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Authors:

Han Bleichrodt & John Quiggin

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Capabilities as menus: A non-welfarist basis for QALY evaluation

HAN BLEICHRODT
Erasmus School of Economics & iMTA/iBMG, Erasmus University Rotterdam, the Netherlands

JOHN QUIGGIN
School of Economics, The University of Queensland, Brisbane, Australia

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Abstract
Quality-Adjusted Life Years (QALYs) are the most widely-used measure of health in cost–effectiveness analysis and cost–benefit analysis. Within a welfarist framework QALYs are consistent with people's preferences under stringent assumptions. Several authors have argued that QALYs are a valid measure of health within an extra-welfarist framework. This paper studies the applicability of QALYs within the best-known extra-welfarist framework, Sen's capability approach. We propose a procedure to value capability sets and provide a foundation for QALYs within Sen's capability approach. We show that, under appropriate conditions, the ranking of capabilities can be represented locally by a QALY measure and that a shadow price for QALYs can be defined.

KEY WORDS: Capability approach, QALYs, Menu-dependent choice.
JEL CLASSIFICATION: D63, D70, I14

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1. Introduction

The provision and funding of health care is one of the most important responsibilities of governments in modern societies. In the absence of effective markets for most health care services, it is necessary to make decisions regarding the amount that should be spent on health care services, and the allocation of those services between individuals and across different kinds of services.

Over recent decades, attempts have been made in many countries to improve the allocation of health care resources using the concepts of cost–effectiveness analysis and cost–benefit analysis that have long been applied in the assessment of public projects in fields such as the provision of infrastructure. The goal of cost–effectiveness analysis (CEA) is to allocate a given quantity of health care resources so as to achieve the maximum feasible benefit, as assessed by some objective such as a social welfare function. Cost–benefit analysis (CBA) incorporates cost–effectiveness analysis and has the further goal of determining whether the benefits of providing a given health care service are of greater value than the opportunity cost of the resources that must be foregone in order to provide the service

The central tool in this approach has been the concept of Quality Adjusted Life Years (QALYs) (Gold et al. 1996). The idea underlying QALYs is to value improvements in health care outcomes in terms of the additional years of life in good health that would be regarded by the recipients as being equally beneficial. The concept of QALYs has intuitive appeal and provides practical guidance in making the difficult trade-offs that are inevitable in any system of health care provision where a government or insurer must decide which services to fund. However, attempts to provide a secure foundation for the QALY approach in terms of economic theories of welfare, which we review in the next subsection, have proved problematic. In particular, the conditions under which CEA and CBA are consistent are stringent and unlikely to hold (Bleichrodt and Quiggin 1999, Bleichrodt and Quiggin 2002, Klose 2002).

A different justification for using QALYs in health policy is based on a questioning of the notion of 'welfarism', which underlies CBA (Culyer 1989, Hurley 2000, Brouwer et al. 2008). In this context, 'welfarism' may be summed up as the idea that policy should aim at maximizing a social welfare function, the only arguments of which are the lifetime utility profiles of the individual members of society. According to the extra-welfarist approach advocated by Culyer and co-authors, social welfare should not be exclusively based on individual utilities but should also take account of other aspects of social welfare.

The best developed alternative to welfarism has been based on the concept of capabilities, put forward by Sen (1985) and further developed by Nussbaum (2000), and Robeyns (2006) among many others. Although the details of the formulation vary, the crucial idea is to distinguish between capabilities, represented as a person's opportunities to achieve well-being, and achieved functionings, the actual outcomes realized by individuals given their capabilities, preferences and social situation.

While the capability approach has been much discussed, there has been less progress in formal theoretical development and empirical application of the approach (Schokkaert 2009). Kuklys and Robeyns (2005) and Fleurbaey (2005) presented formal models, but subsequent development of these models has been limited. Anand (2005) and Cookson (2005) argue that the capability approach should be applied in health economics and could be integrated with a QALY-based analysis. Coast, Smith, and Lorgelly (2008) give a qualified endorsement to this view, but note that the capability approach will require adaptation to the health context.

The aim of this paper is to explore the extent to which a QALY-based evaluation of health services can be used within the capability approach. That is, we study the question of whether QALYs can be justified under a specific extra-welfarist framework. To do so, we must first address an open problem in the capability approach (Schokkaert 2009, Fleurbaey 2009), namely, the question of how capabilities can be valued.

The central idea in our formalization of the capability approach, following suggestions of Sen (1991) and Sugden (1993) is to model capabilities as menus, in the analytical framework developed by Kreps (1979) and extended by Dekel, Lipman, and Rustichini (2001). A similar approach may be found in the literature on valuing freedom deriving largely from the work of Pattanaik and Xu (1990, 2000) and further developed by Puppe (1996), Nehring and Puppe (1999), and Xu (2003). The primary focus of this literature is on the derivation of representation theorems for the evaluation of choice sets, based on a variety of axiomatic conditions. In this paper, by contrast, we seek to apply these results to the evaluation of health status and the allocation of resources to health care.

In the language of the capability approach, we model the capability approach as a two-stage process. In the first stage, a menu, that is, a capability set of possible bundles of functionings, is selected. In the second stage, the individual decision maker selects a functioning from the menu. This modelling strategy permits us to capture an essential part of Sen's capability theory, namely, that people have the freedom to choose the kind of lives they want to lead. Freedom of choice is intrinsically valuable in Sen's theory and our model allows it to be valued as such.

The key contribution of Kreps was to show that, under very weak conditions, any preference ranking over menus may be derived (non-uniquely) from the maximization of a state-contingent utility function over final outcomes, where the 'state' may be taken to encompass the uncertainty the individual perceives about his future preferences. Using this idea, we show that any choice of capabilities may be represented in terms of contingent preferences over final functionings.

We consider the case of an individual with a given initial position determined by genetic endowment, family background, and so on. This individual is representative for some class of individuals with similar initial positions. The individual's capabilities are represented as a set of possible functionings, each of which is itself a vector incorporating consumption of market goods and services along with a range of non-market activities (Schokkaert 2009). The analysis for the optimal capability for the representative individual may be taken as determining the best available capability set of possible functionings. The socially optimal outcome is one in which all individuals have the same capabilities, but, contingent on the realization of individual preferences, make different choices and realize different functionings.

Using this approach, we proceed to consider the relationship between capabilities and QALYs. We derive conditions under which a QALY representation is a local approximation to a ranking over capabilities. Our central result shows that, under appropriate conditions, any ranking of capabilities gives rise to a 'shadow price' for QALYs which is locally consistent with any representation of the given ranking derived from contingent preferences over realised functionings and which can be interpreted as a local approximation of the willingness to pay for a QALY.

In what follows, Section 2 reviews the limitations of QALYs under welfarism. Section 3 describes the basic ideas underlying Sen's capability approach. Section 4 addresses the question how capabilities can be valued. Section 5 provides some formal background for the theory of valuing capabilities that is presented in Section 6. Our model implies that the valuation of capabilities is a two-stage problems where the optimal capability set is selected in the first stage and the optimal level of functionings is chosen in the second-stage. Section 7 shows that the solutions to these stages are well-defined. Section 8 analyzes the role of QALYs within the general framework developed in the previous Sections and shows that, within this framework, the ranking of capabilities can be locally represented by a QALY measure. Section 9 concludes the paper.

2. The limitations of QALYs under welfarism

Before considering the interpretation of health in terms of capabilities, we briefly reconsider the standard QALY approach. The central idea is to value improvements in health in terms of the additional years of life in good health that would be regarded by the recipients as being equally beneficial and to add these improvements across recipients using unweighted summation. Cost–effectiveness then requires that, given two equally costly services, the one that provides the larger improvement in QALYs should be provided first. Given a consistent application of cost–effectiveness analysis to health services with a given budget, there will exist a shadow QALY price p^* (typical values are of the order of €30,000) such that all and only services for which the cost per QALY gained is less than p^* will be approved.

Cost–benefit analysis requires that the benefits of health services should be compared to the consumption opportunities foregone to provide those services. Under a complete cost–benefit analysis, the shadow price p^* should be equal to the marginal social benefit of an additional QALY.

The QALY approach is popular and widely used in practical cost–effectiveness studies, but attempts to found QALYs in economic welfare theory have met with limited success. Early approaches that sought to provide an economic foundation for QALYs involved reasoning based on the expected utility (EU) theory of choice under uncertainty. Axioms on preferences over lotteries, with health profiles as payoffs, were used to derive QALY-maximization as a social objective (Pliskin, Shepard, and Weinstein 1980, Bleichrodt, Wakker, and Johannesson 1997, Miyamoto et al. 1998). This approach encountered a number of difficulties.

First, it became evident that individuals do not, in general, satisfy the assumptions of EU in making choices under uncertainty, even when the payoffs are simple monetary prizes (Starmer 2000). Evidence of violations of EU for health can be found, among others in Llewellyn-Thomas et al. (1982), Oliver (2003), and Bleichrodt et al. (2007). The observed violations of EU entailed a reformulation of the axiomatic basis of the QALY model (Bleichrodt and Quiggin 1997, Miyamoto 1999, Bleichrodt and Miyamoto 2003) to avoid reliance on the EU assumptions. Empirical tests of these reformulations have led to mixed results, but the general finding is that the basic QALY model, in which life-years are multiplied by a quality weight, is too restrictive (Bleichrodt and Pinto 2005, Bleichrodt and Filko 2008).

Second, there was no obvious justification for extending individual preferences for QALY-maximization to a social objective of maximizing the expected QALY benefit derived from health services. Several authors have argued and presented empirical evidence in favour of equity-weighting QALYs (for an overview see Dolan and Tsuchiya 2006) and various alternatives for ‘QALY-utilitarianism’ were put forward, notably including Williams' (1997) fair innings principle. Axiomatic foundations for these alternative models are provided in Bleichrodt, Diecidue, and Quiggin (2004).

Third, the standard welfare-theoretic justifications for QALY-maximization considered individual welfare solely in terms of health status, without taking account of other aspects of welfare such as the consumption of market goods and services. This approach appeared adequate in terms of cost–effectiveness analysis, but proved problematic when broader considerations were taken into account as is typically done in cost–benefit analysis. Bleichrodt and Quiggin (1999) provided necessary and sufficient conditions on individual consumption profiles and preferences for QALY maximization to be consistent with maximization of an individual lifetime welfare objective. As subsequent discussion (Klose 2002, Bleichrodt and Quiggin 2002) made clear, these conditions are very stringent.

3. The capability approach

As emphasized by Robeyns (2006) and Schokkaert (2009), the capability approach (Sen 1985, Sen 1992) is essentially a mode of thinking about normative issues, which specifies an evaluation space. The core characteristic of the capability approach is its focus on what people are able to do and be. Together these doings and beings, which Sen calls *functionings*, constitute a life and make a life valuable. Examples of functionings are being adequately nourished, avoiding premature mortality, and being happy.

A key distinction in the capability approach is the distinction between means and ends. Only ends are important and means are instrumental in reaching the ends. This illustrates that functionings are not equal to commodities. Commodities are objects which a person might use to achieve a valuable life, whereas functionings are an aspect of living itself. The distinction between means and ends is not always obvious, however. Good health, for instance, is a means to be able to work or be happy, but it is also an end in itself.

Functionings are not equal to utility in the classical economic sense of metrics of happiness. For Sen, being happy is just one of many elements of being that are relevant to an overall evaluation of well-being.

The other central concept in the capability approach is the idea of capabilities. The distinction between functionings and capabilities is between the realized and the effectively possible. In choosing what kind of life to live an individual chooses between vectors of functionings. The set of available vectors is the person's *capability set*. The capability set represents the person's opportunities to achieve well-being or, alternatively stated, his freedom. Freedom of choice is central in Sen's theory and having that freedom is of intrinsic value. The focus on the importance of freedom illustrates that the capability approach belongs to the liberal school of thought in political philosophy.

4. Valuing capabilities

Two important questions pervade the application of the capability approach (Fleurbaey 2009). The first concerns the normative issue of whether the evaluation of an individual's situation should be based on capabilities alone or on both capabilities and functionings. This concern is related to the question of whether the appropriate metric for interpersonal comparisons corresponds to achievements or to opportunities. There is an increasing interest in normative economics in opportunity-based theories in which value is attached to the size and richness of an individual's opportunity set (Sen 1992, Arrow 1995, Roemer 1998, Sugden 2004). According to these theories, public policy and theories of justice should be concerned with maximizing individuals' opportunity sets and should not be concerned with preference satisfaction. A normative reason to focus on opportunity sets instead of preference satisfaction is that individuals must take responsibility for how they use their opportunities. A more pragmatic reason is the wealth of behavioral data showing that preferences are unstable and inconsistent and hence an unreliable guide for public policy.

There are several counterarguments against an exclusive focus on capabilities. First, for Sen, functionings are “constitutive of a person’s being” (Sen 1992, p. 39), so functionings should be valued. Second, theories of responsibility are harsh on the losers. Should we punish individuals for the sins of their youth? And if so, how long are individuals deemed to be responsible for past actions? And are there limits to this responsibility (Schokkaert 2009)? The evaluation model we will propose in the current paper takes account both of the richness of the capability set, and of the quality of the functionings in the set. Hence, it values both the individual’s freedom and the achieved functionings and belongs to the category of theories about what Sen calls “refined functionings”.

The second open question concerns the valuation of capability sets: how can we value opportunity sets? If we want to use the capability approach to derive overall conclusions about the impact of specific (health care) programs on welfare, it is necessary to value capability sets. Unfortunately, the capabilities literature has very little to say on how this can be done. Sen (1985) suggests declaring one opportunity set better than another if all individuals involved agree on this, but this suggestion is not very helpful in public policy. It obviously only leads to a partial ordering of opportunity sets and it has paradoxical consequences in the sense that several plausible conditions on social choice cannot jointly hold (Brun and Tungodden 2004). Gaertner and Xu (2006, 2008) suggest ranking capability sets in terms of a standard of living, the development of which over time is uncertain. Their ranking is in terms of a class of distance functions, implying that the resulting ranking of capability sets is, once again, only partial.

A subtle issue in the valuation of capability sets is what the role of individual preferences should be. The capability approach was proposed as an alternative to welfarism with its exclusive focus on individual preferences. According to Sen (1985), some functionings are intrinsically valuable and should not depend on people's preferences. In Sen (1985) he declared his belief that a purely subjectivist view of well-being is "ultimately rejectable" and that "the limits of objectivity extend well into the assessment of well-being". Later contributions have somewhat qualified Sen's position and there is wide agreement that individual preferences have a role to play in valuing opportunities and that it is possible to respect individual preferences while avoiding a return to welfarism (Fleurbaey 2009, Schokkaert 2009).

Once we allow individual preferences to play a role, other problems emerge. Most importantly, do we introduce current preferences or take account of future preferences about which the individual is possibly uncertain? As Schokkaert (2009, p.549) concludes: "The problem of the evaluation of opportunity sets remains open". The purpose of this paper is to provide a solution to this problem that takes account of the above issues. Our solution allows a role for individual preferences while avoiding welfarism, values freedom, and takes account of the uncertainty about future preferences.

5. Theoretical Background

Let us now try to formalize the above discussion. As explained, in Sen's (1985, 1992) capability approach there are two key elements: functionings and capability sets. A *functioning* $\boldsymbol{f}$ is a vector $(f_1,\ldots, f_n)$ summarizing the activities, $j = 1,\ldots, n$, undertaken by an individual.[1] Elements f_j of $\boldsymbol{f}$ include consumption levels for market goods and services, work undertaken in the labor market, and measures of non-market activities such as going for walks, participation in family and social life and so on. Aspects of health quality may be part of the functioning vector if they contribute directly to welfare, but this need not be the case.

Let $\mathcal{F}_j$ denote the set of possible values of functioning activity j, and let $\mathcal{F} = \Pi_{j=1}^{n} \mathcal{F}_j$ denote the set of functionings, which is the Cartesian product of the n different activity sets. We will assume that each of the $\mathcal{F}_n$ is compact. An example is the case where each $\mathcal{F}_n$ is an interval (a closed subset of the real line) which can be represented without loss of generality in the form $[0,M_n]$. Generic elements of $\mathcal{F}$ are denoted f,g,h.

The capability set is the set of *achievable* functionings. This set can be written as

$$C = \mathcal{X}(c(e)). \tag{1}$$

In Equation (1), e is an individual's *initial endowment*. This initial endowment can be spent on goods and services. These goods and services are converted through the function $c(.)$ into a vector of objective characteristics in the Gorman (1959) and Lancaster (1966) tradition. Finally, the objective characteristics are converted through the *technology relation* $\mathcal{X}$ into the set of achievable functionings. Equation (1) captures the notion that goods and services do not carry value by themselves but are a means to the end of producing functionings through the technology relation $\mathcal{X}$.

We analyze the position of a representative individual choosing a social allocation of resources between health care and general resources, and of health care resources between different services. The individual is taken to have an endowment of initial health status and

[1] What we call 'functioning' is also sometimes called 'state of being.'

resources, but not to have specified preferences over achievable functionings. Thus, the individual is not a ‘representative agent’ in the sense used in economic modelling. Rather her position is closer to that of a decision maker seeking ‘reflective equilibrium’ behind a Rawlsian veil of ignorance.

Nevertheless, the modelling framework assumes a given initial health status which may be improved (or in some cases, perhaps made worse) by the use that is made of health care and other resources. One way to extend the above model would be to allow for heterogeneity in initial health status and to make the initial endowment and the technology relation $\mathcal{X}$ individual-specific (for example, by including a genetic endowment). This extension is beyond the scope of the present paper, but it is worth considering some implications.

If the aim of health policy is to provide individuals with equal capability sets, which is what Sen’s theory demands (for example, Sen 1992, p.12), then such differences in initial endowments would act against the idea that social resources should be allocated between individuals so as to maximize aggregate QALYs. Rather, it is consistent with the idea that resources should be allocated to the most disadvantaged. This argument illustrates that the recommendations following from the capability approach are closer to Rawlsian QALY-maximin than to Benthamian QALY-utilitarianism even though there are important differences between Rawls’ theory of justice as fairness and Sen’s capability approach.

A second extension is to allow for individual differences in “producing” functionings from characteristics. That is, to allow for individual-specific differences in the technology relation $\mathcal{X}$. Allowing for such differences would raise a number of complex issues in resource allocation. For example, the question may be raised whether these differences imply that priority should be given to those with the least capacity to benefit from health care. Such a priority would not only be inconsistent with QALY maximization, but also with the aim of equalizing the distribution of health care resources across individuals with similar health conditions, which underlies the extensive literature on the measurement of inequalities in health. We will assume that the initial wealth endowment e can be allocated to health expenditures h and other expenditures m to generate functionings $f = f(q,w)$, where q denotes health and w denotes wealth minus health expenditures. We will define the health cost function $h(q)$ as the expenditure h required to produce health status q.

So, a functioning $f = f(q,w)$ is *feasible* if and only if

$$h(q) + m \leq e \tag{2}$$

The primary question for health policy is to determine the socially optimal choice of q given resource endowments e and the technology as described. This choice may be broken into questions of cost–effectiveness (for given health expenditures h, choose q such that $h(q)$ is maximized), and budget allocation (assuming cost–effective choices of q, determine h).

The central claim of the capability approach is that the best way of approaching these questions is to consider the capability set C arising from particular choices of h and q. We will denote generic capability sets by x,y,z. The elements of the capability sets are feasible functionings and a typical capability set is $\{f^1, f^2, \ldots, f^n\}$. The capability sets represent the individual's opportunities for achieving well-being. The larger the capability set the larger is the number of feasible functionings available to the individual and the larger are the individual's possibilities for achieving well-being. In other words, the capability set represents the individual's freedom of choice, a crucial notion in Sen's theory as explained above.

The literature on the value of freedom distinguishes two approaches to measure freedom: a non preference-based approach and a preference-based approach (Dowding and Van Hees 2009). The first approach originates from Pattanaik and Xu (1990) who derived rules that imply that the value of freedom is determined by the size of the opportunity set. This approach has encountered at least three difficulties (Dowding and Van Hees 2009). First, it ignores the dissimilarity between alternatives. Later attempts to take the diversity of the alternatives in the opportunity set into account have only proved partly successful. Second, it ignores the opportunity aspect of freedom: the set of available may be very large but this is of little use if the alternatives are highly unattractive. Finally, it ignores the psychological costs of choosing and the possibility of negative freedom, adding alternatives may lead to a decrease in freedom because these extra alternatives preclude certain activities (Van Hees 1998). The proposals of Kuklys and Robeyns (2005) and Fleurbaey (2005) to model freedom of choice in the capability approach through the inclusion of a variable that captures the intrinsic value of choice in the utility function over functionings belong to this first approach of valuing freedom.

The second approach to value freedom is based on individual preferences. In this approach individual preferences over alternatives are taken into account in the evaluation of opportunity sets. This approach was adopted by Puppe (1996), see also Nehring and Puppe (1999) and Xu (2003). Our proposal, developed in the next section, belongs to this second approach.

6. The Model

To make the capability approach operational we should find a way to evaluate capability sets. The approach we take is based on Kreps' (1979) model of preference for flexibility, which was later extended by Dekel, Lipman, and Rustichini (2001) and Kopylov (2009). We consider an ordering $\succcurlyeq$ defined over capability sets x,y,z. The ordering $\succcurlyeq$ is assumed to be a *weak order*, that is *transitive* (for all x,y,z, if $x \succcurlyeq y$ & $y \succcurlyeq z$ then $x \succcurlyeq z$) and *complete* (for all x,y, either $x \succcurlyeq y$ or $y \succcurlyeq z$). Strict order $\succ$ and indifference $\sim$ are defined as usual. We assume that the ordering over capability sets is *nontrivial*, i.e. there exist capability sets x and y such that $x \succ y$. A function v *represents* the ordering $\succcurlyeq$ if for all x,y, $x \succcurlyeq y \Leftrightarrow v(x) \geq v(y)$.

In the standard economic model, the ordering $\succcurlyeq$ over capability sets x,y is induced from a preference relation $\succcurlyeq$ over the set of functionings $\mathcal{F}$. More precisely, define $x \succcurlyeq y$ if and only if for all $g \in y$ there exists $f \in x$ such that $f \succcurlyeq g$. The standard economic model cannot, however, incorporate the intrinsic value of freedom, which is central to the capability approach. This follows, because the above definition implies that if $x \succcurlyeq y$ then $x \sim x \cup y$. Hence, the possibility that the capability set $x \cup y$ offers more freedom of choice is not valued.

To incorporate a preference for larger and richer opportunity sets, and thus for freedom of choice, we define the ordering $\succcurlyeq$ over capability sets instead of over functionings. To capture Sen's (1992) idea that freedom of choice is valuable, we assume that $\succcurlyeq$ satisfies *monotonicity*: for all $x,y \in \mathcal{X}$, if $y \subseteq x$ then $x \succcurlyeq y$. In contrast with the standard economic model, $x \cup y$ may be strictly preferred to both x and y. Strictly ordering $x \cup y$ above both x

and y implies that more choice is preferred and, hence, that the freedom of choice is positively (intrinsically) valued.

Another reason why people may value freedom is that they are unsure about their future preferences. For instance, a 20-year old may not be fully sure how important he will consider health to be, relative to income, when he is 50. To account for this preference uncertainty, larger and richer capability sets can be preferred. The different preference relations that are taken into account may also represent what the decision maker considers reasonable preferences (Jones and Sugden 1982, Sugden 1998). Even if the decision maker believes that it is unlikely that he will act on some preferences he may wish to include these preferences in his evaluation because he believes they are reasonable.

A final reason to value freedom of choice is that people's preferences tend to be unstable. The literature on behavioral economics has uncovered many inconsistencies in people's preferences, and individuals may act on different preferences at different times and in different situations. In the presence of such preference instability, the flexibility offered by larger and richer opportunity sets is valuable (Sugden 2004).

Because the ordering is defined over capability sets, the representation presented here is a two-stage model. The first stage determines the capability set while, in the second stage, the optimal functioning is chosen from the selected capability set.

Apart from the idea that freedom is valuable we will impose one more condition, which is again adopted from Kreps (1979). Suppose that adding capability set y to capability set x has no value to a decision maker, for example because the functionings included in y are all of low value to the decision maker compared to what is available in x. The extra opportunities offered by y have therefore no value to him. We can express this as $x \sim x \cup y$.[2] Given this preference, it seems plausible that if we enlarge the individual's capabilities to $x \cup z$ by adding any set z to x, then adjoining y to $x \cup z$ should still have no value to the individual. If the functionings included in y are all of low value to the individual if he has x available, then they should also be of low value when he has the larger set $x \cup z$ available. We will refer to this condition as *irrelevance*: for all $x,y \in \mathcal{C}$, if $x \sim x \cup y$ then for all $z \in \mathcal{C}$, $x \cup z \sim x \cup y \cup z$.

Kreps (1979) showed that if monotonicity and irrelevance jointly hold then $\succcurlyeq$ can be represented by

$$V(x) = \sum_{s=1}^{S} \max_{f \in x} U_s(f), \tag{3}$$

where the different s can be interpreted as states that could reflect the individual's uncertainty about his future preferences and $U_s(f)$ is a real-valued state-dependent utility function defined over functionings.[3] We can also write (3) as

$$V(x) = \sum_{s=1}^{S} \max_{f \in x} \lambda_s v_s(f), \tag{4}$$

where the λ_s are decision weights. In some choice problems these weights can be interpreted as subjective probabilities of the subjective states. Note, however, that when utility is state-dependent these subjective probabilities cannot be uniquely determined and, hence, do not have a clear behavioral interpretation. Moreover, a social ordering may place weight on the

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[2] The fact that we allow for the possibility that $x \sim x \cup y$ shows that larger capabiity sets are not necessarily better.

[3] To be precise, Kreps only showed that (3) holds if the set of functionings is finite. Kopylov (2009) derives (3) for infinite $\mathcal{F}$ from a different set of conditions.

availability of options, even if the probability that these options will actually be selected is zero. One example is the case of reasonable preferences (Jones and Sugden 1982) discussed above. As another example, I may be entirely confident that I would not wish to visit Antarctica. Nevertheless, I may object to a state of affairs in which I am prevented from doing so, either by legal restrictions or by a lack of resources.

To understand (4), imagine that the representative agent chooses a capability set $x \in C$ knowing that at some ex post stage, he will learn what his preferences are. He then chooses the optimal functionings from capability set x according to these ex post preferences once he knows what they are. Ex ante, these preferences are aggregated by summing the maximum utilities across states.

A drawback of Kreps' axiomatization is that the states s are not uniquely defined and that (3) and (4) are essentially ordinal representations. This "problem" was solved by Dekel, Lipman, and Rustichini (2001) by letting menus consist of probability distributions over functionings and was later generalized by Kopylov (2009) to menus as abstract convex compact spaces. Because the non-uniqueness of Kreps' representation is no problem for our subsequent analysis and probability distributions over functionings are intuitively less plausible when considering capabilities, we do not adopt these alternative approaches in this paper. The important thing for our analysis is that by adopting the two plausible axioms of Kreps we can derive a collection of state-dependent utility functions over functionings the sum of which represents the ordering over capabilities.

7. The decision problem

Given (4), in the capability approach we are faced with a two-stage decision problem. In the first stage an optimal vector (q,w) is chosen subject to the restriction that $h(q)+m \leq e$. This choice ensures that the resulting capability set is optimal. The second stage then entails choosing the most preferred functioning from this capability set given the realization of s, encompassing preferences, relative prices and other contingent factors.

The two stages are linked by the relation

$$\mathcal{X}(q,w) = \{f \in \mathcal{F}: f \text{ is feasible given } (q,w)\}.$$

We will show that both the first- and the second-stage problems are well-defined and that solutions to these decision problems exist. We start by analyzing the second-stage solution. To derive the second-stage solution we have to introduce some additional assumptions of a technical nature. First, we assume that a topology exists on the capability set $\mathcal{C}$. The ordering $\succcurlyeq$ is *continuous* with respect to this topology. That is, for any capability set $y \in \mathcal{C}$ the sets $\{x: x \succ y\}$ and $\{x: y \succ x\}$ are both open in the topology on $\mathcal{C}$.

The capability set $\mathcal{X}$ is compact and convex. The convex combination $\alpha x + (1-\alpha)y$, $\alpha \in [0,1]$ of two capability sets $x,y \in \mathcal{X}$ is defined as: for all $f \in$ x, $g \in$ y, $\alpha f + (1-\alpha)g$ belongs to $\alpha x + (1-\alpha)y$. That is, the linear combination $\alpha f + (1-\alpha)g$ is feasible. We further assume that if $(q',w') < (q,w)$ then $\mathcal{X}(q',w') \subset \mathcal{X}(q,w)$. That is, the capability set expands when health and wealth increase. Finally, we assume that $\mathcal{X}$ is an *upper semi-continuous* correspondence. That is, at all (q,w) if $\lim_{n\to\infty}(q^n,w^n) = (q,w)$, $f^n \in \mathcal{X}(q^n,w^n)$, $\lim_{n\to\infty} f^n = f$ implies that $f \in \mathcal{X}(q,w)$. In words, if (q^n, w^n) is a sequence that converges to (q,w) and $f^n \in \mathcal{X}(q^n, w^n)$ is a sequence of feasible functionings that converges to f then f is also feasible given health and wealth (q,w). We can now state our first result, which says that the second-stage solution exists. A proof is in the Appendix.

Result 1: Under the above assumptions $U_s(x) = \max_{f \in x} U_s(f)$ is well defined for each s.

If we further assume that $\mathcal{X}$ is *separable*, i.e. it contains a countable order dense subset then the first-stage problem is also well-defined. The proof is in the Appendix.

Result 2: Under the above assumptions, the first-stage problem is well-defined.

8. A local QALY evaluation

Let us now analyze the role of QALYs within this framework. Using Result 2, we can implicitly define a *capability valuation function* $V(q,w)$. The proof of Result 2 shows that V is continuous, and increases in health and wealth. The health attribute in $V(q,w)$ can consist of various dimensions, such as longevity and mobility. We assume that the set of health states is a subset of $\mathbb{R}^n$. The more general case is presented in the appendix. If the functions U_s are twice differentiable, then, by the implicit function theorem (Rudin 1976, Theorem 9.28), $V(q,w)$ is also twice differentiable. Let $z = (q,w)$. Differentiability of V implies that there exists a linear function $dV: \mathbb{R}^{n+1} \rightarrow \mathbb{R}$ such that

$$\lim_{h \to 0} \frac{V(z+h) - V(z) - dV(h)}{||h||} = 0, \tag{5}$$

where $||h||$ denotes the Euclidean norm of h. The linear function dV can be represented by the gradient vector ∇V of V, in the sense that for all $h \in \mathbb{R}^{n+1}$ $dV(h) = \nabla V \cdot h$.

In the case of differentiable utility, both cost–effectiveness analysis and cost–benefit analysis are consistent with the use of QALY measures in a neighborhood of the optimal (q^*,w^*). For simplicity, suppose that q_1 denotes longevity. Then, normalizing V so that $\frac{\partial V}{\partial q_1} = V_1(q^*,w^*) = 1$, the partial derivative $\frac{\partial V}{\partial q_k} = V_k(q^*,w^*)$ is the increase in health attribute k that would be ranked equally with a unit increase in longevity (with health characteristics q^*). That is, we can compare health gains in terms of a QALY measure. The expression $1/V_w(q^*,w^*)$[4] is the marginal willingness to pay for a unit increase in longevity (with health characteristics q^*). That is, $1/V_w$ is the willingness to pay for a QALY when the individual's initial position is (q^*,w^*) . Thus, we obtain a QALY measure for any local change in health

$$dQ = \nabla V_q * dq, \tag{6}$$

where $\nabla V_q(q^*,w^*) = (\frac{\partial V}{\partial q_1}, \ldots, \frac{\partial V}{\partial q_n})$ is the health gradient vector evaluated at (q^*,w^*). We also obtain a local monetary evaluation

$$dy = \nabla V_q * dq / V_w(q^*,w^*) + dw \tag{7}$$

where dy is a money-metric measure of the equivalent variation associated with the change (dq, dw).

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[4] $V_w = \frac{\partial V}{\partial w}$

Two points are critical here. First, Eqs. (6) and (7) are local approximations and are valid only in a neighbourhood of a given *(q,w)*. They are applicable to valuing alternative health improvements for a representative individual who is initially at an optimal position, given the constraints of the first-stage problem. Eqs. (6) and (7) also imply that QALY-based measures can be used to compare individuals with comparable initial health and wealth (q,w). Eqs. (6) and (7) are not applicable to comparisons between individuals in widely separated initial positions. In those cases, the restrictions on preferences consistent with QALY maximization, derived by Bleichrodt and Quiggin (1999) still apply and alternative measures may have to be used instead. An example is Fleurbaey's (2005) full-health equivalent income measure.

Second, the QALY measure derived in Eq.(6) differs from that usually considered in the literature. The standard QALY measure compares some given health status q with the health vector associated with some given number of years in perfect/full health. By contrast, the evaluation here is undertaken at the health status q^*, the health of the representative individual. In our view, the latter approach may be regarded as an improvement, especially in the light of the capability approach. Even with the best of health care, nutrition and so on, the capabilities of a 70 year-old are not the same as those of a 25-year old. So, if we are comparing interventions that increase longevity and alternatives that increase other aspects of health capability, we would not want to evaluate them against the hypothetical yardstick of an individual living 45 years after age 25, while enjoying the capabilities of a healthy 25-year old.

9. Concluding comments

Attempts to ground QALYs in economic welfare theory have met with limited success. Empirical evidence suggests some support for the use of QALYs in cost–effectiveness analysis. However, the problem of establishing consistency between cost–effectiveness analysis and cost– benefit analysis has proved problematic within a welfarist framework.

This paper has provided a foundation for QALYs within Sen's capability approach. The paper makes two contributions. First, it suggests a general method for valuing capabilities based on Kreps' (1979) model of preference for flexibility. There has been a steady flow of papers advocating the use of capabilities in health policy, but, to the best of our knowledge, little guidance has been provided how capabilities could be valued. This paper suggests a way forward by valuing capabilities as opportunity sets. We then derive within this model the conditions under which QALYs locally represent the ordering over capability sets. To derive a full representation would require similarly stringent conditions as those used by Bleichrodt and Quiggin (1999) to establish the link between cost–effectiveness analysis and cost–benefit analysis.

Our result shows the conditions under which QALYs may justifiably be used to compare the health gains of individuals with similar initial health and wealth. It has been argued that different cost per QALY thresholds should be used, depending on the severity of the health states involved (e.g. Raad voor de Volksgezondheid & Zorg 2007). Our result helps to assess the reasonableness of such a policy. Within the capability approach, a set of rather weak conditions suffices to obtain a local representation result for QALYs and, hence, to use a cost per QALY threshold that depends on the individual's initial situation. Equation (7) then provides a formula for estimating the appropriate cost/QALY thresholds.

There are several topics that our paper leaves unaddressed. The first is to extend our results to the case of individuals with different initial positions and to derive a socially optimal rule for this case. A further extension, to consider differences in individual capacity to benefit from increased capabilities, might then be addressed.

This research is in its early stages. Nevertheless, we believe an analysis based on capabilities may give a better understanding of the principles on which health care resources should be allocated and, in particular, on the strengths and limitations of the QALY approach, than do the standard frameworks of cost–effectiveness analysis and cost–benefit analysis.

Appendix: Proofs

Proof of Result 1:

$\mathcal{X}$ is a correspondence from the set of (q,w) into the set of functionings. $\mathcal{X}$ is upper semi-continuous. The set $\mathcal{F}$ is compact by Tychonoff's theorem (Dugundji 1966, p.224). Continuity implies that the U_s are continuous. It follows by the upper semi-continuous maximum theorem (Berge 1963, p.116) that the function $g(q,w) = \max_{f \in x} \{U_s(f(q,w))$ is well-defined and is continuous from above over the set of (q,w).

□

Proof of Result 2:

Because we now assume convexity of $\mathcal{X}$, we can no longer use (3), because convexity implies that $\mathcal{X}$ is infinite. Instead we use Theorem 4 in Kreps (1979) which shows that $\succcurlyeq$ can be represented by a function $V(u_1, \ldots, u_S)$, which is strictly increasing in each of the u_s and u_s

$= \sup_{f \in x} U_s(f)$. Because $\mathcal{F}$ is compact, $\sup_{f \in x} U_s(f)$ is well-defined. By Tychonoff's theorem, $\mathcal{F}^S$ is compact and the maximization of v is well-defined by the upper semi-continuous maximum theorem. Strict increasingness of v in (q,w) follows from increasingness of the capability set. Continuity of v follows from continuity.

□

Extension to more general health spaces

In the analysis of the main paper, we assumed that health was a subset of $\mathbb{R}^n$. This assumption may be too restrictive, because health states do not correspond directly to subsets of the real numbers. In this appendix we show that Eq.(6) and (7) can also be derived in a more general framework. Let the set of health states be any linear space with a norm $\|\cdot\|$ defined on it. We assume that the capability valuation function V is Fréchet differentiable as in Machina (1982). Fréchet differentiability is the natural notion of differentiability on normed spaces. A real-valued function V on an open subset A of a normed linear space $\mathcal{Z}$ is said to be *Fréchet differentiable* at $z \in A$ if there exists a continuous linear functional T_z on $\mathcal{Z}$ where, given $\varepsilon > 0$, there exists a $\delta(\varepsilon, z) > 0$ such that $|V(z + h) - V(z) - T_z(h)| < \varepsilon \|h\|$ for all $h \in \mathcal{Z}$, $\|h\| < \delta$. Or, alternatively stated:

$$\lim_{y \to 0} \frac{|V(z + y) - V(x) - T_z(h)|}{\|h\|} = 0.$$

An equivalent way of stating this is by writing

$$V(z+h) - V(z) = T_z(h) + o(\|h\|)$$

where o denotes a function which is zero at zero and of a higher order than its argument. By the Hahn-Banach theorem we can extend T_z to $\mathcal{Z}$. The subscript z in T_z serves as a reminder that T will generally be different at different z.

Hence, it follows that

$$\frac{dV}{dz_i} = \frac{dT_z}{dz_i} + \frac{d o \|dz_i\|}{dz_i} .$$

Because the derivative of the higher order term o is zero at zero it follows that

$$\frac{dV}{dz_i} = \frac{dT_z}{dz_i} = \nabla dz_i.$$

Hence, the change in V due to a change in each of the x_i can be written as a linear function of these changes:

$$dV = \sum_{i=1}^{n} \beta_i \, dx_i.$$

Hence, locally we obtain a QALY-type representation.

□

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