Risk & Sustainable Management Group

Risk and Uncertainty Working Paper: R09#2

Research supported by an Australian Research Council Federation Fellowship
http://www.arc.gov.au/grant_programs/discovery_federation.htm

Markets for Influence

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11 September 2009

Abstract

We specify an oligopoly game, where firms choose quantity in order to maximise profits, that is strategically equivalent to a standard Tullock rent-seeking game. We then show that the Tullock game may be interpreted as an oligopsonistic market for influence. Alternative specifications of the strategic variable give rise to a range of Nash equilibria with varying levels of rent dissipation.

1 Introduction

There are many strategic interactions where agents spend resources to dispute some rent or prize. Beginning with the work of Tullock (1967), a large literature, often referred to as the economics of contests, has arise to examine this type of strategic interaction. Konrad (2004) provides a useful summary. One of the most important examples is that of elections, where the resources allocated to campaigning determine candidates' probability of election (Congleton 1986). Other examples include the analysis of patent races, where firms compete by spending a certain amount of money that determines the probability that they make a discovery and win the race (Loury 1979, Nalebuff and Stiglitz 1983), elimination tournaments (Rosen 1986), and the analysis of litigation by assuming that the parties compete by choosing how much to spend on their legal challenge (Farmer and Pecorino 1999).

It is normally assumed in this literature that the probability distribution of outcomes is determined by a success function, with a vector of effort or expenditure levels as arguments. Most commonly, the probability of a particular candidate being elected (or the success probability more generally) is modelled as the ratio between this candidate's expenditure and the total expenditures of all candidates. However, a range of different success functions has been considered. Given a specification of the effort variable, and the success function, the problem is typically represented as a non-cooperative game. The solution is a Nash equilibrium, with expenditure as the strategic variable for each player.

A notable feature of the contest literature is the absence of explicit markets, and therefore, of considerations of industrial organization.[1] It is widely recognized in informal discussion of electoral contests that political contestants may be regarded as entrepreneurs trading in markets for votes, but this insight plays little role in contest-theoretic models of elections.

In this paper it is argued contests should be viewed, not as a separate category from imperfectly competitive markets, but as oligopsonistic *markets for influence*. The influence variable may be interpreted as electoral support, legal expertise, connections within labour markets and so on. Competition between players determines an implicit price for influence, and therefore the expenditure required to acquire a given level of influence. In our framework, the standard Tullock solution corresponds to a Nash equilibrium for firms with market shares as the strategic variable analyzed by Grant and Quiggin (1994).

The isomorphism between contests and oligopoly games has an important implication in this respect; it suggests that the exclusive focus of the Tullock contest literature on effort or expenditure of resources as the strategic variable might be misleading. There is an obvious contrast with oligopoly models, where both prices and quantities (Bertrand or Cournot models) were considered as possible strategic variables even before the game theory revolution that has dominated the field of industrial organization over the last three decades. More recently, a number of papers have proposed alternative strategic variables, such as supply curves (Klemperer and Meyer, 1989) and markups (Grant and Quiggin, 1994). In addition, there have been numerous attempts to motivate the choice of particular strategic variables, for example as outcomes of a multistage game (Kreps and Scheinkman, 1983). These issues have received little or no attention in the contests literature.

In our main result, we show that alternative choices of the strategic variable can yield a range of equilibrium outcomes, from Cournot to Bertrand. As markets for influence become more competitive, the implicit price of influence increases and the net rent shared by purchasers of influence decreases. Thus the analogy between contests and markets is not merely formal, but suggests a range of economic insights.

The determination of the strategy space is of particular interest where the contest market is the product of conscious mechanism design, with the strategies available to players specified by the designer. This point arises naturally when contests are considered as all-pay auctions as in Baye, Kovenock and de Vries (1996). An auction is conducted under a set of rules, which specify the strategies available to the players, and which may be designed to maximize expenditure, to allocate the auctioned item to the player with highest value, or to promote some more general objective such as

[1] Okuguchi (1995) and Szidarovszky and Okuguchi (1997) show that the standard formulation of the Tullock rent-seeking game, where individuals choose effort or resources to win a prize, is strategically equivalent to a Cournot oligopoly game where the elasticity of demand is unitary and firms choose quantity to maximize their profits. This formal identity is used to derive an existence proof, but its implications for the interpretation of contest-theoretic results have received little attention.

social efficiency.

If contests may be viewed as a kind of imperfectly competitive market, it is natural to consider the implications of treating imperfectly competitive markets as a particular kind of contests. This idea has been considered (Fudenberg and Tirole 1987) but there does not appear to have been a systematic consideration of the implications of contest theory for industrial organization. We consider this topic briefly before offering some concluding comments.

2 Contests as Markets for Influence

Our starting point is the most well-known model of contests, namely, the Tullock rent-seeking game. This class of games can be represented by a set of n players, who choose effort levels $e_1, e_2, ..., e_n$ in order to win a prize of fixed value V, and a parameter $R > 0$. Effort levels may be considered as producing a quantity variable, q_i, where the cost function is given by $e_i = c_i(q_i)$. Player i's payoff in this class of games is given by:

(1) $$\pi_i(e_1, e_2, ..., e_n) = V \frac{q_i}{\sum_{j=1}^{n} q_j} - c_i(q_i).$$

The equilibria for this family of games (both symmetric and asymmetric, pure and mixed-strategy) are well-known.[2]

As Okuguchi (1995) and Szidarovszky and Okuguchi (1997) show, a standard Tullock contest characterized by the payoff function

$$V \frac{e_i}{\sum_{j=1}^{n} e_j} - e_i$$

is strategically equivalent to a Cournot oligopoly game with inverse demand function, output and linear cost given by

$$\frac{V}{\sum_{j=1}^{n} q_j}, q_i, c_i(q_i) = q_i$$

The same strategic equivalence applies for more general success functions of the form:

$$\pi_i = \frac{g(e_i)}{\sum_{j=1}^{n} g(e_j)}$$

[2]See, for example, Baye, Kovenock and de Vries (1994). Importantly, Baye and Hoppe (2003) show that this family of games is isomorphic to certain innovation and patent-race games. It follows then that our main result also applies to these other classes of games. That is, there are isomorphisms between oligopoly games and specific innovation and patent-race games.

and cost functions of the form $c_i(q_i) = g^{-1}(q_i)$. If c_i is convex and twice differentiable for all i Szidarovszky and Okuguchi (1997) demonstrate the existence of an equilibrium in pure strategies.

In this paper, we take a different approach to the idea that individual behavior in Tullock contests may usefully be related to the behavior of firms in imperfectly competitive markets. To pursue this idea further, it seems natural to consider more carefully the idea, familiar from public-choice theoretic discussions of political processes, that contests represent a particular kind of market, namely a market for influence. If this analogy is taken seriously, the participants in contests may be regarded as buyers in oligopsonistic markets. To formalize the idea, we need to define concepts analogous to prices, quantities, and supply schedules.

To address this task, we introduce the idea of a price of influence which is given by the inverse demand function

$$p(\theta_1, \theta_2, ..., \theta_n) = \sum_i \theta_i$$

where θ_i is the influence acquired by player i and p is the unit price of influence. In the electoral case, for example, we might adopt the interpretation that p is the price paid by the candidates for each vote and θ_i the total number of voters induced to vote for candidate i. Accordingly, the expenditure for player i is

$$e_i = p\theta_i, i = 1, 2...n. \tag{2}$$

In the standard Tullock contest, the success probabilities are given by

$$\frac{e_i}{\sum_{j=1}^{n} e_j} = \frac{\theta_i}{\sum_{j=1}^{n} \theta_j} \tag{3}$$

where we assume, for simplicity, that $R = 1$ and the prize is normalized to one so that i's payoff is given by $\pi_i - e_i$. One can immediately see that such context is essentially isomorphic to a oligopsony game as described below.

Proposition 1 *A standard Tullock contest characterized by payoff function* $\frac{e_i}{\sum_{j=1}^{n} e_j} - e_i$, $i = 1, ..., n$, *is*

strategically equivalent to a oligopsony game where:

(i) the strategic variable for firm i is the quantity purchased of an input $x_i > 0$*;*

(ii) output is given by the production function $f(x_i) = x_i$*;*

(iii) the (constant) output price is $A - 1$

(iv) A *is sufficiently large that the input supply price* $w = A - \frac{1}{\sum_{i=1}^{n} x_i}$ *is always positive.*

Proof: Each firm i chooses x_i to maximize profits, which can be written as:

$$(4)\ \ \pi_i = (A-1)f(x_i) - wx_i = (A-1)x_i - \left(A - \frac{1}{\sum_{i=1}^{n} x_i} \right) x_i = -x_i + \frac{x_i}{\sum_{i=1}^{n} x_i}.$$

Then replace x_i with e_i. □

As in the analysis of Szidarovszky and Okuguchi (1997), changes in the success function for the Tullock contest are isomorphic to changes in the production technology for the oligopsonistic firm. We will not develop this point, but instead will focus on the choice of strategic variable. The representation of Tullock contests as markets for influence, given in equation (2) suggests three possible choices for strategic variable for player i: the total expenditure e_i as in Proposition **??**, the quantity of influence θ_i, corresponding to a Cournot–Nash equilibrium and the price of influence p, corresponding to a Bertrand equilibrium. It is natural to ask how alternative specifications of the strategic variable affect the proportion of rent dissipated in the contest.

The analogy with oligopoly can help us to answer this question. Grant and Quiggin (1994) show that the equilibrium outcome with revenue as the strategic variable is less competitive (higher price, lower aggregate quantity, higher profit) than the Cournot–Nash equilibrium. This is because (loosely speaking) if one player chooses to deviate by increasing revenue, this entails an increase in their own output and a reduction in the market price, and the Nash assumption that other players will hold revenue constant implies that they must increase quantity. Converse reasoning for the oligopsony case suggests that the outcome of a standard Tullock contest with expenditure as a strategic variable will be more competitive (lower price, higher aggregate quantity, more rent dissipation) than the Cournot–Nash equilibrium. This is because an increase in expenditure by one player raises the market price, and therefore lowers the equilibrium quantity associated with a given expenditure level.

To verify this we first remind the reader that in the standard analysis of Tullock games, player i chooses e_i to maximize (1). The unique (symmetric) Nash equilibrium is well-known and given by $e_i^* = \frac{n-1}{2n} = e^*$ for $i = 1, ..., n$. To see this, note that $\frac{n-1}{2n}$ is the solution to $\frac{\partial \pi_1}{\partial e_1} |_{e_2=e_3=...=e_n=e} = \frac{1}{e_1+(n-1)e} - \frac{e_1}{(e_1+(n-1)e)^2} - 1 = 0$. This implies that the total resources spent by players add up to $\sum_{i=1}^{n} e_i^* = \frac{n-1}{2}$.

Second, consider the Cournot–Nash strategic representation where the candidates choose quantity θ_i to maximize:

$$\pi_i = \frac{p\theta_i}{p\sum_j \theta_j} - p\theta_i.$$

It is easy to see that this representation has a unique symmetric equilibrium where

$$\theta_1^C = \theta_2^C = ... = \theta_n^C = \frac{\sqrt{n-1}}{n\sqrt{n+1}},$$

and consequently

$$p^C = \frac{\sqrt{n-1}}{\sqrt{n+1}}$$

and

$$e^C = \frac{(n-1)}{n(n+1)} \text{ and } \sum_{i=1}^{n} e_i^C = ne^C = \frac{(n-1)}{(n+1)}. \tag{5}$$

This implies less rent dissipation than the standard solution for the Tullock contest as $\frac{(n-1)}{(n+1)} \leq \frac{n-1}{2}$ always holds.

Finally, we consider a strategic representation of markets for influence that is equivalent to a 'Bertrand' model of oligopoly. Under this scenario the candidates compete for voters in the 'prices' space. We impose the standard assumptions in Bertrand competition, where the voters will vote for the candidate who offers the higher price. In the event that both candidates offer the same price, voters are equally split among the two candidates. It is not difficult to see that the Bertrand (auction) logic implies that in equilibrium:

$$p_1^B = ... = p_n^B = 1.$$

That is, any price lower than one leads to 'undercutting'. Under this equilibrium, there is zero profit, that is, full rent dissipation, as

$$\theta_1^B = ... = \theta_2^B = \frac{1}{n} = e_1^B = ... = e_n^B. \tag{6}$$

We summarize this discussion as follows:

Proposition 2 *Consider the following strategic variables for a market for influence*

(i) Expenditure e_i (Tullock)

(ii) Quantity of influence θ_i (Cournot)

(iii) Price of influence p (Bertrand)

Rent dissipation is higher under Bertrand than under Tullock and higher under Tullock than Cournot. Bertrand yields full rent dissipation, regardless of the success function.

The discussion suggests that by considering the full range of strategies available to participants in Tullock contests, it is possible to obtain a wide range of symmetric equilibrium outcomes, just as in the case of oligopoly.

3 Determining the strategy space

In the literature arising from Tullock (1967), a large amount of effort has been devoted to analyzing the implications for equilibrium outcomes of alternative specifications of the contest success function and payoff function. The analysis of strategically asymmetric contests presented above shows that the specification of the strategy space is equally important.

The first possibility is that there exist institutional rules or structures, exogenous to the players that determine the strategies available to them. This is typically the case for actual games of strategy, such as chess; the players are exogenously assigned the White or Black pieces, and the rules of the game specify the strategies available to them.

Second, a one-shot normal-form contest representation of an economic interaction may be derived as the reduced form of an extensive form representation, analogous to the oligopoly models of Dixon (1986) and Kreps and Scheinkman (1983).

Finally, the strategy space for a contest may be the product of conscious mechanism design. For example, in economic environments such as auctions, the strategies available to bidders are specified by the party holding the auction. A sealed-bid all-pay auction gives rise to a Tullock contest, with bid values as strategies, in which the success function awards the prize with probability 1 to the highest bidder (Baye, Kovenock, and de Vries 1996). But the vendor need not choose this auction structure. Other auction rules, specifying different strategy spaces, may yield higher expected revenue, though normally at the cost of inefficiency in allocation of the good (Klemperer 2002).

Similarly, the hierarchical structure of the internal labour markets is the product of design decisions by the owners or senior managers of the firm, possibly constrained by the interventions of unions, governments or other stakeholders. It seems plausible to suppose that owners would prefer contest structures that maximized effort by employees, while managers would have mixed incentives.

As has been shown here, the determination of the strategy space is crucial in determining the outcome of contests. However, this issue has received little attention in the literature on contests. If the strategy space cannot validly be 'read off' from the structure of the game, and, in particular, from the formulation of the success function, it is necessary to examine the economic structure of the contest.

Consider, as an example, the possible takeover of a company with shares that are initially widely held, but where a majority owner could obtain a control premium. Depending on their own financial structure, the organization of the market and the regulatory environment, potential acquirers might pursue a variety of strategies. We will focus on three possibilities: acquirers might choose expenditures on the acquisition project; a price they are willing to pay for control; or a quantity of shares to purchase in anticipation of a proxy war.

In the standard Tullock contest model, the first of these strategy spaces is assumed to apply. However, as shown above, acquirers as a group would prefer the second strategy space, which involves less dissipation of rent. If members of a set of acquirers interacted repeatedly, it would be in their joint interest to set up institutions that facilitated contests of this kind. By contrast, regulators seeking to protect the interests of shareholders in general might prefer a requirement for acquirers to compete on price.

3.1 Imperfectly competitive markets as contests

The interpretation of contests as taking places in markets, which is afforded by the proposition above, may be turned around. Participants in oligopolistic markets may be considered as taking part in a contest for market share. In the case where the elasticity of demand is unitary, this interpretation is represented by the isomorphism given above. More generally, oligopolistic markets may be considered as analogous to contests where the strategic choices of the players determine both the value of the prize (total revenue) and the probability of winning (market share).

One important implication of the contest literature, which has received only limited attention in the industrial organization literature, is that, in determining the rent accruing to participants, the cost function is just as important as the choice of strategic variable. Depending on the cost function, any outcome in the range from perfect competition to joint monopoly pricing may be sustained as a Cournot equilibrium.

The interpretation of oligopolistic markets as contests reinforces a centra point of this paper. The mere fact that an economic interaction can be represented as being (or being isomorphic to) a contest gives no warrant for any particular choice of strategic variables.

4 Concluding comments

In economic terms, a contest may be regarded as taking place in an imperfectly competitive market for influence. Understanding of the relationship between contests and imperfectly competitive markets is hampered by the absence of explicit prices and quantities in the standard contest model. When contests are represented as markets for influence, we derive a natural strategic equivalence between the standard Tullock contest and an oligopsonistic market in which expenditure is the strategic variable. Unlike the corresponding case for oligopoly, this outcome turns out to be less competitive (and hence less dissipative of rent) than the Cournot solution.

In this paper, we have shown that the standard Tullock contest game is strategically isomorphic to an oligopsony game in which input expenditure is the strategic variable. Consideration of this isomorphism indicates some differences in the aspects of the problem considered in the literature on contests, on the one hand, and on imperfectly competitive markets on the other. Analysis of contests has focused on differences in the success function (equivalent to differences in the production technology for the oligopsony case), while the literature on imperfect competition has paid more attention to the determination of the strategic variable. In each case, a range of possible outcomes from complete rent dissipation to sharing of the maximum rent may be obtained in appropriate cases.

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