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Ordinal, nonlinear context-dependence: uncountably many alternatives

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Ordinal, nonlinear context-dependence: uncountably many alternatives

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Abstract

Classical models of ordinal utility, such as those of Debreu and Eilenberg, are context-free in the sense that a single preference relation is primitive. A common theme of behavioural economics is to employ context-dependence as a way of characterising observed behaviour in decision problems and games. Prospect theory, case-based decision theory and decisions with unawareness are all examples. An axiomatic model of linear context dependence, with a cardinal utility function for each context, has been developed relatively recently in Gilboa and Schmeidler's theory of case-based decisions. The present paper seeks to address the need for a corresponding ordinal, nonlinear theory of context-dependent utility.

Keywords
Context-dependence, Utility theory

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2.1 Preferences

The primitives of the model consist of two nonempty sets A and X. Let X denote a set of possible contexts or situations at which the decision maker, Val, might face the problem of deciding amongst certain alternatives. To simplify the exposition, the set A of alternatives is assumed to be the same at each context. In the present, general setting, the question of whether Val is aware of X or her whereabouts in X is left unspecified, but it is assumed that once a context is fixed, some form of ranking of the alternatives according to what she prefers is feasible. That is, for a given context x, and alternatives a and b, she will be able to state whether or not she "strictly prefers" a to b.

Thus for each x in X, Val's preferences are described by a *(context) preference relation* $\succ_x$ which formally is a subset of $A \times A$.[1] This gives rise to a collection of preference relations $\{\succ_x : x \in X\}$, so that the variation of preference, for one alternative *over* another, *across* contexts is explicitly modelled. Where necessary, the more expressive notation $\{(A, \succ_x) : x \in X\}$ is used instead, and brevity favours $\{\succ_x\}_X$ when its status is unambiguous. The term *context preferences* will also refer to this collection of individual context preference relations.[2]

The situation where, for a given context x and pair of alternatives a and b, Val's preferences are such that neither $a \succ_x b$, nor $b \succ_x a$, is denoted by $a \sim_x b$. This situation could just as well be described by $b \sim_x a$. Thus the relation $\sim_x$ is symmetric, and given standard conditions, which are stated below, it is an 'equivalence' relation that characterises indifference between alternatives.

As discussed in the introduction, context preferences exclude any preference statements that Val may in fact be in a position to make regarding pairs of contexts, or indeed between one alternative-context pair and another. This information is intentionally ignored so that no assumption need be made concerning preferences over such objects. n some cases, more can be said

[1] I choose this approach, where strict as opposed to a weak preference relation $\succ_x$ is primitive, because strict preference is unambiguous in its meaning. It is adopted in standard texts such as Fishburn Fishburn (1979) and Kreps Kreps (1988), and convincingly motivated by Adams Adams (1965).

[2] The term context preferences is chosen due to similar terminology being used when state or time preferences are modelled.

about the .[3] Perhaps the best way to understand the approach is in terms of multiple epistemological viewpoints, each pertaining to a context: there are no hidden independence assumptions, indeed the intention is to say something about how preferences vary across contexts

2.2 Axioms and the Context space

Recall that, for each x in X, we have defined $\sim_x$ so that $a \sim_x b$ if and only if strict preference fails to hold in either direction. As a result, the standard assumption of completeness of the weak preference relation $\gtrsim_x := \succ_x \cup \sim_x$ is automatically satisfied, and in this case the following condition is simply the contrapositive of "transitivity of weak preference".[4]

Axiom (Ordering of alternatives (Ord.)).
For all a, b and c in A and x in X, if $a \succ_x c$, then $a \succ_x b$ or $b \succ_x c$.

Axiom (Continuity across contexts (Cac)).
For all a and b in A, and x in X, if $a \succ_x b$, then there exists an open set of contexts O such that $x \in O$, and for every y in O, we have $a \succ_y b$.

Note that continuity has the intuitive appeal that it characterizes the stability of strict preferences. That is stability with respect to perturbations in the context space.

If we were to translate the family of preference relations Γ into a single, incomplete preference relation $\gtrsim^*$ using the following definition:

$$(a, x) \gtrsim^* (b, y) \quad \Leftrightarrow \quad a \gtrsim_x b \text{ and } x = y,$$

then by theorem 2 of Evren and Ok (2011) conditions (Ord.) and (Coa) are sufficient for a multi-utility representation. However, in for the purposes of a characterizing context-dependent preferences, that is not enough, we are interested in a rather special function. For each context, we seek a distinct utility function that characterizes a preference relation at that context. Moreover, as the context varies, the function we obtain should preserve the properties that preferences satisfy as the context varies.

Finally, I introduce an axiom that is not a necessary condition for the representations that are obtained, but it is needed in the sufficiency proofs provided.

[3] In the language of measurement theory (see d'Aspremont and Gevers (2002) for a recent survey) context preferences are low in the information hierarchy.

[4] Recall, completeness says that for all a, b and x, either a is weakly preferred to b at x or vice versa; whereas transitivity says that $a \gtrsim_x b \gtrsim_x c$ implies $a \gtrsim_x c$.

Axiom (Jaffray order-separability across contexts (Sep.)).
There exists a countable subset C of A such that for all alternatives a and b in A with $a \succ_x b$, we have

$$a \succsim_x c \succ_x d \succsim_x b \quad \text{for some } c \text{ and } d \text{ in } C$$

The main theorem holds for topological context spaces that are *perfectly normal.* A space X is said to be *normal* if for every pair of disjoint closed subsets of X there exist disjoint open sets containing A and B respectively. Then a topological space X is perfectly normal if for every set C that is closed in X, there exists a real-valued function f such that $C = f^{-1}(0)$. An equivalent definition is that every such C can be written as a countable intersection of sets that are open in X (Munkres (2000) p.229). The following theorem is a recent, intuitive restatement of Michael's Michael (1956) selection theorem due to Good and Stares (2000). It provides a useful characterisation of perfectly normal topogical spaces.

Theorem 2.1 (Michael's selection theorem). *The following two statements are equivalent.*

1) *X is a perfectly normal topological space.*

2) *If $g, h : X \to \mathbb{R}$ are upper and lower semi-continuous respectively and $g \leqslant h$, then there is a continuous $f : X \to \mathbb{R}$ such that $g \leqslant f \leqslant h$ and $g(x) < f(x) < h(x)$ whenever $g(x) < h(x)$.*

This equivalence is relevant for preferences that are indexed by elements of a context space because if the context space is not perfectly normal then *there exist* $g, h : X \to \mathbb{R}$ that are upper (resp. lower) semi-continuous and $g \leqslant h$, such that *for no* continuous $f : X \to \mathbb{R}$ do we have $g \leqslant f \leqslant h$ and $g(x) < f(x) < h(x)$ whenever $g(x) < h(x)$. This, as we will see in the proof of our theorem and subsequent discussion, would imply that there exists context preferences $\{(A, \succ_x) : x \in X\}$ that are continuous across contexts such that there is no representation $U : A \times X \to \mathbb{R}$ satisfying continuity across contexts.

Note that for countable S the simplex of probability measures Δ on S is a subspace of $\mathbb{R}^S$, which is a 'Hilbert space' under the standard Euclidean metric, so that by Steen and Seebach (1970) p.65, Δ is a metric space. Then by Munkres (2000) p. 229 every metrizable space is perfectly normal. Thus for countable S, Δ is a suitable context space. This is not true for uncountable S, by counterexample 105, on p125 of Steen and Seebach (1970).

Another example of a context space in the literature is any countable product of the discrete space of non-negative integers with the Cartesian product topology as is found in the case-based decision theory of Gilboa and Schmeidler Gilboa and Schmeidler (2001, 2003). By Steen and Seebach (1970) p.121, this is a complete metric space, and so this too is a suitable space of contexts for a nonlinear representation. On the other hand, uncountable products of the nonnegative integers with the product topology are, by counterexample 103 of Steen and Seebach (ibid.), not normal spaces, and so they may be unsuitable depending on preferences.

2.3 Results

The representation we are seeking is of the following form.

Definition 2.1.
$\mathcal{U} : A \times X \to \mathbb{R}$ *is said to be a* context utility representation *of preferences* $\{(A, \succ_x) : x \in X\}$ *if for all* $a, b \in A$ *and* $x \in X$,

$$a \succ_x b \quad \Leftrightarrow \quad \mathcal{U}(a, x) > \mathcal{U}(b, x) .$$

$\mathcal{U}$ *is ordinal if, for any other representation* V *of preferences, there exists a family of strictly increasing functions* $\{f_x : \mathbb{R} \to \mathbb{R}\}_{x \in X}$ *such that for each* x, $\mathcal{V}(\cdot, x) = f_x \circ \mathcal{U}(\cdot, x)$.

Definition 2.2. *[Continuity of the representation]*
A context utility function $\mathcal{U} : A \times X \to \mathbb{R}$ *is said to be*

1) *"continuous across contexts" if, for each* a *in* A, $\mathcal{U}(a, \cdot)$ *is continuous on* X*; and*

2) *"continuous over alternatives" if, for each* x *in* X, $\mathcal{U}(\cdot, x)$ *is continuous on* A.

$\mathcal{U}$ *is "separately continuous" on* $A \times X$ *if both (1) and (2) hold.*

Remark 2.1. *Note that in definition (2.1), the continuity of* $f : X \times \mathbb{R} \to \mathbb{R}$ *on* X *is implied by continuity across contexts of the functions* $\mathcal{U}$ *and* $\mathcal{V}$.

I now state and prove the main theorem, which via Jaffray (1975), is equivalent to Birkhoff (1948) for the case where X is a singleton.

Theorem 2.2. *Let* A *be a nonempty set of alternatives and let* X *be a nonempty perfectly normal topological space of contexts. Then (1) implies (2), where:*

1) context preferences $\{(A, \succ_x) : x \in X\}$ *satisfy (Ord.), (Cac) and (Sep.);*

2) context preferences have an ordinal context utility representation that is continuous across contexts.

The proof of this theorem proceeds via the following steps:
I. Obtain a Cac representation for countable A.
II. Using the representation in (I) to construct a representation for general (uncountable) A that is upper semicontinuous across contexts.
III. Delete the discontinuities of the representation in (II) using a uniform convergence method.

Proof. Part I of the proof follows from theorem 2.2 of O'Callaghan (2013). Parts II and III now follow.
Part II
Let $C := \{c_1, c_2, c_3, \dots\}$ be the countable subset of A satisfying condition (Sep.). By theorem 2.2 of O'Callaghan (ibid.) there exists a function $\mathcal{V} : C \times X \to (0, 1)$ such that for all i, j in $\mathbb{N}$ and x in X,

$$c_i \succ c_j \quad \Leftrightarrow \quad \mathcal{V}(c_i, x) > \mathcal{V}(c_j, x)$$

For each $x \in X$, and $i \in \mathbb{N}$, let $D_i(x) := \{a \in A : a \succsim_x c_i\}$, and let

$$f_i(a, x) := \chi_{D_i(x)}(a)\mathcal{V}(c_i, x),$$

where for each $D \subset A$, χ_D is the indicator function of the set D. That is, $\chi_D(a) = 1$ if $a \in D$, and 0 otherwise.
For each $a \in A$ and x in X, let

$$g(a, x) = \sum_{i \in \mathbb{N}} 2^{-i} f_i(a, x).$$

Introduce the following steps

Lemma 2.1. *For each a in A, $g(a, \cdot) : X \to \mathbb{R}$ is upper semicontinuous (henceforth usc).*

Proof of lemma 2.1. Suppose that $g(a, \cdot)$ is not usc at x'. Then

$$l = \limsup_{y \to x'} g(a, y) > g(a, x') = k,$$

so fix $0 < \epsilon < l - k$. Note that by definition,

$$\{x : \chi_{D_i(x)}(a) = 1\} = \{x : a \succsim_x c_i\}$$

and by condition (Cac), these are closed sets. Then since the finite sum of usc functions on X is also usc on X, the partial sum

$$g_n(a,\cdot) = \sum_{i \leqslant n} 2^{-i} f_i(a,\cdot)$$

is usc for each n. By virtue of the fact that $g_n(a,\cdot)$ is increasing in n and $\lim_n g_n(a,x') = g(a,x') = k$, we have $g_n(a,x') \leqslant k$ for each n. By usc, the set

$$K_n = \{y : g_n(a,y) \geqslant k + \epsilon/2\}$$

is closed, and its complement, $X \backslash K_n$ is open and contains x' for each n. Note that for all y in $X \backslash K_n$, $g_n(a,y) < k + \epsilon/2$. Thus,
Consider the tail sum $\tau_n(a,\cdot) := g(a,\cdot) - g_n(a,\cdot)$. Since $f_i(a,\cdot)$ takes values in $[0,1]$

$$\sup_{x \in X} \tau_n(a,x) \leqslant \sum_{i>n} 2^{-i} = \sum_{i \geqslant 0} 2^{-i} - \sum_{i \leqslant n} 2^{-i} = \frac{1}{1-2^{-1}} - (2 - 2^{-n}) = 2^{-n}.$$

Thus, there exists $n' \in \mathbb{N}$ such that $\tau_n(a,\cdot)$ is uniformly dominated on X by $\epsilon/2$ for all $n \geqslant n'$.
The proof of the lemma then follows by taking any $n \geqslant n'$ and noting that for all $y \in X \backslash K_n$,

$$g(a,y) = g_n(a,y) + \tau_n(a,y) \leqslant k + \epsilon < l,$$

so that $\limsup_{y \to x'} g(a,y)$ cannot equal l, which is the desired contradiction.
□

By Mehta (1998) and $g(\cdot,x) : A \to \mathbb{R}$ is a utility representation of $\succ_x$. Next we need to show that

Lemma 2.2. $g(a,x) > g(b,x)$ *implies that* $\liminf_{y \to x} g(a,y) \geqslant \limsup_{y \to x} g(b,y) = g(b,x)$.

Proof of lemma 2.2. Take any a, b and x such that $a \succ_x b$. Note that

$$l := g(b,x) \equiv \sum \{f_i(b,x) : b \gtrsim_x c_i\}.$$

By (NT) for all $i \in \mathbb{N}$ such that $b \gtrsim_x c_i$, there exists O_i, open in X, such that $x \in O_i$ and for all $y \in O_i$, $a \succ_y c_i$. Since a finite intersection of open sets is open, $\liminf_{y \to x} g(a,y) \geqslant l$ whenever there are only finitely many i such that $b \gtrsim_x c_i$. Suppose there are infinitely many such i. By (Sep.), there exists m and n such that $a \gtrsim_x c_m \succ_x c_n \gtrsim_x b$. In the worst case, $a = c_m$ and $b = c_n$, and (c_n, c_m) is an order gap (an empty "interval" in

$(A, \succ_x)$). But even then, since $c_m \succ_x c_n$, there exists O', containing x, such that $f_m(c_m, y) = 1 > 0 = f_m(c_n, y)$ for all $y \in O'$. By the argument for the finite case, for each $k \geqslant m$ the partial sums satisfy

$$\liminf_{y \to x} g_k(a, y) > g_k(b, x) =: l_k,$$

where $\{l_k\}$ is a monotone sequence converging from below to l. By completeness of the reals, and the fact that for all k

$$\liminf_{y \to x} g(a, y) \geqslant \liminf_{y \to x} g_k(a, y),$$

we have the desired inequality. □

Lemma 2.2 shows that for each a, the set

$$\{r \in \mathbb{R} : \liminf_{x \to y} g(a, y) < r < \limsup_{x \to y} g(a, y) = ga, x)\}$$

is either an open subset of some Debreu-gap in the set $g(A, x)$, or it is empty. This property is crucial for Part III of the proof that now follows.
Part III
First, by *** of Choquet () that the oscillation σ of $g(a, \cdot)$ is usc. This implies that the set

$$\{x : \sigma(a, x) \geqslant k\}$$

is closed. This together with the fact that X is perfectly normal allows us to define a function that vanishes precisely on such sets.

□

3 Motivation

Joint continuity is a condition on the topology preferences themselves, it is not an observable, in contrast with X. In most examples, the modeller will be able to check for herself whether or not X is perfectly normal and decide when it seems reasonable to assume (Cac) and (Sep.) prior to beginning an experiment, say. The joint continuity model is rather different in this respect, for it requires that the modeller make assumptions about . As a result, continuity across contexts, which applies directly to the space X, Example showing that one cannot approximate the lower envelope relative to a with lower semi-continuous functions in the transfinite case.

4 Conclusion

there

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