 0 (respectively, 1) corresponding to the result of the test is 'not true' (respectively, 'true'). We use $r_k(s)$ to denote the k^{th} component of s.

For convenience we denote by s_0 the state $(0, 0...0)$, by s_1 the state $(0, 0...0, 1)$, and so on up to $s_{[|S|-1]}$ for the state $(1, ..., 1)$.

A test interpretation is a function $\pi : T \to 2^S$, where $\pi(t)$ is the set of states in which the test t is true. The state space $S = \{0, 1\}^K$ induces a test interpretation constructed as follows. For each t_k in T_0, set $\pi(t_k) = \{s \in S : r_k(s) = 1\}$. The test interpretation is then inductively extended to tests in T by the rule: for any $t, t' \in T$, $\pi(t \wedge t') = \pi(t) \cap \pi(t')$, and $\pi(\neg t) = S - \pi(t)$.

Conversely, each state $s \in S$ can be identified with a test $t(s) = t_1(s) \wedge ... \wedge t_K(s) \in T$ defined as follows. For each $k = 1, ..., K$ let:

$$t_k(s) = \begin{cases} t_k \text{ if } r_k(s) = 1; \\ \neg t_k \text{ if } r_k(s) = 0. \end{cases}$$

By construction $\pi(t(s)) = \{s\}$ meaning the test $t(s)$ is satisfied only at the state s.

We are interested in the set of *contracts* C, which are constructed inductively from the set of actions A_0 and the set of tests T by taking the closure under the 'if-then-else' construction. That is, we take each a in A_0 to be a contract, and then we require, for any pair of contracts c and c' and any test t in T, that the program '**if** t **then** c **else** c'' should be a contract in C. This contract requires the parties to follow the course of action as determined by contract c if test t is satisfied and follow the course of action as determined by contract c' otherwise.

For any $a \in A_0$, f_a is the unconditional act $f_a(s) = a$ for all $s \in S$. Fix a pair of contracts c and c' in C with associated state-contingent actions f_c and $f_{c'}$. Then for any test t in T, the state-contingent action associated with the contract $c'' =$ '**if** t **then** c **else** c'' is given by $f_{c''}(s) = f_c(s)$ if $s \in \pi(t)$, and $f_{c''}(s) = f_{c'}(s)$ if $s \notin \pi(t)$. It follows from the inductive construction of the set of contracts above that for each c in C, there is an associated 'state-contingent' act $f_c : S \to A_0$.

Conversely, for a given act $f : S \to A_0$, we can define the associated canonical contract c_f with an exhaustive specification given by

$$\textbf{if } t(s_0) \textbf{ then } f(s_0) \textbf{ else if } t(s_1) \textbf{ then } f(s_1) \textbf{ else ...}$$
$$\textbf{else if } t\left(s_{[|S|-2]}\right) \textbf{ then } f\left(s_{[|S|-2]}\right) \textbf{ else } f\left(s_{[|S|-1]}\right)$$

Consider now the individuals' 'ambiguity-free' preferences defined over the set of contracts C. These should be interpreted as the players' preferences over contracts in the absence of any consideration of possible disputes. That is, these are the preferences each player would have, under the assumptions that the other party has the same understanding of the tests used to specify the contract, and that the contract is implemented according to this shared understanding. In the next section, we consider the possibility of a dispute arising from different interpretations of 'ambiguous' tests.

We assume these preferences admit a representation of the following form: there exists for each state s in S a continuous utility function $u_s^i : A_0 \to \mathbb{R}$, such that the following additively-separable function represents the ambiguity-free preferences of individual i:

$$U^i(c) = \sum_{s \in S} u_s^i(f_c(s)). \tag{1}$$

We show in the Appendix that the additive separability across states embodied in expression (1) arises by requiring the preferences to satisfy (along with some other standard properties) the analog of Savage's sure-thing principle. However, as is well-known (see for example Karni, 1985), unless there is some exogenously given structure on the payoffs and their utility, in this formulation, as far as the "ambiguity-free" preferences represented by $U^i(.)$ are concerned, one cannot separate the probability of the state obtaining from the state-dependent utility. One cannot even determine the level of state-dependent utility. More precisely, it is the only the *change* in the state-dependent utility resulting from a change in the action taken in that state that is determined up to a positive scalar. From expression (1) it follows that if $u_s^i(\cdot)$ is a state-dependent utility can be used for the representation in (1) then so can any function $\tilde{u}_s^i(a) = \alpha\tilde{u}_s^i(a) + \beta_s$, with $\alpha > 0$. But notice that for any pair of actions a and a' and any pair of states s and s', we have:

$$\frac{\tilde{u}_s^i(a) - \tilde{u}_s^i(a')}{\tilde{u}_{s'}^i(a) - \tilde{u}_{s'}^i(a')} = \frac{u_s^i(a) - u_s^i(a')}{u_{s'}^i(a) - u_{s'}^i(a')}.$$

We thus define the following equivalence class for state-dependent utilities.

Definition 1 *The state-dependent utility functions* $(u_s)_{s\in S}$ *and* $(\tilde{u}_s)_{s\in S}$ *are* cardinally equivalent *if there exists a positive scalar* $\alpha > 0$ *and vector of constants* $(\beta_s)_{s\in S}$, *s.t.* $\tilde{u}_s(a) \equiv \alpha u_s(a) + \beta_s$ *for all* s *in* S.

In what follows, we shall restrict attention to individuals whose preferences in the absence of ambiguity admit a state-dependent expected utility representation of the form given in (1). We shall identify such a preference relation by its state-dependent expected utility representation.

Definition 2 *Let* $\mathcal{U}$ *denote the set of* state-dependent expected utility functions *defined on the set of contracts* C *that take the form given in* (1).

3 Introducing Ambiguity

Because we have chosen formally identical state spaces for the players, the test interpretation of each player and the language of each player are identical. The distinction and the source of disputes thus arises from the interaction between syntax and semantics. Disputes arise from the players disagreeing about which tests have been satisfied, or, in a semantic rendition, which state of nature applies. In this section we first introduce ambiguity by way of ambiguous tests and show how this makes some contracts 'ambiguous'. We then develop a model of ambiguity averse decision-makers.

3.1 Conclusive and ambiguous tests and contracts

In this section we introduce the notion of ambiguous tests. This notion will be based on a primitive notion of conclusiveness of a test. The idea of conclusiveness of a test t for an individual i with respect to individual $(3-i)$ is that if she finds herself in a position where she assesses that t is satisfied, then she is sure that individual $(3-i)$ will assess t as satisfied also. The set of conclusive tests for individual i will be denoted by T_C^i. We presume that the individuals are mutually cognizant of T_C^1 and T_C^2. The test t is *unambiguous* if it is conclusive for both individuals. The set of unambiguous tests for individuals 1 and 2 is denoted $T_U = T_C^1 \cap T_C^2$.[1]

To ensure that the sets of conclusive tests match our intuition, we assume that T_C^1 and T_C^2 exhibit the following properties.

Properties of Conclusive Tests: For any pair of tests t and t' in T :

(i) the test $t \vee \neg t$ is in T_C^i (that is, all tautologies are conclusive);

(ii) if the test t is in T_C^i then the test $\neg t$ is in $T_C^{(3-i)}$(that is, the negation $\neg t$ is conclusive for the individual $(3-i)$ with respect to i);

(iii) if the tests t and t' are in T_C^i, then the test $t \vee t'$ is in T_C^i (that is, T_C^i is closed under disjunction);

(iv) if $\pi(t) = \pi(t')$ and the test t is in T_C^i, then the test t' is also in T_C^i (two semantically equivalent propositions are either both conclusive or neither)

The next proposition shows that these properties guarantee that any test satisfied in every state or in no state is unambiguous and also that the set of unambiguous tests is closed under negation and conjunction.

Proposition 1 *Fix T_C^1 and T_C^2. If T_C^1 and T_C^2 satisfy the properties of conclusive tests then for each pair of tests t and t' in T:*

(i) *if $\pi(t) = S$ or $\pi(t) = \varnothing$ then $t \in T_U$;*

[1] In a model with more than two individuals, it would be necessary to use the notation $T_U^{1,2}$, since the set of unambiguous tests is specific to the given pair $(1,2)$. In the two-player model presented here, this is unnecessary and superscripts are dropped for simplicity.

(ii) *if* t,$t' \in T_U$*, then* (a) $\neg t \in T_U$ *and* (b) $t \wedge t' \in T_U$

Proof. (i) First, let $\pi(t) = S$. By property *(i)*, the test $t \vee \neg t$ is in T_C^i for $i = 1, 2$. Since $\pi(t \vee \neg t) = S = \pi(t)$, it follows by property *(iv)* and the definition of an unambiguous test that $t \in T_U$. Next, let $\pi(t) = \varnothing$. Then, $\pi(\neg t) = S$, so as just shown above using properties *(i)* and *(iv)*, the test $\neg t$ is in T_U. Then, by property *(ii)*, the test $\neg\neg t$ is in T_C^i for $i = 1, 2$, and so by the definition of an unambiguous test, the test $\neg\neg t \in T_U$. Noting that $\pi(t) = \pi(\neg\neg t)$, it follows from property *(iv)* that $t \in T_U$.

(ii) Let t,$t' \in T_U$. Then, t,$t' \in T_C^i$ for $i = 1, 2$. (a) Consider $\neg t$. By property *(ii)* and the definition of an unambiguous test, $\neg t \in T_U$; (b) Consider $t \wedge t'$. Observe that $\pi(t \wedge t') = \pi(\neg(\neg t \vee \neg t'))$. By properties *(ii)* and *(iii)* and the definition of an unambiguous test, the test $\neg(\neg t \vee \neg t') \in T_U$. Thus applying property *(iv)*, $t \wedge t' \in T_U$. ■

Given that the two individuals are mutually cognizant of T_C^1 and T_C^2 and that they satisfy the four properties listed above, it follows that for any contract of the form '**if** t **then** a **else** a',' if t is an unambiguous test then both individuals anticipate that they will agree whether or not test t has been satisfied. Thus they will agree whether or not the contract calls for action a or for action a'. Suppose, however, the test is conclusive only for individual i and is not conclusive for individual $(3 - i)$. Then i anticipates that, when she has assessed test t is satisfied individual $(3 - i)$ will that agree the contract calls for action a. However, individual $(3 - i)$ believes when he has assessed test t is satisfied, there may be a disagreement with i about whether the contract calls for action a or a'. Conversely, it follows from property (ii) that individual $(3 - i)$ anticipates that when he has assessed test t is not satisfied, individual i will also have assessed that test t is not satisfied and so will agree that the contract calls for action a'. Individual i, on the other hand, anticipates that when she has assessed that test t is not satisfied there may be a disagreement with individual $(3 - i)$ about whether the contract calls for action a or a'.

We can use the test interpretation to derive the set of unambiguous events.

Definition 3 *The set of* unambiguous events $\mathcal{E}_U \subseteq 2^S$ *is given by:*

$$\mathcal{E}_U = \{E \subseteq S : \pi(t) = S \textit{ for some } t \in T_U\} \ .$$

The set of ambiguous events $\mathcal{E}^A = 2^S - \mathcal{E}_U$.

Lemma 2 *The set of unambiguous events $\mathcal{E}_U$ is an algebra of subsets of S that contains S and $\varnothing$. That is, it is closed under taking complements and intersection.*

Proof. Assertion *(i)* of Proposition 1 implies that $\mathcal{E}_U$ contains S and $\varnothing$. Consider any pair of unambiguous events E and E' in $\mathcal{E}_U$. Since they are unambiguous events, there must exist tests t and t' in T_U, such that $\pi(t) = E$ and $\pi(t') = E'$. Assertion *(ii)* of Proposition 1 states that T_U is closed under negation and conjunction, so the tests $\neg t$ and $t \wedge t'$ are also in T_U. Since $\pi(\neg t) = S - E$ and $\pi(t \wedge t') = E \cap E'$, the events $S - E$ and $E \cap E'$ are unambiguous. ■

For each $s \in S$, and for each individual i, we can derive from the set of unambiguous tests for individual i, the collection of possible states the other individual ((3 − i) may have determined as having obtained as follows.

Definition 4 (*Possibilty of Dispute Set for* i) *Suppose $T_C^i \subset T$, is the set of conclusive tests for individual i. For each s in S, define the* possibility-of-dispute for i *associated with state s to be:*

$$D^i(s) := \{s' \in S : \textit{for each } t \in T_C^i,\ s \in \pi(t) \Rightarrow s' \in \pi(t)\}.$$

By construction, the set $D^i(s)$ comprises those states that cannot be distinguished from s by a conclusive test for i being satisfied. Clearly, $s \in D^i(s)$ for each $s \in S$, so $D^i(s) \neq \varnothing$ for each $s \in \Sigma$. We will refer to $\{D^i(s)\}_{s \in S}$ as the *possibility of disputes for i*.

For each $s \in S$ we can define $E(s)$, the smallest unambiguous event containing s, by

$$E(s) := \bigcap_{E \in \{F \in \mathcal{E}_U : s \in F\}} E.$$

We have the following facts which shows that coarsest common-refinement of $\{D^1(s)\}_{s \in S} \cup \{D^2(s)\}_{s \in S}$ is the finest unambiguous partition of S. More specifically, for each state s, the possibility-of-dispute set for i, $D^i(s)$, is a subset of $E(s)$ with equality, if and only if $D^1(s) = D^2(s)$, and $D^i(s)$ is a singleton if and only if the test $t(s)$ associated with the state s is an conclusive test for i.

Lemma 3 *For each $s \in S$: (a) $D^i(s) \subseteq E(s)$ and $D^1(s) = D^2(s) \Rightarrow D^i(s) = E(s)$; (b) $D^i(s) = \{s\}$ if and only if $t(s) \in T^i_C$.*

Proof. (a) First we show $D^i(s) \subseteq E(s)$. Suppose that $s' \in D^i(s)$, but $s' \notin E(s)$. Observe that $E(s) \neq \varnothing$. Hence, there must be some $E \in \{F \in \mathcal{E}_U : s \in F\}$, and $s' \notin E$. Since $E \in \mathcal{E}_U$, there is a test $t \in T_U$ such that $\pi(t) = E$. Also, $s \in E(s)$. Since $s' \in D^i(s)$, it follows from the definition of $D^i(s)$ that $s' \in \pi(t) = E$, which is a contradiction. Hence, we conclude that $D^i(s) \subseteq E(s)$.

Next we show that $E(s) \subseteq D^i(s)$ whenever $D^1(s) = D^2(s)$. Suppose that $s' \in E(s)$, but $s' \notin D^1(s) = D^2(s)$. Then there is some test $t \in T_U$ such that $s \in \pi(t)$ but $s' \notin \pi(t)$. Then $\pi(t)$ is an unambiguous event containing s but not containing s'. Hence $E(s) \subseteq \pi(t)$,and $s' \notin E(s)$, which is again a contradiction. Hence we conclude that $E(s) \subseteq D^i(s)$.

(b) (If) Clearly, $\{s\} \subseteq D^i(s)$ from the definition of $D^i(s)$. Next, since $t(s) \in T^i_C$ and $\pi(t(s)) = \{s\}$, it follows that if $s' \neq s$, then $s' \notin D^i(s)$, that is, $D^i(s) \subseteq \{s\}$.

(Only-if) Since $D^i(s) = \{s\}$, it follows that for each $s' \neq s$, there is a test $t' \in T^i_U$ such that $s \in \pi(t')$ and $s' \notin \pi(t')$. Since T^i_C is closed under conjunction by assertion *(ii)* of Proposition 1, we can take the conjunction of these tests over $S - \{s\}$ to obtain a conclusive test for i, $t^* \in T^i_C$ that excludes everything but s, that is, $\pi(t^*) = \{s\}$. Since $\pi(t(s)) = \{s\} = \pi(t^*)$, it follows from property (iv) of the conclusive test set T^i_C that $t(s) \in T^i_C$. ■

Notice that if a contract is measurable with respect to the *unambiguous partition*, $\{E^i(s)\}_{s \in S}$ although the individuals might disagree about the actual state that has obtained, they will never disagree about which action the contract prescribes. Hence such contracts are viewed as *unambiguous.*

Definition 5 *A contract is* unambiguous *if for all for all $s, s' \in S$, $E(s) = E(s') \Rightarrow f_c(s) = f_c(s')$. We denote by C_U the set of unambiguous contracts.*

3.2 Preferences under ambiguity

We now develop a model the effects of ambiguity has on preferences over contracts. Consider an individual i whose preferences over contracts, in the absence of ambiguity, admit a representation $U^i \in \mathcal{U}$. When individual i believes that the state is s, she considers it possible that the other party may believe any element of $D^i(s)$ has obtained. Hence in terms of a given contract c, this possibility of dispute generates *ambiguity* about the action that will actually be implemented. Depending upon which interpretation is followed, the action might conceivably be any member of the set $\{f_c(s') : s' \in D^i(s)\}$.

We assume that individuals anticipate that a dispute will lead to a 'war of attrition' game in which each player's equilibrium payoff is equal to their security, in this case, the outcome associated with the other player's interpretation.[2] That is, if player i sees state s and $(3-i)$ sees s' then player i's expected payoff in the war-of-attrition equilibrium is $\min\left\{u_s^i(f_c(s)), u_s^i(f_c(s'))\right\}$. If the dispute set $D^i(s)$ contains only two elements, then the player can evaluate the result of a dispute directly.

More generally, given that disputes are resolved by a war of attrition, individual i can do no worse than accept the least favorable action implied by the contract in the set of possible interpretations of the tests by $(3-i)$ at s, that is, in the set $\{f_c(s') : s' \in D^i(s)\}$.

Hence, one possible way to model the potential loss from a dispute and the resulting war of attrition is to assign a decision weight to this worst-case outcome. This reasoning corresponds to one of the most commonly applied models of ambiguity averse preferences, the ε-contamination model.[3]

If we let ε_s^i be the decision-weight she assigns to the ambiguity she faces in state s, then her ε-contaminated subjective expected utility $V_s^i(c)$ of contract c in state s is given by

$$\left(1-\varepsilon_s^i\right) u_s^i(f_c(s)) + \varepsilon_s^i \min_{s' \in D^i(s)} u_s^i(f_c(s')). \tag{2}$$

In what follows, we refer to an increase in ε_s^i as a greater aversion of i to ambiguity in state s. We let $V^i(c) = \sum_{s \in S} V_s^i(c)$ denote her *ex ante* expected utility from contract c. A contract c is *ex ante efficient* if there is no other contract c' such that $V^i(c') \geq V^i(c)$ for $i = 1, 2$, with a strict inequality for some i.

Depending on the degree of concavity of the state-dependent utility functions $u_s^i s$ compared to the decision-weights $\varepsilon_s^i s$, the ambiguity may lead players to prefer incomplete risk sharing to

[2] We thank Roger Myerson for the suggestion that disputes could be modelled as wars of attrition.

[3] The approach here can viewed as a state-dependent extension of Kopylov (2008).

possibly ambiguous contracts.

4 Concluding comments

We have provided a formal model for incorporating linguistic ambiguity into decision making. The ambiguity in our model arises from the bounded rationality of the players which is expressed as limited abilities to perform tests over the possible contingencies. As a result, players have limited descriptions of the possible states of the world available to them. Even when they use the same language, their interpretations may differ.

Contracting is modelled using a multi-player version of the test-based contingent plans described in Blume et al. (2006). In this context, ambiguity can affect incentives for risk sharing, and the desirability of contracts.

The representation of ambiguity proposed here suggests new approaches to a range of issues in contract theory, and potentially broader applications in agency theory. The standard principal-agent problem is one where contracting is limited to some observable unambiguous characteristics like output, rather than a full set of characteristics including effort levels which may be ambiguous. The framework developed here suggests the possibility of an endogenous choice between contracts over different characteristics, where the choice of the contractual variables chosen depends on the level of ambiguity and potential gains from risk sharing. While this application would require overcoming some new technical details involving the appropriate treatment of tests, the benefit would be the development of a theory of contracting in which the terms of the contract, over which the parties actually bargain, plays the central role.

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A Axiomatization of state-dependent additively-separable ambiguity-free preferences.

Let $\succsim^i$ denote individual i's (dispute-free) preferences over contracts. Consider the following three axioms.

Ordering Axiom The relation $\succsim^i$ is complete and transitive.

Act-equivalence Axiom For any pair of contracts, c and c' in C, if $f_c = f_{c'}$ then $c \sim c'$.

***Sure-thing Principle*:** For any four contracts c, c', c'' and c''' in C, and any test t in T,

$$\begin{aligned} & \textbf{if } t \textbf{ then } c \textbf{ else } c'' \succsim^i \textbf{if } t \textbf{ then } c' \textbf{ else } c'' \\ \Rightarrow \quad & \textbf{if } t \textbf{ then } c \textbf{ else } c''' \succsim^i \textbf{if } t \textbf{ then } c' \textbf{ else } c'''. \end{aligned}$$

The first axiom is the standard ordering axiom. The second requires any two contracts that induce the same act over actions must come from the same indifference class. This seems natural in a setting in which we assume the agent understands the language in which contracts are written and the logical implications of its terms and attendant requirements. The third axiom is the analog of Savage's *sure-thing principle.*

The fourth axiom is a continuity assumption to ensure a numerical representation of preferences exists. Before stating it, we need to define what it means for a sequence of contracts to converge to a limit. We do this inductively. First, we define the notion of convergence for constant acts directly from the notion of convergence of actions in the set A_0, and then we extend it inductively to all contracts via the 'if..then..else' construction.

Definition 6 (Convergence of Sequences of Contracts) *The (countably infinite) sequence of constant acts $\langle a_n \rangle$ converges to the constant act $\bar{a}$, if the corresponding sequence of actions converge to the corresponding action, that is, $\lim_{n\to\infty} a_n = \bar{a}$. For any sequence of tests $\langle t_n \rangle$ and any pair of sequences of contracts $\langle c_n \rangle$ and $\langle c'_n \rangle$, the sequence of contracts $\langle c''_n \rangle$, where $c''_n =$ '**if** t_n **then** c_n **else** c'_n' is said to converge to $\bar{c}'' =$ **if** $\bar{t}$ **then** $\bar{c}$ **else** $\bar{c}'$, if $\langle c_n \rangle$ and $\langle c'_n \rangle$ converge to $\bar{c}$ and $\bar{c}'$, respectively, and there exists finite m, such that $t_n = \bar{t}$ for all $n > m$.*

Continuity of preference can now be expressed in the standard manner of requiring that there are no 'jumps in preference at infinity'.

***Continuity*:** For any pair of sequences of contracts $\langle c_n \rangle$ and $\langle c'_n \rangle$, that converge to $\bar{c}$ and $\bar{c}'$, respectively, if $c_n \succsim^i c'_n$ for all n, then $\bar{c} \succsim^i \bar{c}'$

Finally, we require a minimum amount of non-degeneracy for the preferences with respect to the states in S. Formally, we require at least three states to be 'essential'.

Definition 7 *Fix $\succsim^i$. A state s in S is essential for $\succsim^i$ if there exists a pair of actions a and a' in A_0 and a contract c in C, such that*

$$[\textbf{\textit{if}}\ t(s)\ \textbf{\textit{then}}\ a\ \textbf{\textit{else}}\ c] \succ^i [\textbf{\textit{if}}\ t(s)\ \textbf{\textit{then}}\ a'\ \textbf{\textit{else}}\ c].$$

We now have all the pieces for the representation result.

Theorem 4 (State-Dependent Expected Utility Representation) *Fix $\succsim^i$. If there are at least three essential states then the following are equivalent.*

1. *The relation $\succsim^i$ satisfies ordering, act-equivalence, sure-thing principle and continuity.*
2. *There exists for each state s in S a continuous utility function $u^i_s : A_0 \to \mathbb{R}$, such that the following additively-separable function represents $\succsim^i$:*

$$U^i(c) = \sum_{s \in S} u^i_s(f_c(s)) \tag{3}$$

Moreover, the functions $u^i_s(\cdot)$ are unique up to multiplication by a common positive scalar $\alpha > 0$, and the addition of a state-dependent constant β_s.

Proof. Sufficiency of axioms. Consider the preference relation $\succsim^i_{\mathrm{F}} \subset A_0^{|S|} \times A_0^{|S|}$ over acts, induced by $\succsim^i$: $c \succsim^i c'$ implies $f_c \succsim_{\mathrm{F}} f_{c'}$. Consider a pair of acts, $f \succsim^i_{\mathrm{F}} f'$. By construction, there exists a pair of contracts c and c' such that $f_c = f$, $f_{c'} = f'$ and $c \succsim^i c'$. Now for any pair of contracts $\hat{c}$ and $\hat{c}'$, such that $f_{\hat{c}} = f$ and $f_{\hat{c}'} = f'$, it follows from act-equivalence that $\hat{c} \sim c$ and $\hat{c}' \sim c'$, and so by ordering we have $\hat{c} \succsim^i \hat{c}'$. Hence it is enough to obtain a representation $\hat{U}^i(f)$ of $\succsim^i_{\mathrm{F}}$, since we can then set $U^i(c) := \hat{U}^i(f_c)$.

It is straightforward to show that continuity of $\succsim^i$ implies that $\succsim^i_{\mathrm{F}}$ is continuous in the product topology of $A_0^{|S|}$; and that the sure-thing principle for $\succsim^i$ implies that $\succsim^i_{\mathrm{F}}$ satisfies the sure-thing principle for acts: that is, for any four acts f, f' f'' and f''', and any event $E \subset S$, if $f(s) = f''(s)$ and $f'(s) = f'''(s)$ for all $s \in E$, and $f(s) = f'(s)$ and $f''(s) = f'''(s)$ for all $s \notin E$ then $f \succsim^i_{\mathrm{F}} f'$ implies $f'' \succsim^i_{\mathrm{F}} f'''$. Hence by Debreu (1960, Theorem 3) it follows there exists an additive representation for $\succsim^i_{\mathrm{F}}$ as given in (3). Proof of necessity of axioms is omitted. ■

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